Proceedings of the Edinburgh Mathematical Society (1999) **42**, 425–432 ©

BOOK REVIEWS

NATHANSON, M. B. *Additive number theory: the classical bases* (Graduate Texts in Mathematics Vol. 164, Springer-Verlag, Berlin-Heidelberg-New York-London-Paris-Tokyo-Hong Kong, 1996), xiv + 342 pp., 0 387 94656 X (hardback), DM78.

Additive Number Theory has one great advantage over many other branches of mathematics: many of its most important results and conjectures are accessible to every mathematician. Every positive integer is the sum of four squares (Lagrange's Theorem). Is every positive even integer the sum of at most two primes? (Goldbach's Conjecture). Simplicity and clarity itself. While you might sympathise with the sentiments quoted in Arnold [1] ("Prime numbers are made to be multiplied, not added"), you cannot but appreciate the accessibility of such a statement. Barriers of abstraction: there aren't any. Definitions and concepts depending, Russian-doll-like, upon other definitions and concepts which depend on . . . . Not in additive number theory! But of course additive number theory is a deep subject. Its depth lies in the intricacy, sophistication and delicacy of its technical ideas, its "tricks". (A trick is a technical idea whose origins you don't know).

What of the book under review? "Everything in this book is a generalisation of Lagrange's Theorem" (page 7). "The subject of this book is additive bases" (page 49). A set of integers is a (finite) additive basis (for the positive integers) if there is some integer h such that every non-negative integer can be written as the sum of at most h members of the set. So Lagrange's Theorem says that the set of squares forms a basis with $h = 4$. Goldbach's conjecture says that the primes (with 1 replacing 2) form a basis with $h = 2$ for the positive even integers.

This book is divided into two major parts: Waring's Problem and The Goldbach Conjecture. These two topics are used as themes to introduce a wide range of techniques of the subject, both elementary and sophisticated. In the part on Waring's problem, among the topics covered are the classical theory of quadratic forms, Hermite polynomials and, most importantly, the circle method. Under the umbrella of The Goldbach Conjecture, we are again introduced to topics at very different levels, from the simple covering congruences to intricate sieving techniques, finishing with Chen's Theorem. This is a technical *tour de force*, showing that every sufficiently large integer is the sum of a prime and a number which is the product of at most two primes. The author has done well to present it in a readable manner.

Throughout, the author's style is direct and unfussy. The book, intended for non-experts in the field, succeeds very well in making the ideas and techniques of the subject accessible, at least for this non-expert. The book is very carefully written, and the author's policy of including all the details makes it a pleasant book to pick up and read. It is well laid out and typeset, apart from some overambitious "subscript" clauses in minute type in quite a few places (especially page 247) which I could read only under strong light! Overall, however, I heartily recommend this book as an excellent way into the technical delights of additive number theory.

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REFERENCE

1. V. I. ARNOLD, Will mathematics survive? Report on the Zurich Congress, *Math. Intelligencer* **17** (1995), no. 3, 6–10.