DEFINITION AND PRACTICAL REALIZATION OF THE REFERENCE FRAME IN THE FK5 – THE ROLE OF PLANETARY DYNAMICS AND STELLAR KINEMATICS IN THE DEFINITION

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ABSTRACT

The formulation of the fundamental reference system to be represented by the FK5 includes the determination of the equinox and equator on the basis of planetary dynamics and the application of the new expressions for the general precession in longitude adopted in the IAU (1976) System of Astronomical Constants. The role of hypotheses which entered the determination of the lunisolar precession is explained. Results are presented for the equinox and equator of the FK5 which are based on observations of the Sun, planets, and lunar occultations.

CONSTRUCTION OF INERTIAL SYSTEMS AND DYNAMICAL DETERMINATIONS OF THE GENERAL PRECESSION IN LONGITUDE

In assuming that the motions in the solar system are governed by the gravitational forces of its members only – i.e., that no external forces have to be taken into account – the law of gravitation allows one to determine inertial planes and directions. The orbital planes, their inclinations and nodes, and the directions to the perihelia have – aside from secular perturbations which can be computed – fixed (inertial) positions and may be employed for the formulation of an inertial system. Anding (1905) has shown how a purely dynamical reference system can be constructed. Bauschinger (1922) has followed Anding's ideas in order to find out in how far the star catalogue PGC by Boss (1910) represents an inertial system. He applied the comparisons which Newcomb (1895) had made between the secular variations of the elements of the four inner planets derived from observations and those resulting from the planetary masses, and he took into account the relativistic motions of perihelia predicted by Einstein's theory. Bauschinger came to the following remarkable conclusion: "The star catalogue by Boss can be considered an inertial system in the sense of Newtonian mechanics and of the law of gravitation corrected for the Einsteinian motion of perihelia provided that Newcomb's (1898) precession is corrected by + 0$''\!.$86 and that a secular motion of 1$''\!.$16 of the equinox on the equator with respect to Newcomb's equinox is taken into account" (translated from German).

E. M. Gaposchkin and B. Kołaczek (eds.), Reference Coordinate Systems for Earth Dynamics, 331–340.

It has been known for a long time that precession should be determined from the moments of inertia of the earth's body, but these are not known with sufficient accuracy. Hence, determinations based upon observations of the Sun and planets have been considered of great importance, because they are free of hypotheses. However, the attempts made by Brouwer (1950) and Clemence (1966) on the basis of observations of the Sun, Mercury and Venus failed to yield any reliable corrections; the uncertainty was at least as large as the correction $\Delta p = +1''.12$ per tropical century (to Newcomb's general precession) which resulted from these determinations. A considerable improvement was achieved by Laubscher (1976) who applied the dynamical method to the optical observations of Mars made from 1751 to 1969. These observations were reduced to the system of the FK3 which is very near to the FK4 system in the zodiacal zone. They yielded the following values for 1850

$$\begin{aligned} \Delta p &= +1''.21 \pm 0''.44, \\ \Delta e &= +1''.12 \pm 0''.29, \\ \Delta \varepsilon &= -0''.14 \pm 0''.11; \end{aligned} \tag{1}$$

Δp is the correction to Newcomb's value of the general precession per tropical century, Δe, a correction to the centennial proper motions in right ascension of the FK3, and $\Delta \varepsilon$, a correction to Newcomb's value of the obliquity of the ecliptic for 1850. The errors are standard deviations; they are large in Δp and Δe. In discussions on the revision of Newcomb's precession Laubscher's determination was duely considered. It was not taken into account in the new system of constants, because there remained uncertainty about the effect of correlations between the unknowns as explained by Laubscher himself.

HYPOTHESES INCLUDED IN THE DETERMINATION OF THE GENERAL PRECESSION ADOPTED BY THE IAU

In the IAU (1976) System of Astronomical Constants the new value of the general precession in longitude is the result of a revision of Newcomb's precession in two parts. The change consists of the following components: a correction $\Delta p_1 = +1''.10$ per tropical century at 1900 to Newcomb's lunisolar precession, and a correction to Newcomb's planetary precession arising from the adoption of new values of planetary masses. The correction to the lunisolar precession was determined by Fricke (1967) from proper motions of FK4 stars, and the expressions for precession quantities based upon the IAU (1976) System were developed as a function of the constants by Lieske et al. (1977). The motivating ideas for changing precession were described by Fricke (1977 a), who also presented the basic material for the determination of precession and a review of methods and results (Fricke, 1977 b).

The last mentioned paper contains full information on the assumptions involved in the determination of the lunisolar precession from fundamental proper motions. Before entering a discussion on kinematical hypotheses it should be pointed out that the fundamental proper motion components of each star consist of the following parts:
(a) a parallactic motion of each star due to the solar motion with respect to the stars under consideration;
(b) a rotation due to precessional errors and the rotation of the assembly of stars considered;
(c) a deformation (shear) of the velocity field of the stars leading to terms produced by non-rigid rotation in which the assembly of stars take part; and
(d) a residual motion due to the peculiar motion of the stars and internal errors of the motions.
This list is complete and free of assumptions. The crucial point is the decomposition of the angular rotation vector (ω_1, ω_2, ω_3) resulting from (b). The decomposition involves the hypothesis that the rotation vector describes the accumulative effect of three parts of different origin:
(a) precessional corrections (Δp_1 to the lunisolar precession, $\Delta\lambda$ to the planetary precession);
(b) a common correction Δe to the fundamental proper motions μ_α due to erroneous determinations of the vernal equinox, and
(c) galactic rotation (the rotation parameter Q).

They allow one to determine from the rotation vector $\vec{\omega}$ the following three unknowns

$$\Delta n, \quad \Delta\lambda + \Delta e, \; Q,$$

according to the equations

$$\begin{aligned} \omega_1 &= -\,0.868\, Q \,, \\ \omega_2 &= -\,0.188\, Q - \Delta n \,, \\ \omega_3 &= +\,0.460\, Q + \Delta n \cot \varepsilon - (\Delta\lambda + \Delta e) \,, \end{aligned} \tag{2}$$

where Δn, the correction to the general precession in declination, is related to the correction Δp_1 to the lunisolar precession by $\Delta n = \Delta p_1 \sin \varepsilon$. One notices that not more than three unknowns can be determined from the components of the velocity vector. The proper motions of distant stars in FK4 and FK4 Sup yield the values (per tropical century)

$$\begin{aligned} \Delta p_1 &= +\,1''.10 \pm 0''.15, \\ \Delta\lambda + \Delta e &= +\,1''.20 \pm 0''.16, \\ Q &= -\,0''.23 \pm 0''.06. \end{aligned} \tag{3}$$

The errors are standard deviations. In adopting the correction $\Delta\lambda = -\,0''.03$ to planetary precession determined dynamically by Laubscher (1972) and Lieske et al. (1977) with an accuracy better than $0''.01$, one obtains the centennial corrections

$$\begin{aligned} \Delta p &= +\ 1''.13 \pm 0''.15 \\ \Delta e &= +\ 1''.23 \pm 0''.16 \end{aligned} \qquad (4)$$

to Newcomb's general precession in longitude and to the proper motions in right ascensions of the FK4, respectively. The quantity Δe tells us that all μ_α of the FK4 are too small and that, hence, the equinox of the FK4 (zero point of right ascensions) is in motion with respect to the true (dynamical) equinox.

The decomposition of the angular velocity vector and the formulation of Eqs. (2) is based on the following hypotheses:
(1) no other rotation than those selected in (a) to (c) contributes to the angular velocity vector;
(2) with the coefficients of B the Oort-Lindblad model of galactic rotation was adopted;
(3) even if the decomposition is well-founded, one may suspect that the interpretation of Δe is hypothetical, because Δe may also indicate either a rotation of the assembly of stars under consideration about the earth's axis of rotation or a rotation of the planetary system with respect to the stars;
(4) finally, least-square solutions are based on a Gaussian distribution of the residuals.

In referring to the hypotheses in the order of its numbering the following comments can be given:
(1) no other physically comprehensible or spurious rotation (originating from errors of observation) can be seen;
(2) in an investigation of the same material of proper motions Du Mont (1977) has replaced the Oort-Lindblad model by the three-dimensional method proposed by Ogorodnikov (1932) and Milne (1935) with the result that no additional significant information was found;
(3) the quantity Δe is a correction to the fundamental proper motions in right ascensions. For clarity, the name "non-precessional motion of the equinox" previously also used by the author should be replaced by "a motion of the catalogue equinox (zero point of fundamental right ascensions) with respect to the dynamical (true) equinox". Fricke (1979) has shown that we are dealing with a zero point error of the proper motions. It occured in Newcomb's FC (1899) and was not eliminated later. The zero point error is due to an error (magnitude equation) of observations of bright stars made for the determination of the equinox in the 19th century. An alternative interpretation of Δe proposed by Balakirev (1980) consists in a real rotation of the plane of the ecliptic about an axis lying in this plane. He found that the axis of rotation is nearly identical with the nodal line of the galactic plane on the ecliptic, and that the period of rotation is about 3×10^8 years. Balakirev has conceded that no physical explanation can be given. The arithmetic is correct but the interpretation fantastic in view of the disillusionment by facts that do not allow us to assume that equinox determinations were free of systematic errors.

In this connection it deserves mentioning that Van Woerkom (1943) has investigated a precessional motion of the invariable plane of the planetary system with respect to the galactic plane due to the attractive force of the galaxy on massive planets. The effect turned out so small that it need not be considered.

(4) The residuals of the proper motions, which resulted from the determination of precessional corrections etc., were investigated by Fricke (1967). It was found that no systematic effects can be recognized in the residuals. Because the distances of the stars were approximately known, parallax factors were applicable, and the remaining residuals are not much greater than the errors of the fundamental proper motions. The assumption of an arbitrary distribution of the residual proper motions has often been considered a fundamental hypothesis underlying determinations of precession. This is no longer correct.

DYNAMICAL DEFINITION OF THE EQUINOX AND EQUATOR AND ITS REALIZATION IN THE FK5

As mentioned before, the systems of all fundamental catalogues in the series from Auwers' FC to the FK4 are defined by the dynamics of the planetary system as far as the equinox and equator are concerned. Pioneer work was done by Bessel (1830) who determined the equinox from observations of the Sun made by Bradley from 1750 to 1762 and his own observations made at Königsberg from 1822 to 1835. On this basis Bessel determined the mean and apparent places of 36 "Maskelyne Stars" from 1750 to 1850 presented in his famous "Tabulae Regiomontanae", which was widely used as a fundamental reference coordinate system in the 19th century. Newcomb (1872) has extended Bessel's equinox determination in employing all available observations of the Sun from 1756 to 1869, as far as they were reduced by the observers to give fundamental right ascensions of clock stars. The difference between α(obs) and α(Bessel) turned out to be $+0\overset{s}{.}014$ at 1840, and no significant secular variation was found. Newcomb (1882) made use of his equinox determination in fixing the zero point of right ascensions of equatorial stars in his "Catalogue of 1098 Standard Clock and Zodiacal Stars. This zero point is commonly called Newcomb's equinox N_1.

At the beginning of this century deficiencies of Newcomb's N_1 became apparent from new observations of the Sun and planets carried out with improved transit circles equipped with impersonal micrometers. One noticed that all right ascensions required a negative correction ΔN_1, and Newcomb's error was explained as the consequence of the imperfect methods of observing (eye and ear, key, hand driven travelling threads of micrometers). On the basis of observations of the Sun and planets made with impersonal micrometers a correction $\Delta N = -0\overset{s}{.}050$ was adopted for the FK3. Hence, the equinox of the FK3 is $N_1 - 0\overset{s}{.}050$, and it was adopted independent of

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time. After careful consideration this equinox was maintained in the FK4, although the equator of the FK4 was determined from the observations of the Sun and planets from 1900 to 1958 which indicated a correction $\Delta\delta = -0\overset{''}{.}017$ at 1928 to the equator of the FK3. This correction was applied.

In view of the demand of an improved fundamental catalogue (FK5) and of a new set of high-precision lunar and planetary ephemerides it became clear that in the formulation of the FK5 system every effort has to be made in order to ensure that the FK5 corresponds as closely as possible to the dynamical reference frame. The FK5 equinox and equator have to correspond to the dynamical equinox and to the dynamically determined equator. In the latest status report on the FK5 Fricke (1980) has given detailed information on the methods applied for determination of the equinox and equator and on results so far achieved. Hence, a brief summary may be sufficient here.

(1) Definition of the equinox: The vernal equinox is the average location of the ascending node of the earth's moving mean orbit on the equator. This definition is consistent with the common practice of comparing observations of the Sun with the places computed from the ephemeris, and it is in accordance with Newcomb's definition.

(2) Motion of the FK4 equinox: Three different sources have provided evidence that the correction E to the FK4 equinox increases with time according to

$$E(T) = E(T_o) + \dot{E}(T - T_o) , \tag{5}$$

where $\dot{E}$ is the time derivative of E. The sources are (a) the FK4 proper motions, (b) the observations of lunar occultations, and (c) the equinox determinations $E(T_\nu)$ for different mean epochs T_ν from about 1900 to 1970. They yield the following results for $\dot{E}$:

(a) FK4 proper motions; after application of the new value of the general precession in longitude the centennial proper motions require a correction Δe such that

$$(\mu_\alpha)_{FK4} + \Delta e = (\mu_\alpha)_{dyn} \tag{6}$$

is fulfilled, where

$$\Delta e = \dot{E} = + 1\overset{''}{.}23 \pm 0\overset{''}{.}16 \tag{7}$$

(b) The lunar occultations from 1820 to 1970 analysed by Van Flandern (private communication) have resulted in

$$E(T) = + 0\overset{''}{.}65 + 1\overset{''}{.}31 \ (T - 19.50) , \tag{8}$$

where $\dot{E} = + 1\overset{''}{.}31$ and T is counted in centuries.

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(c) Available are 35 equinox determinations which have yielded discrete values for $E(T_\nu)$ from 1900 to 1970 at the mean epochs of the respective observations. The following list gives information on the objects, the number of values E, and the observatories or authorities.

Objects	Determinations	Observatories or Authorities
Sun	22	Washington (8), Cape (6), Greenwich (4), Breslau (1), Herstmonceux (1), Ottawa (1), Pulkovo (1)
Mercury	1	Duma, Fricke
Venus	1	Duma, Fricke
Mars	3	Duma, Glebova, Niimi
Minor Planets	5	Branham (2), Duma, Kristensen, Orelskaya
Lunar Occultations	2	Morrison, Van Flandern
Various Objects: (optical, radar, laser ranging)	1	Standish

A least-squares solution based on all 35 values of E has yielded

$$\dot{E} = +1\overset{''}{.}28 \pm 0\overset{''}{.}15 \quad \text{(m.e.)} \tag{9}$$

The agreement of the results given in Eqs. (7) to (9) is excellent; it is better than we had expected in view of the independent sources and the differences between the methods of determination of $\dot{E}$. Hence, the mean value of the results (7) to (9), which is

$$\dot{E} = +1\overset{''}{.}276 = +0\overset{s}{.}085 \quad \text{per century} \tag{10}$$

appears to be the most likely value of the correction to the motion of FK4 equinox.

(3) Correction to the FK4 equinox: According to Eq. (5) the value $E(T_o)$ is the correction to all right ascensions of the FK4

$$\alpha_{FK4} + E = \alpha_{dyn} , \tag{11}$$

at the equinox and epoch T_o. In adopting the view that $E(T_o)$ is best determined from the most recent observations of different type , we have employed all equinox determinations $E(T_\nu)$ from 1950.0 to 1970.0 leading to the weighted average value

$$E = +0\overset{s}{.}042 \pm 0\overset{s}{.}003 \quad \text{at} \quad T = 1958.5. \tag{12}$$

From Eqs. (10) and (12) we obtain

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$$E = + 0\overset{s}{.}035 \pm 0\overset{s}{.}003 + (0\overset{s}{.}085 \pm 0\overset{s}{.}010)\ (T-19.50), \qquad (13)$$

where T is counted in centuries. Hence, the zero point correction is $E = + 0\overset{s}{.}035$ at 1950, while previously Fricke (1980) reported the preliminary value $+ 0\overset{s}{.}031$ which resulted from all determinations from 1900 to 1970. From Eq. (13) we conclude that in the transition from FK4 to FK5 the following operations have to be made in order to achieve that the FK5 equinox is "at all times" as nearly as possible identical to the dynamical equinox

$$\begin{aligned} \alpha_{FK4} + 0\overset{s}{.}035 &= \alpha_{FK5} \quad \text{at } 1950.0, \\ (\mu_\alpha)_{FK4} + 0\overset{s}{.}085 &= (\mu_\alpha)_{FK5} , \end{aligned} \qquad (14)$$

where in μ_α the change in the general precession in longitude has to be taken into account. These results were reported in March 1980 to the astronomers engaged in the construction of new lunar and planetary ephemerides.

(4) Definition of the equator and its practical realization: The equator is dynamically defined as the plane of symmetry of the ecliptic in declination. Its determination may most easily be explained in considering observations of the sun only. Such observations will indicate differences $\Delta\delta$ between observed declinations of the Sun and those computed from the ephemeris

$$\Delta\delta = (\delta_\odot)_{obs} - (\delta_\odot)_{comp} . \qquad (15)$$

If the observations are reduced to the FK4 and if the FK4 equator represents the plane of symmetry, the annual mean value of $\Delta\delta$ will be zero. In practice, one will find

$$\langle\Delta\delta\rangle = - D , \qquad (16)$$

where D is the correction which has to be applied to the equatorial declinations of the FK4 in order to find the dynamical equator. This classical method does not take into account that a catalogue equator may be tilted with respect to the dynamical equator as emphasized by Duma (1978). However, a tilt of the FK4 equator must become apparent in the comparisons between absolute observations of equatorial declinations and the FK4. The modern absolute observations (see section 4) don't indicate any significant tilt, a fact, which is plausible, because terms $\Delta\delta_\alpha$ have always formed an essential part in the construction of the fundamental catalogues from the FC to the FK4. Furthermore, the equator point corrections D as determined from all suitable recent observations of the Sun and planets do not indicate a significant change of the position of the FK4 equator. No change is therefore intended in the FK5.

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In future, one should expect that the equator will be determined by absolute radio-interferometric measurements of the declination of extragalactic radio sources. The position of the vernal equinox, however, will remain to rest upon the application of the dynamical method, and the knowledge of the equinox point will be required as long as planetary ephemerides are needed.

EXTENSION OF THE DYNAMICAL SYSTEM TO THE POLES AND TO FAINTER STARS

The extension of the system from the equator to the poles or vice versa is established by absolute or quasi-absolute observations of stars in both coordinates. After the completion of the FK4 about 30 catalogues of observations of this category have become available; they will contribute to the elimination zonal and regional systematic errors of the FK4. The analysis of these catalogues is based on the methods for the determination of systematic differences developed by Brosche (1966) and Schwan (1977) and refined and tested by Bien et al. (1978). The latest paper describes precisely the procedure applied for the FK5. Good progress has been made in the analyses; thus the completion of the FK5 system can be expected in 1982.

For the determination of differential corrections to the positions and proper motions of the FK4, there are many modern catalogues of observation available, and for the extension of the system to fainter stars we exploit all suitable observations of about 50,000 stars (including the stars in FK4 and FK4 Sup) that have become available since about 1900. So far, about 630,000 catalogue positions have entered the data-processing for FK5. Our criteria for selecting new fundamental stars, however, will not allow us to make use of all these observations. The criteria are: (a) reduction to the FK4 must be possible free of errors depending on the magnitude (this criterion sets the magnitude limit $m_V \simeq 9.2$ for the FK5); (b) stars with best observational history; (c) no nearby stars and no visual binaries or doubles; (d) fit into a uniform distribution on the sky and in spectral types.

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