A CORRECTION TO MY PAPER "ON THE NON-COMMUTATIVITY OF PONTRJAGIN RINGS MOD 3 OF SOME COMPACT EXCEPTIONAL GROUPS"

SHÔRÔ ARAKI

This note is a correction of an error of the author's paper mentioned in the title. (The reference [1]). The proof of the Prop. 6 of [1], Chap. II, p. 247, is an error. And the propositions and formulas in pp. 247-249 of [1] depending on this Prop. 6 must be corrected. All notations are referred to [1].

1. We continue the discussion of [1, p. 246]. The singular planes of Q are partially ordered by the ordering of associated planes in P. Give a linear order in Q compatibly with this partial order. Then any subsequence Q_k of length k gives a $2k$-dimensional sub E_6-cycle $\Gamma(Q_k)$ of $\Gamma(fP)$. The totality of these $2k$-dimensional E_6-cycles forms an additive basis of $H_{2k}(\Gamma(fP): Z)$. The dual cohomology class of $\Gamma(Q_k)$ is $y_{i_1}^{(\varepsilon_1)} \cdots y_{i_k}^{(\varepsilon_k)}$ for $Q_k = \{q_{i_1}^{(\varepsilon_1)}, \ldots, q_{i_k}^{(\varepsilon_k)}\}$ where $\varepsilon_s = 0$ if ρ_{i_s} is a long root of F_4 and $\varepsilon_s = 1$ or 2 if ρ_{i_s} is a short root.

Now the Prop. 6, Chap. II of [1], must be corrected as follows:

Proposition 6. *The $2k$-cycles $f_P\Gamma(P)$ and*

$$\sum x_{i_1} \cdots x_{i_k} (\Gamma(P)) \cdot \Gamma(\{q_{i_1}^{(\varepsilon_1)}, \ldots, q_{i_k}^{(\varepsilon_k)}\})$$

represent the same class in $H_{2k}(\Gamma(fP): Z)$, where the summation runs over all subsequences $\{q_{i_1}^{(\varepsilon_1)}, \ldots, q_{i_k}^{(\varepsilon_k)}\}$ of length k of Q.

Since $f_P^*(y_{i_1}^{(\varepsilon_1)} \cdots y_{i_k}^{(\varepsilon_k)}) = x_{i_1} \cdots x_{i_k}$ by (11) of [1, p. 246], a standard argument proves this proposition immediately. The crucial of the erroneous statement of the Prop. 6 lies in what the author had overseen that the $2k$-cohomology class such as $(x_2)^2 x_3 \cdots x_k$ is not necessarily zero in general.

The Prop. 7 of [1, p. 547] should be corrected as follows:

Proposition 7. *$\Omega f_*(P_*) = \sum x_{i_1} \cdots x_{i_k}(\Gamma(P)) \cdot \{q_{i_1}^{(\varepsilon_1)}, \ldots, q_{i_k}^{(\varepsilon_k)}\}_*$ where Ωf_* denotes the homology map induced by Ωf and the summation is the same as in*

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the Prop. 6.

The proof is entirely the same as in the proof of the Prop. 7 of [1, p. 247].

2. The formula (12) of [1, p. 247] is correct as is easily seen from the corrected Prop. 7.

The formulas (12′) and (12″) of [1, p. 248] are incorrect. If we compute by making use of the Prop. 4.2 of [2, Chap. III], then we see that the cohomology rings $H^*(\Gamma(P_5^1(F_4)) : Z)$ and $H^*(\Gamma(P_5^2(F_4)) : Z)$ have the relations

$$(*) \qquad x_1^2 = 0, \qquad x_2(x_1 + x_2) = 0$$

among others, and the cohomology fundamental classes are $x_1x_2x_3x_4x_5$ for both rings. Then the corrected Prop. 7 and the relations (∗) prove the following corrections of the formulas (12′) and (12″):

(12′)

$$\Omega f_*(P_{5*}^1(F_4)) = P_{5*}^1(E_6) + P_{5*}^2(E_6) - P_{5*}^3(E_6) + \{(\mu_4', 1), P_4^1\}_* + \{(\mu_4, 1), P_4^2\}_*$$

$$(13') \qquad \Omega f_*(P_{5*}^2(F_4)) = \{(\mu_3 - \varphi_3', 1), P_4^1\}_* + \{(\mu_3 - \varphi_3', 1), P_4^2\}_* - P_{5*}^3(E_6).$$

The same argument as in pp. 248–249 above the Prop. 8 of [1], with the corrected (12′) and (12″), prove the following corrected formulas of (13) and (14):

$$(13) \qquad \Omega f_*(P_{5*}^1(F_4)) = P_{5*}^1(E_6) + P_{5*}^2(E_6) + P_{5*}^3(E_6),$$

$$(14) \qquad \Omega f_*(P_{5*}^2(F_4)) = P_{5*}^3(E_6).$$

The Prop. 8 of [1, p. 249] is exact and the Prop. 8′ is false as is easily seen from the formula (12) and the corrected formulas (13) and (14). We can state the Prop. 8 in a more stronger form as follows:

PROPOSITION 8″. *The homology map* Ωf_* *is injective in* deg $\leqq 10$ *for any coefficients.*

In the discussion in Chap. III of [1] only the Prop. 8 is refered from pp. 247–249 so that no more related corrections are needed.

3. We can prove the above proposition in its most general form.

The diagram of the symmetric space E_6/F_4 is of type A_2 and all roots have multiplicity 8 ([3], p. 422). The K-cycles of [2] describing the additive basis of $H_*(\Omega(E_6/F_4); Z_2)$ are all iterated 8-sphere bundles over 8-spheres, whence in particular orientable. Then $H_*(\Omega(E_6/F_4); Z)$ has no torsion and $H_i(\Omega(E_6/F_4); Z)=0$ if $i\not\equiv 0 \pmod 8$ by [2].

The spectral sequence associated with the fibration $\Omega(E_6)\to\Omega(E_6/F_4)$ (fibre $\Omega(F_4)$) is collapsed for any coefficients since odd degree homologies vanish for all three involved homology groups. Hence $\Omega(F_4)$ is totally non homologous zero in $\Omega(E_6)$ for any coefficients, i.e., we obtained the

PROPOSITION. *The homology map* $\Omega f_*: H_*(\Omega F_4: G)\to H_*(\Omega E_6: G)$ *is injective in all degrees and for any coefficient group* G.

A related question will be discussed elsewhere.

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Kyusyu University.