AN ORIGIN OF THE VARIATIONS OF ASTRONOMICAL REFRACTION.

Haruo Yasuda and Rikinosuke Fukaya
Tokyo Astronomical Observatory, Mitaka, Tokyo, Japan.

Abstract. There exists an empirical relation between the anomalous refraction and the atmospheric density in the surface layer. From the relations the variations of scale height for each night can be determined by the temperature and pressure in the surface layer. A correction term to the refraction table is derived in an analytical expression.

1. EMPIRICAL RELATION BETWEEN THE ATMOSPHERIC DENSITY AND THE REFRACTION ANOMALY.

When the stars north and south of the zenith are observed with a meridian circle, there exists a relation

$$\phi_s - \phi_n = - \Delta r \ (\tan z_s + \tan z_n) \qquad (1)$$

between an observed latitude, ϕ, and a correction to the constant of refraction, Δr, under the assumption that the difference between ϕ_s and ϕ_n is due to refractions. Subscript, s and n, show the south and north of a zenith respectively and z is an absolute zenith distance. When the southern and northern stars have the same absolute zenith distances, the relation (1) is written in the form of

$$\phi_s - \phi_n = - 2\Delta a_o \tan z \frac{\rho}{\rho_o} . \qquad (2)$$

Here Δa_o is the correction to the astronomical refraction constant adopted in the Pulkovo Table, since the refractions applied to Tokyo observations are equivalent to those taken from the Pulkovo Table 4th Ed.. ρ_o is the air density of the standard air. From the observations of FK4 stars during the period, 1972 - 1976, Fukaya, one of the authors obtained a relation

$$a_o = \underset{\pm 0.005}{0''049} \sin (T + \underset{\pm 3^\circ}{277^\circ}) + \underset{\pm 0.005}{0''024} \sin (2T + \underset{\pm 13^\circ}{202^\circ}), \qquad (3)$$

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where T is a fraction of a year in a degree of arc and its epoch is January 15. The value of ρ is calculated from the measured temperature and pressure in our observing pavilion. An annual term in (3) shows that the measured temperatures are too high in winter and too low in summer. It coincides with an annual term of the grounding inversion of temperature, 1.39°C sin (T + 295°), at the Tateno Aerological Observatory. The observatory situates at the distance of about 65 km in the north-east direction from our observatory.

Using the observations of equatorial and circumpolar FK4 stars, Fukaya also obtained an empirical relation of

$$\delta(\Delta r) = \underset{\pm\ 1.8}{8''\!.6}\ \Delta\rho \quad . \tag{4}$$

See Figure 1. Here ρ is calculated in a unit of 10^{4} gr/little from the temperature and pressure measured in an observing pavilion, and $\Delta\rho$ is designated as a residual of ρ for each observing tour from a smoothed average of densities for five tours around it. $\delta(\Delta r)$ is a residual of r in second of arc from its smoothed average for the same tour as used in the calculation of $\Delta\rho$. The standard deviations of $\Delta\rho$ and $\delta(\Delta r)$ are $\pm$ 0.0125 gr/little and $\pm$ 0''.09/$\tan^{-1}$1. They show the differences between the stationary atmosphere adopted in the Refraction Table and the dynamical atmosphere at Mitaka.

Being calculated by the use of aerological data at the height of 1 km, there exists a relation of

$$\delta(\Delta r) = \underset{\pm 1.8}{7''\!.7}\ \Delta\rho_1 \quad , \tag{5}$$

while, being calculated by the aerological data at the height of 5 km, there is no longer a proper correlation between them. See Figure 2.

The practical diminution law of the air density at Mitaka differs from that adopted in the Refraction Table. It is concluded that (1) Δr is quite due to the atmospheric density below the height of 5 km, and that (2) Δr may be calculated by the use of the density in the surface layer.

2. INTERPRETATION OF THE EMPIRICAL LAW.

To demonstrate qualitatively such anomalous refraction as $\delta(\Delta r)$, we shall express the refraction in such an analytical form as is used in the textbook of Newcomb (1906).

Teleki (1967) set up the following relations from the aerological data ;

$$\rho = \rho_o \ e^{-as} \quad ,$$

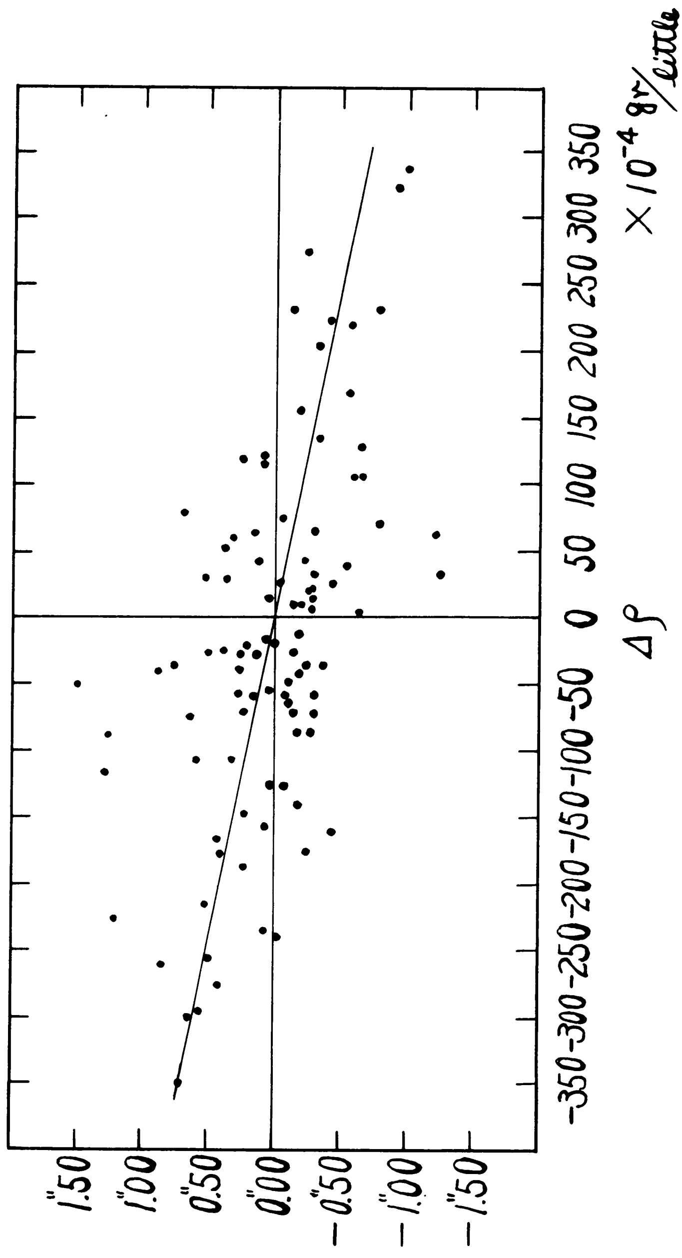

Fig. 1 The relation between the refraction anomaly $\delta(\Delta r)$ and the variation of the air density at the observing site $\Delta\rho$

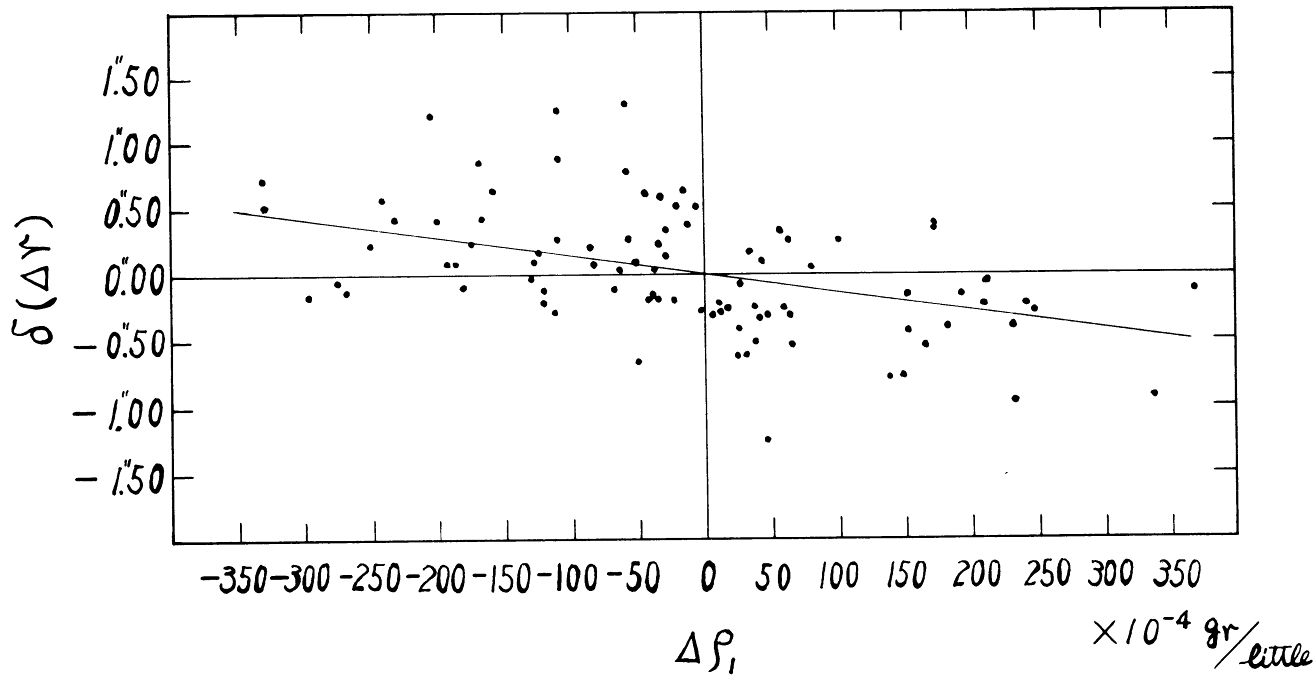

Fig. 2 The relation between the refraction anomaly $\delta(\Delta r)$ and the variation of the air density at the height of 1 km $\Delta\rho_1$

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$$a = \left[\frac{\Gamma_h - \Gamma}{T_s} \right] r_o \quad , \qquad (6)$$

$$s = (r/r_o) - 1 = \frac{H}{r_o} \quad ,$$

where r and r_o are the radius of curvature of an equidensity layer and of the geoid at the station respectively, T_s the mean temperature of the layer, and H the height of the layer. Γ is the vertical gradient of an air layer.

The rigorous differential equation of astronomical refraction can be expressed by

$$dR = - \frac{d\mu}{\mu} \frac{\sin z}{\sqrt{(\mu r/\mu_o r_o)^2 - \sin^2 z}} \quad . \qquad (7)$$

From the Dale-Gladstone equation and the first equation of (6), the refractive index of the air, μ , is expressed by the formula of

$$\mu = 1 + c\rho_o \, e^{-as} \quad , \qquad (8)$$

where the coefficient c is a constant depending on the wave length and characterizing a given air medium. From (6) and (8), we have

$$\frac{\mu r}{\mu_o r_o} = (1 - \alpha\omega) (1 + s) = 1 + 2u \qquad (9)$$

and

$$d\mu = - c \rho_o \, d\omega \qquad (10)$$

where the constant α and the variable ω are

$$\alpha = \frac{c\rho_o}{1 + c\rho_o}$$

and $$\omega = 1 - \frac{\rho}{\rho_o}$$

Substituting (9) and (10) in (7) and integrating from the point of observation to the outer limit of the atmosphere, we have

$$R = \alpha m \tan z \qquad (11)$$

where

$$m = \int_0^1 \frac{d\omega}{(1 - \alpha\omega) \sqrt{1 + 2u \sec^2 z}} \quad . \qquad (12)$$

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This general formula for the astronomical refraction coincides with Newcomb's formula except of a term of $1 - \alpha\omega$.

From the equations (11) and (12), we have, keeping the accuracy of the refraction R to $1 \cdot 10^{-2}$ arc second,

$$R = \left(1 + \alpha - \frac{1}{a} \right) \tan z - \alpha\left(\frac{1}{a} - \frac{1}{2}\alpha \right) \tan^3 z \qquad (13)$$

in the case of zenith distances to 70°. α can be calculated by the temperature and pressure at the earth surface and contributes only to the mean value of the corrections to the Pulkovo Table at a given station. Taking only the differential correction of a, we have

$$\Delta R = \frac{\alpha}{a^2} \Delta a \left(\tan z + \tan^3 z \right) .$$

According to the difinitions of Δr and $\delta(\Delta r)$ in (1) and (4), we have

$$\Delta r = \frac{\alpha}{a^2} \Delta a . \qquad (14)$$

Substituting (4) in (14), we finally get

$$\Delta a = 1.08 \cdot 10^4 \Delta\rho \qquad (15)$$

where $\Delta\rho$ is expressed in (gr/cm^3). The numerical coefficient of (15) is calculated by putting

$$a = 850 \text{ and } \alpha = 0.28 \cdot 10^{-3}.$$

At a standard pressure 760 mm Hg, the change of temperature by $20^{\circ}C$ is equivalent to that of $0.9 \cdot 10^{-4}$ gr/cm^3 in air density and causes the change of a by 1.0. The results do not greatly differ from the seasonal variation of a in Teleki's paper (1967). The increment of Δa by 1.0 is equivalent to that of scale height by 9 gpm. The amplitude of the seasonal variation of a is expected to be about 1.0 at Tokyo. See Figure 3.

3. CONCLUSION.

The variation of the density distribution of atmospheric layers below the height of 3 or 5 km is likely the most effective triggers to the variation of refraction. From the empirical relation (4), the value of Δa at Tokyo can be calculated by the relation (15). In practice, after a correction term (14) has been applied to the refraction calculated by the Refraction Table, the correction to the refraction constant should be determined in conventional ways, since its origin is due to the difference between the mean atmospheric density at a given station and that adopted in the Refraction Table.

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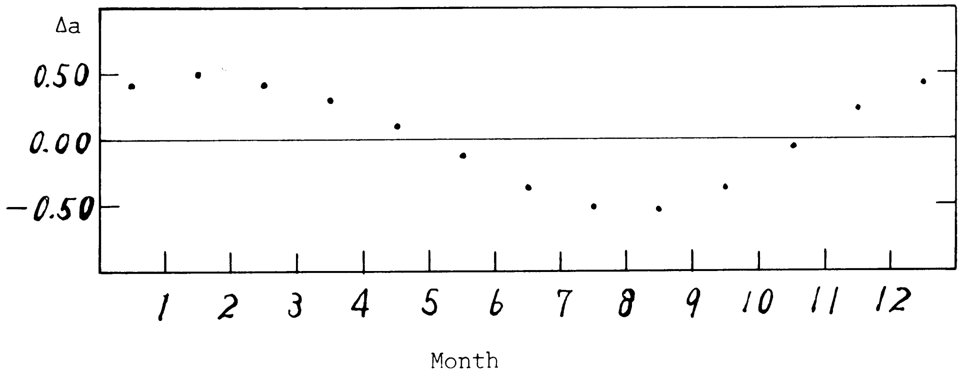

Fig. 3 The seasonal variation of a at Mitaka.
The value of a is assumed to be 850.

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DISCUSSION

E. Tengström: remarked that the secular change of the atmosphere is also one of the sources which can influence the refractional calculation during the long catalogue work.

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