THE SOLUTION OF OVERLAPPING PLATES BY ITERATION

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The recomputation of the plate constants for an astrographic zone requires a general catalogue of the positions of all of the comparison stars. Then on each plate the ideal coordinates, X_{ik} and Y_{ik}, of each star, insofar as they depend upon its individual position in the comparison star catalogue, are given by

$$D = \cos \Delta\delta - \cos \delta_* \cos \delta_o \ (1 - \cos \Delta\alpha)$$
$$D\,Y = \sin \Delta\delta + \cos \delta_* \cos \delta_o \ (1 - \cos \Delta\alpha)$$
$$D\,X = \sin \Delta\alpha \cos \delta_*$$

where $\Delta\alpha$ and $\Delta\delta$ are the differences from the plate center, and $\underline{i}$ and $\underline{k}$ refer to the $\underline{i}$th star on the $\underline{k}$th plate. Let p_{kn} and q_{kn} represent the plate constants of the $\underline{k}$th plate and the $\underline{n}$th polynomial term. The equations of condition (EQCD) for the least squares solution of the plate constants are then of the form

$$\sum_n p_{kn} \omega_i^{(n)} = X_{ik} \quad \text{and} \quad \sum_n q_{kn} \omega_i^{(n)} = Y_{ik} \qquad (1)$$

where ω_i represents the plate measures, x_i and y_i, the diameter, d_i, etc.

For example (omitting $\underline{i}$ and $\underline{k}$)

$$p_0 + p_1 x + p_2 y + p_3 d + p_4 d x = X.$$

We shall refer to these EQCD as Type "O" (Original).

After the least squares solutions are completed, we may substitute into Equa. (1) and determine new values of X and Y for each star, which now depend upon the whole system of the comparison star catalogue in this neighborhood of the sky and upon the individual plate measures of the $\underline{i}$th star. When these values are used in Equa. (1) we shall call them Type "C" (Computed). To repeat the least squares solution with all the EQCD of Types "O" and "C" would be entirely redundant, as it would yield exactly the same values of the plate constants as did the first solution. However, there are now Type "C" values of $X_{ik'}$ and $Y_{ik'}$ available for the same star on the overlapping $\underline{k}'$th plate. The vector, $X_{ik'} \bar{i}_{k'} + Y_{ik'} \bar{j}_{ik'} + 1\bar{k}_{k'}$, referred to the $\underline{k}'$th plate center may be transformed to the vectorial coordinate system of the $\underline{k}$th plate center by means of an orthogonal rotation matrix. The resulting vector $\overline{V} = V_x \bar{i}_k + V_y \bar{j}_k + V_x \bar{k}_k$ must be reduced to the plane of the $\underline{k}$th plate by division by V_z, giving $X_{ik} = V_x/V_z$ and $Y_{ik} = V_y/V_z$.

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The EQCD using these values of X_{ik} and Y_{ik} will be called Type "P" (Projection or overlap). These latter EQCD impose the conditions that the ideal coordinates of each ith star as given by the plate constants of the kth plate will agree with the position given by the plate constants and measures of the ith star on the overlapping k'th plate. Thus the eventual solution for the plate constants from all the EQCD of Types "O", "C", and "P" will yield the exactly equivalent result as would a massive, simultaneous solution by the overlap method.

The simplicity of operation and programming, and the reduction of storage capacity requirements in the computer at any one time, are obvious. All the calculations with respect to the left hand side of all the EQCD are done only once. The convergence of the successive solutions should offer no serious problem, because the EQCD of Type "O" remain constant, those of type "C" tend to reinforce the solution from the previous iteration, and only those of Type "P" may be expected to vary from one iteration to the next and to induce oscillations. Furthermore the residuals of the Type "P" EQCD offer a valuable cross-check against gross errors of the input data. If field stars are introduced for the purpose of strengthening the magnitude terms in the range of fainter magnitudes, they will have EQCD of Type "C" and "P" only.

We propose to exploit this simple iterative method to its fullest, using the Bordeaux zone of the Astrographic Catalogue. All of the necessary input material is already available on magnetic tapes except the AGK 3 and AGK2 revised catalogues of the comparison stars.