Cultures of Creativity
Mathematics and Physics

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The cultures here in question are those of mathematics and of physics that I shall interpret with the goal of exploring different modes of creativity. As case studies I will consider two scientists who were exemplars of these cultures, the mathematician Henri Poincaré (1854-1912) and the physicist Albert Einstein (1879-1955). The modes of creativity that I will compare and contrast are their notions of aesthetics and intuition. In order to accomplish this we begin by studying their introspections.

Henri Poincaré and Edouard Toulouse

Although a scientist's introspection can help to unravel creativity, it must be handled carefully, within a web of mutually confirming historical data. This is the situation with Poincaré's famous introspection of 1908, "Mathematical Invention," that focuses on a mathematical discovery he made some twenty-seven years earlier in 1881.[1] Can we trust his recollection? Had Poincaré ever actually probed his *own* thought processes before 1908? Could it be that on the spur of the moment he thought up the scenario presented in "Mathematical Invention," as had the Belgium chemist August Kekulé in 1890 with his snake dream? It turns out that Poincaré's 1908 introspection agrees with conversations and psychological tests performed on him in 1897 by the French psychologist Edouard Toulouse – published in 1910 with Poincaré's *imprimatur*[2] – as well as with archival material of Poincaré that I had the good fortune to have discovered in 1976 in Paris.[3]

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Toulouse was Chief of Medicine at the asylum of Villejuif and Director of the Laboratory of Psychology, *l'Ecole des Hautes Etudes* in Paris. In 1895 he began to study creativity. Among those who agreed to be interviewed by him were, besides Poincaré, Emile Zola, the sculptors Jules Dalou and Auguste Rodin, and the composer Camille Saint-Saëns. The best observations turned out to be for Zola and Poincaré who was interviewed last. Toulouse's book contains the most complete psychological profile ever undertaken face-to-face with a major scientist.

A Word About the Mathematician

Presently there are only informative biographical sketches of Poincaré, and so Toulouse's interviews add much to the little we know, in addition to the unpublished material from the Poincaré archives.

Poincaré's *curriculum vitae* is a 110-page book. He published on the order of 500 papers, 30 books, and received numerous honorary degrees and every scientific prize except the Nobel Prize – for which a great deal of lobbying was done on his behalf. Poincaré took all philosophical, scientific and mathematical knowledge to be his province. Besides being one of the greatest mathematicians in the history of that discipline, he made significant contributions to every branch of physics and astronomy, and formulated the unique viewpoint of the philosophy of science called conventionalism. All of this in addition to occupying the apex of the pyramid that constituted the structure of French science.[4]

We may now turn to Toulouse's data obtained through interviews with and psychological tests of Poincaré as well as Poincaré's 1908 introspection.

The Scientific Creativity of Henri Poincaré

Toulouse described Poincaré's problem-solving methods as "intuitive, rapid and spontaneous."[5] When writing a scientific paper Poincaré neither used notes, nor had he any definite overall plan or goal in mind, nor even any idea of whether the problem at hand was soluble. This is research at its most fundamental, as Toulouse reports from an interview with Poincaré:

> "[Poincaré] does not make a grand plan when he writes a paper. Ordinarily he starts it without knowing where he will conclude .... Starting is generally easy. Then he seems led by his work and has not the impression of any willful effort. At that moment it is difficult to distract him. *When he searches, he often writes a formula automatically in order to awaken some association of ideas.* If the starting is painful, M.H. Poincaré does not persist and abandons the started work, in contrast to Zola who persists.(...)In certain work, M.H. Poincaré proceeds by sudden blows, taking and abandoning a subject. *During intervals he assumes ... that his unconscious continues the work of reflection.* Cessation of work is difficult if there is not a sufficiently strong distraction, especially in the case when the work is not judged complete.... It is for this reason that M.H. Poincaré never does any important work in the evening in order not to trouble his sleep .... This is an uncommon method of work in scientific matters and it constitutes a character well suited to the mental activity of M.H. Poincaré."[6]

Note Poincaré's complete confidence in his power of unconscious thought. In fact, Toulouse implies that Poincaré knew just how to activate areas of long-term memory when he observes that Poincaré "writes a formula automatically in order to awaken some association of ideas."

Toulouse goes on to report that his results on Zola and Poincaré are poles apart, and entirely unexpected:

> "The one [Zola's] was an intelligence that was willful, conscious, logical, methodical, and seemingly made for mathematical deduction: it gave birth entirely to a romantic world. The other [Poincaré's] was spontaneous, little conscious, more taken to dream than to the rational approach and seemingly throughout apt for works of pure imagination, without subordination to reality: it triumphed in mathematical research. And this is one of the surprises (...) that arises from direct studies touching on the deepest mechanisms ...."[7]

Zola worked like a scientist and Poincaré like a writer or poet. Toulouse had fallen into the classical stereotyping of poets and scientists. His report of Poincaré's dream-like approach to his mathematical research is important because this state of mind opens up the boundaries of thought beyond the restrictions imposed by conscious deliberations. Although Poincaré was a remarkably clear observer, Toulouse found that his memory of visual images was not good and that in his highly creative research Poincaré usually "neglected visual imagery altogether."[8]

Poincaré on Intuition, Aesthetics and Mental Imagery

Just as in his science, Poincaré was as meticulous as possible when defining terms in his philosophy. When working as a physicist, he defined intuition as something abstracted from our senses. But when working as a mathematician, he adhered to his cautionary remark that "intuition is not necessarily founded on the evidence of the senses; the senses would soon become powerless"[9] – because, for example, we cannot imagine multidimensional spaces or figures. Rather, in this case, he took recourse to a *process* definition of intuition which, in Poincaré's view, was more appropriate to creativity in general:

> "[T]o make geometry, or to make any science, something else than pure logic is necessary. To designate this something else we have no word other than *intuition*."[10]

What is not logical in a demonstration is intuition, defined as a process described as a catalyst for thought.

What Poincaré referred to as "sensible intuition"[11] was precisely the role of mental imagery in mathematics, the ingredient in mathematical proof, as he writes in his 1908 introspection "Mathematical Invention," that is not of a "simple juxtaposition of syllogisms," but of their "order ... the feeling, so to speak, the intuition of this order, [the] ability to perceive the whole of the argument at a glance."[12] While we have no visual imagery of the steps in a mathematical proof, we do have some sense, *beyond logic*, of what form a mathematical proof ought to take or of what approach or tactic is best for an overall strategy of attack: "intuition is this faculty."[13]

Because for Poincaré "invention is selection," how does the unconscious select and assemble the appropriate combination of mathematical facts? Here Poincaré's definition of intuition enters as the ingredient of creativity other than logic:

> "The rules that guide choices are extremely subtle and delicate, and it is practically impossible to state them in precise language: they must be felt rather than formulated. Under these conditions how can we imagine a sieve capable of applying them mechanically?"[14]

Some of these rules pertain to aesthetics and intuition. We may consider them as instantiated in the mind in a non-language like representation, referred to as an analog representation. For example, mental imagery is encoded in an analog representation.[15]

Nor can the choice of the appropriate combination of facts be articulated: "This too is most mysterious [because the] useful combinations are the most beautiful, I mean those that can charm the special sensibility that all mathematicians know."[16] And, not unexpectedly, Poincaré wrote: "Among the combinations we choose, the most fruitful are often those which are formed of elements borrowed from widely separated domains."[17] What I call *network thought* is of the essence here. Network thinking occurs when concepts from "widely separated domains" are combined with proper choice of mental image or metaphor to catalyze the nascent moment of creativity. This necessarily nonlinear thought process can occur unconsciously, and not necessarily in real time. Concepts combine like light rays focusing at a point. We are reminded here of Poincaré's description of scientific creativity as the "process by which the human mind seems to borrow least from the exterior world, by which it acts, or appears to act, only by itself and on itself."[18]

The mathematician's "special aesthetic sensibility" or sensible intuition plays the role of the "delicate sieve" that filters out all but the few combinations that are "harmonious" and "beautiful." Liberation of thought in the unconscious "permits unexpected combinations of mathematical facts."[19] This is what Toulouse referred to as the "play of associations" that leads to scientific invention. This, too, is a point of modern psychological research which indicates the freedom in the unconscious for ranging over and activating information in long-term memory. "In a word," Poincaré concludes, "is not the subliminal self superior to the conscious self?"[20]

Regarding aesthetics, Poincaré is among the most quotable of scientists, because he actually used it as a guideline for scientific research:

> "The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful."[21]

Poincaré Introspects

Most analyses of Poincaré's 1908 introspection focus on the cycle of conscious thought/unconscious thought/illumination/verification.[22] But in light of Toulouse's observations, a hitherto unnoticed point emerges from Poincaré's introspection: Poincaré's critical and most far-reaching illuminations occurred in periods that involved unconscious work with either *no* predefined path and/or *no* set goal. Since Poincaré reported to Toulouse that this is the creative mode upon which he explicitly relied for his research, I will focus on this aspect of Poincaré's introspection.

The background to Poincaré's mathematical research in 1881 is as follows.[23] Poincaré's 1879 doctoral dissertation dealt with a restricted class of differential equations; but he believed that his results were generalizable to very much more complex situations. He was not sure, however, whether this would actually work out.

Poincaré began by trying to prove that the solutions he sought did not exist. He was not completely successful and this bothered him. After a sleepless night, he awoke with the realization of how to establish one class of these hybrid functions which he quickly generalized and named Fuchsian functions after the German mathematician Lazarus Fuchs whose work on differential equations had been of some importance to Poincaré.

In the scenario of Poincaré's scientific invention the key part goes thus. He journeyed from Caen, where he was on the faculty during 1879-1881, to a geological conference at nearby Coutances. The "vicissitudes of the journey made me forget my mathematical work."[24] That is, he *consciously* forgot the work. About to embark on a sightseeing drive at Coutances he stepped up into the carriage when the "idea came to [him], though nothing in [his] former thoughts seemed to have prepared [him] for it."[25] The idea came from network thinking on the unconscious level: the groups of transformations that leave Fuchsian functions unchanged or invariant could be obtained from an apparently unrelated branch of mathematics, non-Euclidean geometry. As Poincaré recollected this realization in 1883: Up until then, it seemed to everyone that the subject of non-Euclidean geometry "only had interest for the philosopher without any use for the mathematician."[26] Immediate written verification was unnecessary and he resumed conversing with the other passengers. Back at Caen, Poincaré verified the result "*à tête reposée.*"[27] As Mozart wrote to his father regarding his work on *Idomeneo*, "everything is composed, just not copied out yet."[28]

In summary, throughout his invention of Fuchsian functions Poincaré focused on groups of transformations that left these functions unchanged (invariant). A key step in Poincaré's invention of Fuchsian functions occurred when he suddenly realized – an illumination following unconscious thought without a predetermined path – that these groups could be obtained from considerations based on an apparently unrelated discipline, non-Euclidean geometry. Groups are mathematical quantities that play a key aesthetic role in mathematics and physics. So Poincaré actually practiced what he preached concerning intuition and aesthetics, namely, that his "special aesthetic sensibility" selected the most aesthetic solution which involved bringing into play concepts from a discipline that heretofore was considered to have no relation to the analysis of functions that solve differential equations: "Among the terms which have exercised the most happy influence, I would point out those of group and invariable."[29]

Poincaré and Relativity

Poincaré's and Einstein's lives overlapped, and at one time so did their research. They were the key players in a classic episode in the history of ideas. In 1905 both men were aware of the same empirical data and theoretical problems. Independently and simultaneously they both produced the same mathematical framework for dealing with these issues. Whereas Poincaré interpreted the mathematical framework as an improvement of an already existing electron theory, Einstein interpreted it as a new theory of space and time. The instrumental factor in this episode is their modes of mental imagery. In order to explore this point we now turn to Albert Einstein.[30]

We are fortunate that Toulouse published his extensive psychological data on Poincaré because no psychologist ever did this for Einstein.[31] Biographical studies of Einstein and Poincaré indicate dramatic contrasts in their school careers, personal lives and early research efforts, in addition to those attributable to the intellectual cultures in which they lived and worked. Were I to write a biography of Poincaré, I would alternate chapters between his scientific work and other parts of his life. The scientific life was dominant, which is not unusual for scientists and, as far as we know, there never was any reflection of one on the other. With Einstein it was altogether different, at least through 1909. As organized as Poincaré's life was, Einstein's was the opposite. Einstein's life is the stuff of which movies are made. The story is worth sketching out because it offers a further opportunity to compare and contrast Einstein and Poincaré.

A Portrait of the Physicist as a Young Man

Einstein's rebellious attitude against the authoritarian teaching methods at the Gymnasium he attended in Munich led eventually to a situation so intolerable that the fifteen year old boy left in 1894 without a diploma. He essentially was a high school dropout.

Until the fall of 1895 Einstein traveled through Northern Italy. As had been the case for Goethe some hundred years before, the Italian sunshine and landscape impressed the young man, freeing him of the *Sturm und Drang* of the Munich years. During this period, however, Einstein did not neglect his love of science. By this time he knew the integral and differential calculus, self-taught at about age 13. In the summer of 1895 Einstein wrote his first scientific essay which he sent to his maternal uncle Caesar Koch. The essay demonstrates Einstein's deep knowledge of advanced topics in electromagnetic theory. Even so, there are no signs of genius in the essay. Yet in retrospect the perseverance and self-discipline needed to learn difficult subjects is an indication of things to come. Einstein was an autodidact.

As Einstein had promised his parents prior to withdrawing from the Gymnasium, he was also preparing himself for the entrance examination to the *Eidgensssische Technische Hochschule* (ETH) of Zurich. Whereas Poincaré went from a stellar stay at the *lycée* in Nancy directly to *l'Ecole Polytechnique,* Einstein failed the entrance examination to the ETH due to deficiencies in foreign languages, biology and history. These subjects require a form of learning that Einstein detested: rote learning. Owing to Einstein's excellent grades in the mathematics and physics portions of the

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entrance examination, one of the ETH's eminent professors encouraged him to attend lectures at the ETH if Einstein stayed in Zurich. Instead Einstein decided to spend a year at a preparatory school in the Swiss canton of Aarau, in order to correct the deficiencies that had caused him to fail the entrance examination.

The strong impression made on Einstein by the cantonal school was due to its unpretentiousness and to its seriousness which was in no way dependent on a teacher's authority. The school also emphasized the power of visual thinking, a mode of thought to which Einstein found himself disposed. Sometime during his sojourn in Aarau during 1895-1896, Einstein realized a thought experiment in highly visual terms over which he would ponder tenaciously for 10 years, until he realized that it contained the "germ of the special theory of relativity." He flourished in Aarau, passing with the highest grade average in his class and gained admission to the ETH.

But Einstein's educational experience at the ETH between 1896-1900 was bittersweet. Almost immediately, difficulties arose. The role of visual thinking was de-emphasized and the outdated physics curriculum focused on applications. Einstein liked neither the subject matter nor being coerced to memorize large quantities of what to him was unessential material. So in the evenings at home and during cut classes, he studied the masters of theoretical physics like Ludwig Boltzmann and Hermann von Helmholtz. From them, he learned the kind of physics not taught at the ETH, as well as the importance of visual thinking in the making of a scientific theory. As Boltzmann wrote in a book that Einstein undoubtedly read as a student: "Unclarities in the principles of mechanics [derive from] not starting at once with hypothetical mental pictures but trying to link up with experience at the outset."[32]

Einstein In Love

Poincaré's correspondence during his years at *l'Ecole Polytechnique* reveal a bon vivant student life. Since most of our picture of Poincaré as student is gleaned from letters to his mother, it is not surprising that there are no recorded female involvements. Einstein, too, was an active member of student café society. We do know that he had a number of correspondences with female friends. Then there was the tumultuous love affair with the only female member of his class, Mileva Maric. They met in October 1896 and until August 1899 were just close friends. During 1899 and 1902, their love affair had all the trappings of Romeo and Juliet with a considerable amount of *La Bohème* too.

We can imagine that at least from 1899 through 1902 the dynamics of Einstein's personal life drove his scientific work, as is often the case in the lives of artists, musicians and writers. The love letters have recently been published, and in the main they resemble the ones that you or I may have written. They are important in retrospect because they were of course written by Albert Einstein. Can you recall ever having seen a book of love letters written by a scientist?[33]

In August 1899 Einstein writes Mileva (they were separated much of the time owing to the opposition from both sets of parents) of his concerns over the present state of the "electrodynamics of moving bodies," which would be the title of the 1905 relativity paper. Despite suggestive phrases such as this one in their correspondence, it is straightforward to conclude that Mileva mostly acted as a sounding board for Einstein's ideas. There is no evidence that Mileva should have shared with him the accolades for the special theory of relativity. Suffice it to say that the couple who could not live without each another married in 1903 and found they could not coexist in peace. Albert and Mileva separated in 1914 and divorced in February 1919. Later that year he married a distant cousin, Elsa, whom he had known since 1912.

Einstein recalled that his independence of thought was not appreciated by the professors at the ETH. This situation, in conjunction with a personality conflict with a major professor, led to his being refused letters of recommendation upon graduation. Whereas Poincaré was accepted for advanced studies at the prestigious *l'Ecole des Mines*, while also pursuing advanced mathematical study at the University of Paris with the famous mathematician Charles Hermite, Einstein was the only one of four students in his class who passed the final examination to be refused a position as assistant to a professor at the ETH (Mileva failed twice and so never received a university diploma). Whereas Poincaré eagerly pursued mathematical problems after graduation, Einstein recalled in 1946 that it took him a year to recover from the ETH and to reacquire his taste for scientific research.

During the years 1900 to 1902, Einstein had only intermittent employment and was denied positions as assistant to several major physicists. As Einstein wrote to Mileva on 4 April 1901, "Soon I will have honored all physicists from the North Sea to the southern tip of Italy with my offer."[34] He persevered: In 1901, Einstein submitted a doctoral thesis to the University of Zurich which was rejected, but he succeeded in publishing his first paper in the prestigious German physics journal the *Annalen der Physik.*

Finally, through the intervention of the father of a college friend Marcel Grossmann, Einstein obtained a (provisional) position as third class technical expert at the Swiss Federal Patent Office in Bern. In reply to someone's comment that he might be bored in this position, Einstein wrote to Mileva in February 1902, "certain people find everything boring – I am sure that I will find it very nice and I will be grateful to Haller [the Director of the Patent Office] as long as I live."[35] And he was.

Albert Einstein's Creativity

The Bern period (1902-1909) was the most creative of his life. While working at the Patent Office eight hours a day, six days a week, he published on the order of 50 papers. Although between 1901 and 1904 he had published five papers in the *Annalen*, nothing prepared us for what would happen in 1905. After all, in 1905 Albert Einstein was a middle level junior civil servant with an academic record that was distinctive in retrospect only by its lack of distinctness. His score on the cumulative final exam at the ETH was 4.91 out of six, good but not superlative; he had failed once to obtain a Ph.D. and was denied letters of reference from his undergraduate school. Yet at eight week intervals, starting in March 1905, Einstein submitted three papers to the *Annalen* that changed the course of physics in the twentieth century, not to say life itself on our planet. May we not say that with Freud and Picasso, Einstein "created" the twentieth century?

In the first of the 1905 trio of papers, Einstein proposed that light has a particle nature. The second one solved the problem of why dust particles in the air perform an erratic dance, known as Brownian motion. An offshoot of this paper was a means for demonstrating the existence of atoms. The third one was the special relativity paper. These three papers appeared in the Volume 17 of the *Annalen*, which is so valuable that it is customarily removed from library shelves and placed in a safe. Einstein published one more paper in 1905 which contained a result he had overlooked in the relativity paper. He couldn't believe it himself – energy and mass are equivalent, $E = mc^2$, a result that is not even an equation any longer. It has become the signature of the twentieth century.

Contrary to Poincaré's discovery of Fuchsian functions, Einstein's early research results were at first appreciated mostly for the wrong reasons, if at all, the 1905 paper on special relativity included. Did he not work in the Patent Office until 1909? That Einstein had an *Annus Mirabilis* in 1905 became clear only in retrospect from the 1920s when *all* of his contributions from that year were duly acknowledged. Special relativity was not recognized as an achievement until 1911.[36] Whereas Poincaré clearly exhibited a certain genius in mathematics by age 17, there was no forewarning of Einstein's creative outburst in 1905.

In order to gain further insight into Einstein's creative thought we must come to grips with his various definitions of intuition and aesthetics.

Einstein Introspects on Intuition and Aesthetics

One of Einstein's definitions of intuition was a feel for nature: "There is no logical path leading to these laws [of nature], but only intuition, supported by sympathetic understanding of experience."[37] According to Einstein there is no logical path between necessarily imprecise experimental data and exact statements of a scientific theory. The scientist's only guide is intuition "resting on" a particular understanding of what are good data. This resembles Poincaré's process definition of intuition because it enables something to happen, while emphasizing an intuitive disposition toward good data.

Central to Einstein's creative thinking was his visual imagery, which is based on another definition of intuition as visualization. In the German cultural-scientific environment the term *Anschauung* is rendered either as intuition or visualization, consistent with its roots in Kantian philosophy. By visualization is meant the visual imagery abstracted from phenomena that we have actually witnessed in the world of sense perceptions.[38] Many scientists in the German-cultural environment believed that creative thinking occurs in visual imagery and words follow; this statement can be found in a revealing letter of Einstein written on 17 June 1944 to Jacques Hadamard,[39] and in Einstein's 1946 introspection entitled, "Autobiographical Notes."[40]

That our creative thinking is essentially nonverbal seemed reasonable to Einstein, for how could "we 'wonder' quite spontaneously about some experience"?[41] Einstein was as specific as he could be on the meaning of "wondering," which "seems to occur when an experience comes into conflict with a world of concepts which is already sufficiently fixed within us."[42] For example, Einstein recalled that as a boy of 5 or 6 he saw a compass and "wondered" at how its needle stayed fixed on magnetic north as if held by an unseen hand. This image remained in his mind. It became the basis of his preference for field-theoretic formulations of physics, of the sort pioneered by Faraday and Maxwell. This representation of nature is an abstraction of the way phenomena occur in the world we live in: action by contact.

It is in Einstein's use of the term "wonder" that his three definitions of intuition fuse. The first time that Einstein used the term *Anschauung* extensively was in his 1909 paper "On the Development of Our Intuition [*Anschauung*] of the Existence and Constitution of Radiation."[43] As the title indicates, Einstein explored the clash between the *Anschauungen* or intuitions of the time-honored wave mode of light and the particle mode, or light quantum, that he had invented in 1905. As he had written in a letter to his friend Conrad Habicht in the spring of 1905, he considered light quanta to be *"sehr revolutionär"* because they were what Einstein referred to in 1946 as a "wonder," since their characteristics conflict with already formed concepts, that is, the visual images of *Anschauung*.[44]

Aesthetics

Poincaré's notion of aesthetics was of a formal mathematical sort, in which he explored results in mathematics by seeking ways to alter variables in equations in such a way as to leave the equation itself unchanged, or invariant. Einstein, on the other hand, concerned himself with symmetries in how theories ought to *represent* nature.

Imagine dropping a stone into a still pond of water. Circular waves move outward from the stone's entry point. This is the representation used for how an electron produces spherical light waves. All scientists in 1905 judged the visual representation of particle and wave (discontinuity/continuity) side-by-side to be aesthetic, and to carry no hint of contradiction. To Einstein alone, however, there was an asymmetry in this tension between discontinuity/continuity or particle/wave – the asymmetry of redundancy in representation. Why burden a theoretical representation with particle and wave side-by-side? As Einstein put it in his 1905 paper on the constitution of light, this "profound formal distinction" science makes is wrong.[45] This aesthetic discontent was precisely Einstein's reason in 1905 for proposing that light can be *represented* as a particle.

Although we can visualize light moving through space as *either* a wave or a particle, we cannot visualize or even imagine light as both wave *and* particle. Consequently, it is no wonder that for more than two decades physicists puzzled over and resisted Einstein's light quanta. Their criticisms were based not on any empirical data, but the problem of visually representing light and particularly how certain optical phenomena that had traditionally been the hallmark of light could be explained with light quanta. For example, no visual representation could be constructed for how light quanta produce interference, whereas one existed for light waves, namely, comparison with how water waves produce interference.

Yet another example of Einstein's unique use of aesthetics appears in the very first sentence of the 1905 relativity paper: "That Maxwell's electrodynamics – the way in which it is usually understood – when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena is well known."[46]

One asymmetry was the redundancy in explanation of Maxwell's theory for electromagnetic induction, which Einstein developed with a highly visual thought experiment. As a master of understatement, Einstein's phrase – "is well known" – is far from the truth, which is that to *everyone* else two explanations for the generated current in electromagnetic induction were perfectly natural.

Such notions of aesthetics – combined with Einstein's "feel" or intuition for the proper subset of data to use to bridge the gap, in one fell swoop, toward the exact statements of physical theory – enabled him to formulate the special relativity theory. Great innovations occur in just this way: in an explosion of thought possible only with analog representations of mental imagery and aesthetics. This occurred as well with Poincaré's discovery of Fuchsian functions. But in physics this process required Einstein to permit his scientific speculations to take him beyond the immediate world of sense perceptions in which Poincaré was grounded, and where, for example, there seems to be no relativity of time. Through thought experiments and a unique interpretation of aesthetics Einstein, alone, found it possible to *redefine* the concept of intuition to a level of abstraction higher than in the perceptual bound mechanics and electrodynamics of 1905. This was why Einstein discovered relativity theory in 1905, whereas Poincaré only reformulated an already existing electron theory.

* * *

This case study of Poincaré and Einstein has permit us to identify differences between creativity in the cultures of mathematics and physics.

Poincaré considered intuition and aesthetics the critical guidelines for unconscious network thinking. He deemed that what is not logical in a mathematical proof to be of intuition. This process definition of intuition can be described as a catalyst for creative thought. Poincaré himself spoke of his *nonvisual* mode of mental imagery as a "sensible intuition," which is consistent with Toulouse's finding that in his creative work Poincaré neglected visual imagery altogether. Sensible intuition is the faculty for perceiving the whole of a mathematical demonstration at a glance. Aesthetics is a guideline in network thinking because it acts as a "delicate sieve" that filters out all but the most "beautiful" combinations of mathematical facts. For a mathematician the concepts of aesthetics and beauty have definite connotations, such as invariance and symmetry. Poincaré's recollection of his great mathematical invention in 1881, in conjunction with his published papers and unpublished manuscripts, and Toulouse's results, indicate that in his research he actually *relied* on aesthetics and intuition to guide his network thinking.

Particularly interesting support for network processing is that Poincaré's most far-reaching illuminations occurred when he was working on the cutting edge of scientific research with no predefined path or set goal. From what we know about activation of mental networks of information in whatever coding they may be in, only the freedom of unconscious thought will do in this situation. Similarly for Einstein who was also facile enough to bring to bear information from diverse areas on physics problems.

Einstein's introspection of 1946 in conjunction with his published papers, correspondence and archival documents indicate that his creative mode was also network thinking but of a sort that emphasized visual images. Consider the two thought experiments that were essential to his discovering the special and general theories of relativity. The 1895 thought experiment explored what it is like to catch up with a point on a light wave. This experiment led, ten years later, to the special theory of relativity. What is interesting is that in his "Autobiographical Notes" Einstein describes the thought experiment as something that he "had already hit upon at the age of sixteen."[47] Perhaps his use of the word "hit" denotes the suddenness of this realization, occurring in a burst of thought.

We are luckier with Einstein's recollection of the 1907 thought experiment that led to the general theory of relativity. In this case the thought experimenter leaps off the roof of his house while simultaneously dropping a stone. He observes that he and the stone fall together freely with no relative acceleration. While discussing his work in a seminar at Kyoto, Japan, in 1922, Einstein was reported to have recalled that this experiment emerged suddenly while he was day dreaming at the Patent Office.[48] Day dreaming is akin to unconscious thought in that it knows no boundaries. Consequently, there is a "play of associations," to use

Toulouse's terminology. In a reminiscence of 1919, Einstein referred to the 1907 thought experiment as the "happiest thought of my life."[49]

Einstein linked intuition with visual imagery which differed radically from Poincaré who related it to a nonvisual "sensible intuition." Here I refer to the important term in German philosophy and science, *Anschauung*. Einstein used this concept of intuition in his great thought experiments of 1895 and 1907 as well as in his early work on the nature of light. Einstein's notion of aesthetics also differed from Poincaré's which was most essentially useful to mathematicians. For Einstein aesthetics played a key role in deciding how physical theories ought to represent nature.

A principal problem that emerges toward unraveling scientific creativity is how strands of thought networks interact with different modalities of mental imagery.

I will conclude by saying a word about this problem because it bears on creativity in art as well as in science. Introspections by other scientists and artists bear a striking similarity to Poincaré's emphasis on unconscious processing of information, and to Einstein's emphasis that creative thinking is essentially nonverbal. Their statements about creative thinking bear a strong resemblance to those of artists.

Joan Miró described his creative thinking thus: "I begin painting and as I paint the picture begins to assert itself, or suggest itself, under my brush ... The first stage is free, unconscious ... the second stage is carefully calculated."[50] Picasso introspected along the same lines in 1935: "A picture," he explained, "is not thought out and settled beforehand ... An idea is a starting point, nothing more ... That which I think about a great deal, I find I have always had complete in my mind."[51] Similarly, the Abstract Expressionist painter Mark Rothko, in a vein similar to Poincaré's, writes in 1947: "Neither the action nor the actors can be anticipated, or described in advance. They begin as an unknown adventure in an unknown space. It is at the moment of completion that in a flash of recognition, they are seen to have the quantity and function which was intended. Ideas and plans that existed in the mind at the start were simply the doorway through which one left the world in which they occur."[52]

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There are numerous quotations of this sort by artists. The main point I want to make is that art and science practiced at their deepest levels are adventures into the unknown. Exploring this process of high creativity is a problem in which, as Toulouse wrote in 1910, "we touch the foundation of the mind."

Notes

1. H. Poincaré, "L'Invention mathématique," in *Science et méthode*, Paris, 1908, pp. 43-63.
2. E. Toulouse, *Enquète médico-psychologique sur la supériorité intellectuelle: Henri Poincaré*, Paris, 1910.
3. Among these materials, which had disappeared since Poincaré's death in 1912, figure manuscripts and letters that considerably augmente our knowledge of the history of physics and mathematics, but also of the history of science in France. These archives have been placed at the Université Nancy-II, publication center of several volumes of correspondence and manuscripts.
4. For biographical details as well as further discussion of Toulouse's psychological data see A.I. Miller, *Insights of Genius: Imagery and Creativity in Science and Art*, New York, 1996.
5. E. Toulouse, (note 2 above), p. 136.
6. Ibid., pp. 145-146 and 187, my emphasis.
7. Ibid., p. 200.
8. But I am not claiming that Poincaré considered visual imagery to be ancillary to scientific research. Quite the contrary. For example, Poincaré's pioneering research on dynamical systems was ingeniously carried out using topological concepts in two and three dimensions in lieu of rigorous and far too difficult investigations of the exact solutions of certain differential equations. Such rigorous investigations, concluded Poincaré, would have obscured essential points. He was right. See H. Poincaré, "Sur le problème des trois corps et les équations de la dynamique," in *Acta Mathematica*, vol. 13, 1890, pp. 27-40.
9. H. Poincaré, "Du rôle de l'intuition et de la logique en mathématiques," in *La valeur de la science*, 1905; all references are from the 1970 edition, Paris, p. 33.
10. Ibid., p. 32, emphasis in the original.
11. Ibid., p. 40.
12. H. Poincaré, (note 1 above), p. 47.
13. H. Poincaré, (note 8 above), p. 22.
14. H. Poincaré, (note 1 above), p. 56.
15. For discussion of language-like and analog representations see A.I. Miller, (note 2 above), chapters 8 and 9.
16. H. Poincaré, (note 1 above), p. 58.
17. Ibid., p. 49.
18. Ibid., p. 43.

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19. Ibid., p. 49.
20. Ibid., p. 56.
21. H. Poincaré, "Prefatory to Translation," in *The Value of Science*, trans. G.B. Halsted, New York, 1958, p. 8.
22. See, for example, M. Boden, *Creative Mind: Myths and Mechanisms*, London, 1990; and R. Penrose, *The Emperor's New Mind: Concerning Computers, Minds, and the Laws of Physics*, Oxford, 1989.
23. For more details see A.I. Miller, "Scientific Creativity: A Comparative Study of Henri Poincaré and Albert Einstein," in *Creativity Research Journal*, vol. 5, pp. 385-418.
24. H. Poincaré, (note 1 above), p. 51.
25. Ibid.
26. See A.I. Miller, (note 23 above), p. 418.
27. H. Poincaré, (note 1 above), p. 52.
28. W. Hildesheimer, *Mozart*, trans. M. Faber, New York, p. 238.
29. H. Poincaré, "L'avenir des mathématiques," in *Science et méthode*, (note 1 above), p. 30.
30. For discussion of this episode see A.I. Miller, "Why did Poincaré not Formulate Special Relativity in 1905?," in J.L. Greffe, G. Heinzmann and K. Lorenz (eds.), *Henri Poincaré: Science and Philosophy*, Berlin, pp. 69-100. Some scholars interpret the situation to the effect that both men arrived at the special theory of relativity and, consequently, Poincaré ought to share the accolades with Einstein. Existing archival and primary sources render the co-discovery claim insupportable.
31. Einstein discussed scientific creativity beginning in 1916 with his colleague at the University of Berlin, the Gestalt psychologist Max Wertheimer. But Wertheimer's subsequent published analysis of Einstein's discovery of special relativity turns out to be a reconstruction according to the guidelines of Gestalt psychology and is lacking in historicity. See A.I. Miller, *Imagery in Scientific Thought: Creating 20th Century Physics*, Cambridge, 1986, chapter 5.
32. L. Boltzmann, "Lectures on the Principles of Mechanics," in B. McGuiness (ed.), *Ludwig Boltzmann: Theoretical Physics and Philosophical Problems*, trans. R. Foulkes, Boston, 1974, p. 225.
33. J. Renn and R. Schulmann (eds.), *Albert Einstein, Mileva Maric: The Love Letters*, trans. by S. Smith, Princeton, 1992.
34. J. Stachel, D.C. Cassidy, and R. Schulmann (eds.), *The Collected Papers of Albert Einstein: Volume I, The Early Years, 1879-1902*, Princeton, 1987, p. 284-285.
35. Ibid., pp. 332-333.
36. These issues are discussed in A.I. Miller, *Albert Einstein's Special Theory of Relativity: Emergence (1905) and Early Interpretation (1905-1911)*, MA, 1981.
37. A. Einstein, "Prinzipien der Forschung," (How I see the World) in *Mein Weltbild*, Berlin, 1934, p. 109.
38. For further discussion see A.I. Miller, (note 3 above).
39. J. Hadamard, *The Psychology of Invention in the Mathematical Field*, New York, 1954, pp. 142-143.
40. A. Einstein, "Autobiographical Notes," in P.A. Schilpp (ed.), *Albert Einstein: Philosopher-Scientist*, IL, 1949, pp. 2-94.
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42. Ibid.

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46. A. Einstein, "Zur Elektrodynamik bewegter Körper," in *Annalen der Physik*, vol. 17, 1905, pp. 891-921.
47. A. Einstein, (note 40 above), p. 53.
48. J. Ishiwara, *Einstein Kyôzyu-Kôen-roku*, Tokyo, 1971.
49. A.I. Miller, "Albert Einstein's 1907 *Jahrbuch* Paper: The First Step from SRT to GRT," in J. Eisenstaedt and A.J. Kox (eds.), *Studies in the History of General Relativity*, Boston, 1992, pp. 319-335, quote on p. 325.
50. N. Stangos (ed.), *Concepts of Modern Art*, London, 1991, p. 130.
51. From an interview of 2 December 1933 of Picasso with Daniel-Henry Kahnweiler, in *Le Point*, vol. XLII, October 1952, p. 24; reproduced in *Huit entretiens avec Picasso*, Paris, 1988.
52. H.B. Chipp (ed.), *Theories of Modern Art: A Source Book by Artists and Critics*, Los Angeles, 1968, p. 548.