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Great Circle v Rhumb Line Cross Track Distance at Mid-Longitude – The Final Answer

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KEY WORDS

1. Great circle. 2. Rhumb line.

I wrote a piece in the Forum of May 2004 on Great Circle v Rhumb Line Cross Track Distance at Mid-Longitude in which I set out a recent JAA ATPL examination question, offered 3 possible methods of solving it without the use of spherical trigonometry, but suggested that they were all quite time-consuming under examination conditions, and wondered if anyone knew of a quicker and more elegant solution – possibly the one that the JAA ATPL examiners were looking for. Two excellent, simple and creative solutions were offered in the January 2005 'Forum', one from Dr John Ponsonby, based on solid geometry, and the other from Sqn Ldr Peter Hoare, based on the properties of the Polar Gnomonic projection. Unfortunately, both were beyond the scope of what would be expected of ATPL candidates and I thought that they were probably not the solutions that the examiners were seeking.

This little conundrum appears to continue to intrigue readers of the *Journal*. Wei-Kuo Tseng and Hsuan-Shih Lee published 2 methods based on vector analysis, polar/Cartesian conversion, and the Gnomonic projection in the January 2007 Forum and the Canadian mathematician Michael Earle has written to me to point out that this problem is simply a particular case of finding the vertex of a Great Circle from the Great Circle equation.

However, I now think that I have found the examiners' model answer. Flight Crew Licensing used to be national – i.e., totally under the control of the CAA, who set the syllabus and ran the exams. Now it is European; there is a Joint Aviation Authority of which the UK is a member, but does not control. The JAA exams include questions from all over Europe and this particular one was set by the French.

We had a student who had been through the Belgian system here recently and he told me that it is a normal part of the French ATPL syllabus to teach calculation of cross-track RL/GC difference. "*I can't see why, because I cannot imagine any airline pilot ever needing this knowledge in any practical situation, but there you are – I'm not*

French!'' He also gave me the formula they teach. It is approximate, but over reasonably short distances gives an accuracy of less than a mile – which is close enough for exams. The formula is:

$$\textit{Cross-track distance} = \textit{Rhumb Line distance} \times \textit{Conversion Angle}/230$$

Looking through it, I realised that it is simply Method 1 in my original article, which I worked out from first principles, reduced to a formula. In Method 1, I found the distance to the half-way point (which was 150 nm in that example). The Great Circle distance would have been more accurate but the RL distance gives a reasonable approximation. So my figure was half the total RL distance. I then multiplied by half the conversion angle. At small angles (which is what we are considering) half the sine of x is very close to the sine of half x. Substituting these changes into Method 1, we get:

$$\textit{Cross-track distance} = \tfrac{1}{2}\ \textit{Rhumb Line distance} \times \tfrac{1}{2}\ \textit{sine Conversion Angle}$$

which is:

$$\textit{Cross-track distance} = \textit{Rhumb Line distance} \times \textit{sine Conversion Angle}/4$$

Using the '1 in 60 rule' or, more precisely, the '1 in 57·3 rule' (because this is a more accurate conversion of degrees to radians), I can remove the ***sine*** in the above equation and replace it with ***1/57·3***. This now makes the equation:

$$\textit{Cross-track distance} = \textit{Rhumb Line distance} \times \textit{Conversion Angle}/229{\cdot}2$$

Obviously, 230 is a simpler number to remember and is acceptably accurate and the problem can easily be solved in 2 minutes – if you've been shown the formula!!! It would be nice, if the JAA are going to set questions not normally taught in the UK syllabus, for them to give us a set of teaching references!