SOME RECOLLECTION OF THE INTERNATIONAL CONGRESS OF MATHEMATICIANS

Lee Lorch[1]

The International Congress of Mathematicians (ICM) held its quadrennial session August 16-26, 1966, this time at Moscow University. This was the first occasion on which it had assembled in the USSR. The proceedings are expected to be published within a year, incorporating the 83 invited addresses, statistics of the Congress and awards, but not abstracts of the 1870 contributed papers. These abstracts and other materials (including English translations made by the American Mathematical Society of Russian invited addresses from copy supplied beforehand by the authors) were distributed at registration.

In attendance were about 4300 ordinary members, including 114 women, and 200 associate members (i.e., relatives of ordinary members). They came from 54 countries: about 1500 from the USSR, 725 from the USA, 300 from Great Britain, 300 from France, 230 from the German Democratic Republic, and smaller numbers from other countries, including about 60 from Canada. Socialist countries other than the USSR and the GDR contributed about 500, including delegations from Cuba, North Korea and North Vietnam. Approximately a dozen countries (African and Asian) made their first ICM appearances. From every point of view (except associate members) this was the largest ICM. The previous session (Stockholm 1962) had 2107 ordinary members and 984 associates, the largest ICM to that date.

Invitations had been sent to the mathematical organizations of all countries and regions, whether or not they are on diplomatic speaking terms with the USSR. Arrangements were made and publicized for the issuance of visas en route for those coming from places without diplomatic relations.

Neither Mainland China nor Taiwan were represented. Taiwan had sent people to the 1962 Stockholm Congress. Mainland China has never sent anyone to an ICM, presumably in keeping with its policy of not participating in organizations which give any sort of recognition to Taiwan. The ICM operates under the aegis of the International Mathematical Union, an organization of 41 members, one of them the Taiwanese mathematical association. The ICM secretariat told me that it received no communication from either part of China, although it did receive about 80,000 letters (including some 200 containing purported proofs of Fermat's Last Theorem and the like!), from the rest of the world.

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1 This is a fuller version of a "meeting report" published in Science (vol. 155, 1967, pp. 1038-1039), after editorial abridgment.

FIELDS MEDALS[1]

The enormous growth of mathematics was reflected in the increase from two to four of the number of Fields Medals awarded. A sort of "Nobel prize" for younger mathematicians, two went to Americans: Professor Paul J. Cohen (Stanford) for his work in the foundations of mathematics, and Professor Stephen Smale (Berkeley) for his contributions to differential topology. Professor M. F. Atiyah (Great Britain) received one for his contributions to topology and partial differential equations, Professor A. Grothendieck (France) for his to algebraic topology.

Grothendieck did not attend the Congress. The Organizing Committee, which had included him (and the other medalists) among the 83 distinguished scholars invited to give special addresses, informs me that it received no explanation from him. The other three medalists received their awards at the opening ceremonies held in the Palace of Congresses at the Kremlin, although one of them had to witness the proceedings from the rear, having been delayed at the Greek frontier because of his car, and at the entrance to the Palace by lack of credentials.

There appeared to be universal agreement that all four eminently deserved their awards. But there was also substantial feeling that the awarding committee might well have recognized in this fashion the achievements of at least one of what the President of the International Mathematical Union, Professor G. de Rham (Switzerland), characterized in his closing address to the ICM as "the abundance of brilliant young Soviet mathematicians," especially since the number of such awards is not fixed. No Soviet mathematician has ever received a Fields Medal.

Of the work of Professor Smale some mention has already been made in Science (October 7, 1966) in the course of an article devoted mainly to his difficulties with the House Committee on Un-American Activities, other Congressmen and agencies. For his scientific work he was awarded in January, 1966, a Veblen Prize by the American Mathematical Society.

THE CONTINUUM HYPOTHESIS

Given present-day specialization in mathematics, it is likely that only Professor Cohen' s work, being in the foundations of mathematics, is in an area with which all mathematicians feel they should have a nodding acquaintance.

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[1] The Fields Medals were initiated at the 1924 Congress held in Toronto. The President of that Congress was Professor J. C. Fields (1863-1932), F. R. S., F. R. S. C., of the University of Toronto, who presented a fund to subsidize the awards. The first Fields Medals were presented at the 1936 (Oslo) Congress.

This work, published in summary form in Proc. Natl. Acad. Sci., 50 (1963), 1143-1148, 51(1964), 105-111, is in the theory of sets and is concerned with the problem of counting infinite collections. Two sets are regarded as having an equal "number" of elements if the elements of one can be put into one-to-one correspondence with those of the other.

The set P of all positive integers can clearly be put into such correspondence with the set Q of all positive even integers by associating with each element of P its double in Q and conversely. The founder of the theory of sets, G. Cantor (Germany), denoted this common transfinite cardinality by aleph-null. He showed that no infinite set has fewer than aleph-null elements, but that many have more, for example, the set of all real numbers. He showed also that there is a next larger transfinite cardinal, aleph-one, and conjectured that there are precisely aleph-one real numbers, i.e., sets of integers. This became known as the continuum hypothesis.

What Cohen has established is that the continuum hypothesis can be neither proved nor disproved on the basis of the standard structure (axioms) of the theory of sets. Moreover, his work showed that none of the additional axioms that have been proposed can be of any assistance in resolving this question.

Perhaps subsequent investigations will reveal new principles on the basis of which Cantor's continuum hypothesis can be settled, perhaps not. On this point intense controversy now centres.

FOURIER SERIES

Cantor was led to his studies in the theory of sets by his earlier work on trigonometrical series. In this subject too there was presented a solution of its most celebrated problem On the eve of the Congress, Professor L. Carleson (Sweden) published (Acta Math., 116 (1966) 135-157) a proof of the famous conjecture of N. Lusin (USSR) that the Fourier series of a periodic continuous function (more generally, even only an L_2 function) converges, except possibly on a set of measure zero. This proof, now undergoing intense study by specialists everywhere, was contrary to the expectations of many leading authorities who had come to believe that Lusin's conjecture was wrong. About 40 years ago, A. Kolmogorov (USSR) had constructed a Lebesgue integrable function whose Fourier series diverges everywhere. This famous example does not, of course, conflict with Carleson's result, since Kolmogorov's function is not continuous, nor even L_2.

In a paper that appeared immediately after the ICM, J.-P. Kahane (France) and Y. Katznelson (Israel) showed that Carleson's result is "best possible" (Studia Math. 21 (1966), 305-306). They proved that, given an arbitrary set of measure zero, there exists a continuous periodic function whose Fourier series diverges on the given set.

A set of measure zero (equivalent to the concept of zero probability) is a set on which one can change arbitrarily the values assumed by a (Lebesgue) integrable function without altering the value of the integral.

OLEVSKII'S RESULT

The failure of a Fourier series to reproduce for all values its generating function, even when that generating function is continuous (a fact known since 1876) naturally has led mathematicians to consider the problem of constructing, if possible, systems analogous to the Fourier trigonometric system $\{1, \sin x, \cos x, \ldots, \sin nx, \cos nx, \ldots\}$ which have the property that the Fourier series constructed from them will reproduce continuous generating functions. Systems of great importance having this property were brought to light, but none of them possessed all the fundamental properties of the trigonometric Fourier sequence.

At the Congress, a young Soviet mathematician, A. M. Olevskii, showed that nothing better can be done. More precisely, he proved that there exists no uniformly bounded, orthonormal system such that the Fourier series (with respect to that system) of an arbitrary continuous function must always reproduce that function everywhere. Together with related interesting results, he has published this in the Izvestiya of the Academy of Sciences of the USSR, Math. Series, 30 (1966), 387-432.

CONGRESS MISCELLANY

There were, as noted earlier, nearly 2,000 separate presentations to the Congress. I have mentioned only three. Others of outstanding importance were presented in a bewildering variety of fields, beyond my competence to discuss or evaluate. Pedagogical questions also received serious attention. A separate section devoted to them attracted able scholars and teachers at various university and school levels from a number of countries.

From the work which I have described, the Moscow ICM would seem to be characterized more by the solution of famous problems than by the indication of new directions. Those able to evaluate other work presented may provide a different impression.

The most important new paths will probably result from the informal discussions among the 4300 mathematicians who gathered from 54 countries. This represented the first large-scale contact between the mathematical communities of the USSR and non-socialist countries, undoubtedly the most valuable contribution of the Congress.

Perhaps in expectation of this, the official program incorporated a number of discussion periods free from formal presentations. These were used to the full and supplemented by a large number of unscheduled individual and group discussions.

At a time when everybody writes and hardly anyone reads, the spoken word and private letter are displacing the printed page as the most effective communication of current ideas.

Another value of the Congress, simply by virtue of its existence, is that it assembled enough mathematicians in one place so that nearby areas could schedule highly specialized conferences for much smaller groups (say about 300 each) to present and discuss research in tightly knit topics. Czechoslovakia, Finland, Hungary, Italy and Poland were sites of such gatherings, either just before or just after the ICM.

The holding of a scientific Congress is clearly regarded as a great event in the USSR. A special stamp was issued by the postal authorities; the Soviet press carried extensive accounts both of the ICM and on the subject of mathematics itself, before, during and after the Congress. For example, both Academician I. G. Petrovskiĭ, Rector of Moscow University and President of the ICM, and Dr. V. G. Karmanov, Secretary of the ICM Organizing Committee, published feature-length articles on mathematics.

There were interviews with both Soviet and foreign mathematicians. In one such, Fields Medalist Cohen expressed high praise for Moscow University, for Soviet mathematical life generally and characterized the organization of the Congress as "perfect". He added that the participants had "every opportunity for fruitful work, to see Moscow and the life of Soviet people".

Indeed, there was the closest personal contact between Soviet and foreign mathematicians. They were housed in the same dormitories at the University or hotels in town. Moscow mathematicians had social gatherings in their homes to which they invited foreign mathematicians, including many Americans. The young Soviet mathematicians threw a huge party and entertainment for all the young mathematicians at the Congress; some of the older ones crashed the affair.

The opening ceremonies included a splendid performance of Shostakovich' s exciting ballet, "The young lady and the hooligan".

Fourteen different excursions were available to members and associate members. Two concerts by prize-winning artists were presented. Theatre and other tickets could be booked directly from the ICM office.

The University restaurants had interpreters present at all times to assist in ordering meals. There was also an interpreters' room where one could go at any time to borrow an interpreter to help with some specific task or even to act as an unpaid guide for sight-seeing purposes. Mostly, they appeared to be University students gathered for the occasion.

The Science article (October 7, 1966) on Smale points out that the attack on him by various U. S. Congressmen, motivated by his opposition to U. S. government policy regarding Vietnam, came on the eve of the ICM and that responses there by Smale led to further attacks. The initial attack caused a good deal of resentment at the ICM. Many Americans present joined in cables of protest which were sent so quickly that other U. S. participants complained that they had not had sufficient time to add their own signatures. There was an enormous amount of corridor discussion of this matter; none appears to have been favourable to the U. S. Congress.

Vietnam was present in other ways as well. There was a vigorous display (in English), which I have seen, up-dated, also on my post-ICM visits to the University.

Four Hanoi mathematicians attended. In response to numerous questions, they prepared a statement on current academic life in North Vietnam. This reported that scientific life is expanding, although many academic and research centres have been evacuated to rural lecture halls and laboratories which professors and students have constructed themselves from bamboo stalks and palm leaves; ten new centres will open soon.

Two journals publish mathematical research, one in Vietnamese, the other in English, French, German and Russian. Reprints from the latter (Acta Scientiarum Vietnamicarum; sectio scientiarum Mathematicarum et Physicarum) were available. The research was of a level which would have merited publication in the established journals abroad, and the printing is also of high standard.

Even the "new math" appears to be found in North Vietnam. Nearly every province, the Hanoi mathematicians wrote, has special classes "reserved for secondary school students particularly gifted in mathematics." A monthly journal is published for them.

In closing this report, it may now be particularly appropriate to recall the words of the late Professor O. Veblen, after whom the American Mathematical Society named its research prize in geometry. As President of the ICM in 1950, when it met in the USA, he concluded his address with these words:

"To our non-mathematical friends we can say that this sort of a meeting, which cuts across all sorts of political, racial and social differences and focuses on a universal human interest will be an influence for conciliation and peace."

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