J. Plasma Physics (2009), *vol.* 75, *part* 1, *pp.* 15–18.
doi:10.1017/S0022377808007708 Printed in the United Kingdom

Letter to the Editor

Excitation of ion wakefields by electromagnetic pulses in dense plasmas

P. K. SHUKLA†

Institut für Theoretische Physik IV, Ruhr-Universität Bochum,
D-44780 Bochum, Germany
(ps@tp4-ruhr-uni-bochum.de)

(Received 7 October 2008 and accepted 21 October 2008, first published online 28 November 2008)

Abstract. The excitation of electrostatic ion wakefields by electromagnetic pulses in a very dense plasma is considered. For this purpose, a wave equation for the ion wakefield in the presence of the ponderomotive force of the electromagnetic waves is obtained. Choosing a typical profile for the electromagnetic pulse, the form of the ion wakefields is deduced. The electromagnetic wave-generated ion wakefields can trap protons and accelerate them to high energies in dense plasmas.

There are many proposals [1–5] for exciting high-phase speed intense electrostatic wakefields by electron bunches and laser beams in an unmagnetized electron–ion plasma [2, 4, 5]. Large-amplitude electron plasma waves are capable of accelerating electrons to extremely high energies, as demonstrated experimentally [6–14]. Possible applications of collective plasma accelerators lie in producing beams of energetic electrons, protons, and gamma rays, as well as femtosecond pulses and compact radiation sources for medicine. Plasma-based charged particle acceleration schemes are also holding promises for extremely high-energy charged particles and radiation sources from astrophysical plasmas as well.

However, in very dense plasmas, such as those in astrophysical environments [15–18] and in the next-generation intense laser–solid density plasma experiments [19–23], there might appear novel effects at the nanoscale owing to the presence of the new electron pressure law and the quantum force involving the Bohm potential [24–28]. This happens because in dense quantum plasmas the electrons degenerate and they follow the Fermi–Thomas distribution.

In this letter, we consider the excitation of electrostatic ion wakefields by electromagnetic (EM) waves [29–32] in a very dense plasma. For this purpose, we use the quantum fluid model for the degenerate electrons and derive the ion wakefield equation in the presence of the ponderomotive force of the EM waves. The profile of the ion wakefield is deduced by assuming a given EM pulse shape. The relevance of our investigation is also mentioned.

† Also at Department of Physics, Umeå University, SE-90187 Umeå, Sweden; Scottish Universities Physics Alliance Department of Physics, University of Strathclyde, Glasgow, Scotland; Centre for Fundamental Physics, Rutherford Appleton Laboratory, Chilton, Didcot, UK; GoLP/Instituto de Plasmas e Fusão Nuclear, Instituto Superior Técnico, 1049-001 Lisboa, Portugal; School of Physics, University of KwaZulu-Natal, 4000 Durban, South Africa.

Let us consider a very dense unmagnetized electron–ion plasma in the presence of the EM fields. At equilibrium, we have $n_{\mathrm{e}0} = Z_{\mathrm{i}} n_{\mathrm{i}0}$, where n_{j0} is the unperturbed number density of the particle species j ($j =$ e for the electrons and i for the ions) and Z_i is the ion charge state. The EM fields are given by $\mathbf{E}_w = -(1/c)\partial \mathbf{A}/\partial t$ and $\mathbf{B}_w = \nabla \times \mathbf{A}$, where c is the speed of light in vacuum and $\mathbf{A}$ is the vector potential. The frequency of the EM waves is

$$\omega = \left(k^2 c^2 + \omega_{\mathrm{pe}}^2\right)^{1/2}, \tag{1}$$

where $\mathbf{k}$ is the wave vector, $\omega_{\mathrm{pe}} = (4\pi n_{\mathrm{e}0} e^2/m_{\mathrm{e}})^{1/2}$ is the electron plasma frequency, e is the magnitude of the electron charge, and m_{e} is the electron mass.

The electron quiver velocity in the EM field is

$$\mathbf{u}_{\mathrm{e}} = \frac{e\mathbf{A}}{m_{\mathrm{e}} c}. \tag{2}$$

The ponderomotive force associated with the EM waves reads

$$m_{\mathrm{e}} n_{\mathrm{e}0} \langle \mathbf{u}_{\mathrm{e}} \cdot \nabla \mathbf{u}_{\mathrm{e}} \rangle + \frac{e n_{\mathrm{e}0}}{c} \langle \mathbf{u}_{\mathrm{e}} \times \mathbf{B}_{\mathrm{w}} \rangle \equiv \frac{n_{\mathrm{e}0} e^2}{2 m_{\mathrm{e}} c^2} \nabla |\mathbf{A}|^2, \tag{3}$$

where the angular bracket denotes an ensemble average over the period $2\pi/\omega$. We have used the gauge $\nabla \cdot \mathbf{A} = 0$.

The ponderomotive force pushes electrons locally and produce the space charge electric field ($-\nabla\phi$, where ϕ is the wake potential) and the density perturbation ($n_{\mathrm{e}1}$) in our dense plasma. The equation of motion for the inertialess electrons is

$$\frac{n_{\mathrm{e}0} e^2}{2 m_{\mathrm{e}} c^2} \nabla |\mathbf{A}|^2 = e n_{\mathrm{e}0} \nabla\phi - m_{\mathrm{e}} V_{\mathrm{e}}^2 \nabla n_{\mathrm{e}1} + \frac{\hbar^2}{4 m_{\mathrm{e}}} \nabla \nabla^2 n_{\mathrm{e}1}, \tag{4}$$

where $V_{\mathrm{e}} = (2\pi\hbar/\sqrt{3} m_{\mathrm{e}})(3 n_{\mathrm{e}0}/8\pi)^{1/3}$, and $\hbar$ is the Planck constant divided by 2π. The second and third terms in the right-hand side of (4) are associated with the pressure law [33] (e.g. $p_{\mathrm{e}} = (4\pi^2\hbar^2/5m_{\mathrm{e}})(3/8\pi)^{2/3} n_{\mathrm{e}}^{5/3}$ for non-relativistic degenerate electrons, where n_{e} is the electron number density) and the Bohm potential [24], respectively, in dense plasmas. The electrons are coupled with ions via the space charge electric field. The ion density perturbation n_{i1} is determined from [34]

$$\frac{\partial n_{\mathrm{i}1}}{\partial t} + n_{\mathrm{i}0} \nabla \cdot \mathbf{u}_{\mathrm{i}} = 0, \tag{5}$$

where the ion fluid velocity $\mathbf{u}_{\mathrm{i}}$ is obtained from

$$\frac{\partial \mathbf{u}_{\mathrm{i}}}{\partial t} = -\frac{Z_{\mathrm{i}} e}{m_{\mathrm{i}}} \nabla\phi. \tag{6}$$

In (6) we have neglected the ion ponderomotive force, the ion quantum force, and the ion pressure, since they are smaller by a factor $m_{\mathrm{e}}/m_{\mathrm{i}}$ in comparison with those acting on the electrons. Here m_{i} is the ion mass.

The Poisson equation is

$$\nabla^2\phi = 4\pi e (n_{\mathrm{e}1} - Z_{\mathrm{i}} n_{\mathrm{i}1}). \tag{7}$$

Combining (5)–(7) we obtain

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{\mathrm{pi}}^2\right) \nabla^2\phi = 4\pi e \frac{\partial^2 n_{\mathrm{e}1}}{\partial t^2}, \tag{8}$$

where $\omega_{\mathrm{pi}} = (4\pi Z_i^2 e^2 n_{\mathrm{i}0}/m_{\mathrm{i}})^{1/2}$ is the ion plasma frequency.

We can now eliminate n_{e1} from (8) by using (4), obtaining the driven (by the ponderomotive force of the EM waves) ion wakefield equation

$$\frac{\hbar^2}{4m_e^2}\left(\frac{\partial^2}{\partial t^2}+\omega_{pi}^2\right)\nabla^4\phi - V_e^2\left(\frac{\partial^2}{\partial t^2}+\omega_{pi}^2\right)\nabla^2\phi + \omega_{pe}^2\frac{\partial^2\phi}{\partial t^2} = \frac{\omega_{pe}^2 e}{2m_e c^2}\frac{\partial^2|\mathbf{A}|^2}{\partial t^2}. \quad (9)$$

In the limit $|\partial^2\phi/\partial t^2| \ll \omega_{pi}^2\phi$, (9) reduces to

$$\left(\frac{\partial^2}{\partial t^2} - V_i^2\nabla^2 + \frac{\hbar^2}{4m_e m_i}\nabla^4\right)\phi = \frac{e}{2m_e c^2}\frac{\partial^2|\mathbf{A}|^2}{\partial t^2}. \quad (10)$$

where $V_i = (m_e/m_i)^{1/2}V_e$.

In one space dimension, (10) in the moving frame $\xi = x - V_g t$ can be written as

$$\left(\frac{\partial^2}{\partial \xi^2} + K_q^2\right)\phi = \frac{2em_i V_g^2|\mathbf{A}|^2}{\hbar^2 c^2}, \quad (11)$$

where $V_g = k_x c^2/\omega_0$ is the group velocity of the EM pulse, $\omega_0 = (k_x^2 c^2 + \omega_{pe}^2)^{1/2}$, k_x is the wave number along the x-axis in a Cartesian coordinate system, and $K_e = 4m_e m_i(V_g^2 - V_i^2)/\hbar^2 > 0$.

The solution [35–37] of (11) is

$$\phi = \frac{2em_i V_g^2}{K_q\hbar^2 c^2}\int_\xi^0 |\mathbf{A}|^2(\xi')\sin[K_q(\xi-\xi')]\,d\xi', \quad (12)$$

where the boundary conditions, $\phi = \partial\phi/\partial\xi = 0$ at $\xi = 0$ have been used.

Let us suppose that the EM pulse is given by $\mathbf{A} = \mathbf{A}_0\sin(\pi\xi/L_0)$ for $-L < \xi < 0$, and $\mathbf{A} = 0$ otherwise [35–37]. Here L is the pulse length, which is shorter than approximately $2\pi c/\omega_{pe}$. Hence, (12) yields

$$\phi = C_q\frac{|\mathbf{A}_0|^2}{4}\left\{1 - \frac{1}{\left(K_q^2 - 4\pi^2/L^2\right)}\left[K_q^2\cos(2\pi\xi/L) - \frac{4\pi^2}{L^2}\cos(K_q\xi)\right]\right\}, \quad (13)$$

where we have used the notation $C_q = 2em_i V_g^2/\hbar^2 c^2$.

For $K_q L \ll 1$, (13) gives

$$\phi \simeq \frac{C_q K_q^2|\mathbf{A}_0|^2}{8}g(\xi), \quad (14)$$

where we have used the notation

$$g(\xi) = \xi^2 - 2\left(\frac{L}{2\pi}\right)^2\left[1 - \cos\left(\frac{2\pi\xi}{L}\right)\right]. \quad (15)$$

The functional $g(\xi)$ maximizes at $\xi = L$. Thus, the maximum value of the wake potential, deduced from (14) is

$$\phi_m \simeq (C_q/8)(K_q L|\mathbf{A}_0|)^2. \quad (16)$$

To summarize, we have considered the excitation of the ion wakefields by large-amplitude EM waves in a very dense plasma with degenerate electrons. Specifically, we have used the relevant momentum equation for the latter, as well as the ion continuity and momentum equations together with the Poisson equation to derive the governing equation for the ion wakefield in the presence of the ponderomotive force of the EM waves. In a stationary moving frame, the profile of the ion wakefield is determined provided that the shape of the high-frequency EM vector potential is prescribed. It is found that even weak short EM pulses are capable of exciting sizable ion wakefields. The latter can be exploited for accelerating protons in dense plasmas, such as those in compact astrophysical objects [15–18] (e.g. interior of white dwarfs), in the next-generation laser–solid density plasma interaction experiments [20–23], in free electron lasers [38], and in plasmonic devices [33].

References

[1] Rosenbluth, M. N. and Liu, C. S. 1972 *Phys. Rev. Lett.* **29**, 701.
[2] Tajima, T. and Dawson, J. M. 1979 *Phys. Rev. Lett.* **43**, 267.
[3] Berezhiani, V. I., Tskhakaya, D. D. and Shukla, P. K. 1992 *Phys. Rev.* A **46**, 6608.
[4] Mendonça, J. T. 2001 *Theory of Photon Acceleration*. Institute of Physics, Bristol.
[5] Bingham, R., Mendonça, J. T. and Shukla, P. K. 2004 *Plasma Phys. Control. Fusion* **46**, R1.
[6] Mangles, S. et al. 2004 *Nature (London)* **431**, 535.
[7] Geddes, C. G. R. et al. 2004 *Nature (London)* **431**, 538.
[8] Faure, J. et al. 2004 *Nature (London)* **431**, 541.
[9] Bingham, R. 2007 *Nature (London)* **445**, 591.
[10] Bingham, R. 2008 *Phil. Trans. R. Soc. A* **366**, 1749.
[11] Leemans, W. P. et al. 2006 *Nature Phys.* **2**, 696.
[12] Maltis, N. H. et al. 2006 *Nature Phys.* **2**, 749.
[13] Blumenfeld, I. et al. 2007 *Nature (London)* **445**, 744.
[14] Lu, W. et al. 2007 *Phys. Rev. ST Accel. Beams* **10**, 061301.
[15] Chabrier, C., Douchin, F. and Potekhin, A. Y. 2002 *J. Phys.: Condens. Matter* **14**, 9133.
[16] Chabrier, G., Saumon, D. and Potekhin, A. Y. 2006 *J. Phys. A: Math. Gen.* **39**, 4411.
[17] Schlickeiser, R. and Shukla, P. K. 2003 *Astrophys. J. Lett.* **599**, L57.
[18] Harding, A. K. and Lai, D. 2006 *Rep. Prog. Phys.* **69**, 2631.
[19] Glenzer, S. H. et al. 2007 *Phys. Rev. Lett.* **98**, 065002.
[20] Son, S. and Fisch, N. J. 2005 *Phys. Rev. Lett.* **95**, 225002.
[21] Marklund, M. and Shukla, P. K. 2006 *Rev. Mod. Phys.* **78**, 591.
[22] Malkin, V. M. et al. 2007 *Phys. Rev.* E **75**, 026404.
[23] Shukla, P. K. and Eliasson, B. 2007 *Phys. Rev. Lett.* **99**, 096401.
[24] Gardner, C. L. and Ringhofer, C. 1996 *Phys. Rev.* E **53**, 157.
[25] Manfredi, G. 2005 *Fields Inst. Commun.* **46**, 263.
[26] Shukla, P. K. and Eliasson, B. 2006 *Phys. Rev. Lett.* **96**, 245001.
[27] Shaikh, D. and Shukla, P. K. 2007 *Phys. Rev. Lett.* **99**, 125002.
[28] Shukla, P. K. 2006 *Phys. Lett.* A **352**, 242.
[29] Yu, M. Y., Spatschek, K. H., and Shukla, P. K. 1974 *Zh. Naturforsch.* A **29**, 1736.
[30] Shukla, P., Yu, M. Y. and Spatschek, K. H. 1975 *Phys. Fluids* **18**, 265.
[31] Shukla, P. K., Yu, M. Y., Rahman, H. U. et al. 1984 *Phys. Rep.* **105**, 227.
[32] Murtaza, G. and Shukla, P. K. 1984 *J. Plasma Phys.* **31**, 423.
[33] Marklund, M., Brodin, G., Stenflo, L. and Liu, C. S. 2006 *Europhys. Lett.* **84**, 17006.
[34] Shukla, P. K. and Stenflo, L. 2006 *Phys. Plasmas* **13**, 044505.
[35] Tang, T. et al. 1990 *Phys. Fluids* B **2**, 1390.
[36] Shukla, P. K. 1993 *Phys. Fluids* B **5**, 3088.
[37] Shukla, P. K. 1994 *Phys. Scr.* **T52**, 73.
[38] Serbeto, A., Mendonça, J. T., Tsui, K. H. and Bonifacio, R. 2008 *Phys. Plasmas* **15**, 013110.