Remarquons que l'accord des résultats déduits des galaxies et des étoiles du GC est en moyenne tout-à-fait satisfaisant. Les résultats entre des clichés et des galaxies pris séparément sont naturellement d'un accord moindre.

Plusieurs conclusions peuvent être tirées de ces investigations. Il est évident, en premier lieu, que pour assurer une haute précision il est indispensable d'avoir 4 ou 5 paires de clichés pour chaque champ. L'élimination de l'erreur dite 'équation de magnitude est obligatoire pour les étoiles du catalogue. La durée d'exposition des clichés ne doit pas dépasser une heure. Il est recommandé de choisir des étoiles de magnitude photographique 12 à 14 comme étoiles de repère pour la détermination des constantes du cliché et pour le rattachement statistique des galaxies aux étoiles du catalogue. Ces étoiles de repère doivent être choisies dans les environs des galaxies et des étoiles du catalogue.

Des astrographes à champ visuel de 25 degrès carrés et plus, ainsi que des instruments du type de l'astrographe normal, que beaucoup de pays possèdent, pourraient servir à ce travail. L'accroissement du nombre des instruments permettrait de diminuer la différence des époques des clichés et accélèrerait l'achèvement de l'œuvre. Il est essentiel aussi que, dans les champs contenant les galaxies, soient déterminés les types spectraux, les couleurs et les grandeurs stellaires des étoiles de repère. Comme nous l'avons déjà dit, il est absolument indispensable d'étendre cette activité à l'hémisphère austral.

LITTERATURE

(1) *H.A.* **88**, no. 2, 1932.
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10. THE PRECISION OF THE DETERMINATION OF STAR MOTIONS WITH RESPECT TO THE EXTRA-GALACTIC NEBULAE

By C. D. Shane and S. Vasilevskis

The programme of proper motions with the 20-inch astrograph of the Lick Observatory was conceived and planned by Dr W. H. Wright, formerly Director of the Lick Observatory. Not only did he formulate the work in considerable detail but he obtained the grant to purchase the telescope and planned it as a precision instrument admirably adapted to its purpose. A paper published in the *Proceedings of the American Philosophical Society*, Vol. **94**, no. 1, 1950, describes the programme as formulated by Dr Wright.

At the time of Dr Wright's retirement the telescope was still in the process of adjustment and M. Shane, with the assistance of M. Wirtanen, undertook to obtain the first series of photographs. Of 1246 fields north of declination $-23°$, all but about 100 have now been photographed. The plates are centred 5° apart in declination and 5° or less in right ascension. The magnitude of the faintest detectable star is 19 while nebulae are recorded as identifiable objects to magnitude 18·3. All plates are taken with a grating that gives first-order spectra four magnitudes fainter than the central images. In addition, there is on each plate a one-minute exposure displaced about one minute of arc in right ascension from the main exposure of two hours' duration. The one-minute exposure together with the grating images provide means to connect stars over a range of eight or more magnitudes without using over-exposed images.

Recently M. Vasilevskis has undertaken a study of the precision one may expect in proper motions determined from these plates. Since the observatory does not have any measuring engine that will accommodate the 17×17-inch photographs on the regular programme, a pair of plates 10 inches square was taken of each of two selected areas, Nos. 15 and 59. The results presented in this report are based on his measures of these plates.

On each pair of plates, approximately 25 faint nebulae having well-defined images were measured. Approximately 25 stars of average magnitude 12·5 were likewise measured. The star measures were made on the first-order grating images of the two-hour exposures. We shall call this co-ordinate system, 'System No. I' In addition, the central images of the stars were measured on the one-minute exposures, with the co-ordinates designated 'System No. II' The two co-ordinate systems were related to one another by means of the linear six-constant formula. The probable errors indicate that the mean of 25 stars can be transferred from one system to another with a probable error of approximately 0ʺ.02 in each co-ordinate.

The reduction constants between the two plates of each selected area were calculated from the measures for the nebulae and stars separately. These constants were in excellent agreement. If we compare the four sets of reduction elements (two in x and two in y) we find a probable error in the displacement between 25 stars and 25 nebulae as measured on two plates, of 0ʺ.02. If, on the other hand, we use the calculated probable errors of the reduction elements, the probable error of the displacement should be 0ʺ.05. This appears to be a case in which the external probable error was less than the one derived internally. If we take the larger value, 0·05, for the probable error in measuring the displacement on a plate between 25 nebulae and 25 stars, two plates taken eight years apart should yield a probable error in relative motion equal to 0ʺ.006 which is the order of magnitude of the galactic rotation. On this basis, it can be seen that 100 pairs of plates with a time interval of 25 years should give a value of the galactic rotation 30 times its probable error. This conclusion omits the effect of galactic latitude but if the plates were taken in latitude 60°, the accuracy would be reduced only by one-half. On the other hand, it seems probable that at least 50 nebulae with suitable images can be found on the average plate, thus decreasing the error by $\frac{1}{\sqrt{2}}$. It thus appears that with the large number of fields at our disposal and an interval of twenty or more years, a very satisfactory determination of the galactic rotation can be made.

If we consider the errors in the proper motions of bright stars, about 8·5 magnitude, we note that the probable error in the direct comparison between two plates is 0ʺ.11 Since, however, the first-order grating images of the one-minute exposures must be measured, we must take account of a probable error of 0ʺ.03 for the bridge between co-ordinate Systems Nos. I and II, and 0ʺ.04 for the reference system of the nebulae. We thus get a probable error of 0ʺ.12 for an individual difference in position for a bright star. In 25 years the proper motion should thus be determined with a probable error of 0ʺ.005

The proper motion of a medium bright star, magnitude about 12·5, can be determined with higher accuracy. Each star would be measured twice, once on the one-minute exposure and its first-order images on the two-hour exposure. Taking account of the increased number of measures and applying the error of the bridge to the one-minute exposure measures, we find a probable error in relative position of 0ʺ.09. The proper motion determined over an interval of 25 years should thus be 0ʺ.0036. Faint stars in the magnitude range 16 to 17 would have probable errors between those for the bright and medium stars. Only one image would be measured but these stars would be connected directly with the nebulae and with no error introduced by the bridge. Although one purpose of the programme is to eliminate the effect of systematic errors in the determination of the constant of precession, we have as yet made no study of the accuracy that can be expected. The results of this study may be summarized briefly as follows:

(1) We may hope to measure the galactic rotation with quite satisfactory precision with an interval of 25 years between sets of plates.

(2) Individual proper motions of stars can be referred to the external galaxies yielding, with interval of 25 years, proper motions whose probable errors will be of the order of 0ʺ.004 or 0ʺ.005.

Tables 1 and 2 contain respectively the constants with their probable errors for the reduction from System No. II to System No. I and plate constants with their probable errors for the reduction from one plate to another one of the same field.

https://doi.org/10.1017/S0251107X00032909 Published online by Cambridge University Press

Table 1

Plate constants with their probable errors for reduction from System II to System I

Plate no.	*x* (*a*)		(*b*)		(*c*)		P.E. of U.W.
	″		″		″		″
1	−0·03 ± 0·02		+2·33 ± 0·02		−52·16 ± 0·01		±0·07
2	+0·01	3	+2·27	3	−49·80	2	11
3	−0·06	2	+0·79	2	−51·84	2	7
4	+0·03	3	+0·80	3	−51·62	2	9

Plate no.	*y* (*d*)		(*e*)		(*f*)		P.E. of U.W.
	″		″		″		″
1	−2·35 ± 0·04		+0·03 ± 0·04		+0·68 ± 0·03		±0·14
2	−2·25	3	+0·05	3	+0·26	2	11
3	−0·83	4	−0·01	4	+0·52	2	11
4	−0·75	3	+0·06	3	+0·18	2	10

(Scale and orientation constants are to be multiplied by *x* and *y* in units of 100 mm.)

Table 2

Plate constants with their probable errors for reduction from one plate to another one of the same field

SA 15

Objects	*x* (*a*)		(*b*)		(*c*)		P.E. of U.W.
	″		″		″		″
Nebulae	−1·12 ± 0·08		+0·36 ± 0·08		−14·42 ± 0·06		±0·26
Stars	−1·08	3	+0·38	3	−14·46	2	11

Objects	*y* (*d*)		(*e*)		(*f*)		P.E. of U.W.
	″		″		″		″
Nebulae	+0·14 ± 0·07		−1·10 ± 0·07		−14·59 ± 0·05		±0·22
Stars	+0·13	3	−1·04	3	−14·62	2	11

SA 59

Objects	*x* (*a*)		(*b*)		(*c*)		P.E. of U.W.
	″		″		″		″
Nebulae	+0·84 ± 0·06		−0·35 ± 0·06		−18·81 ± 0·04		±0·18
Stars	+0·80	2	−0·27	2	−18·80	2	7

Objects	*y* (*d*)		(*e*)		(*f*)		P.E. of U.W.
	″		″		″		″
Nebulae	+0·45 ± 0·06		+0·93 ± 0·06		−1·90 ± 0·04		+0·18
Stars	+0·50	5	+0·86	5	−1·92	3	15

(Scale and orientation constants are to be multiplied by *x* and *y* in units of 100 mm.)

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Appendix

In addition, two plates from S.A. 35 have been measured and reduced.

Reduction from System II to System I

	x			
Plate no.	(*a*)	(*b*)	(*c*)	P.E. of U.W.
	″	″	″	″
5	−0·02 ± 0·02	+1·42 ± 0·02	−51·13 ± 0·01	± 0·07
6	−0·10 2	+1·37 2	−52·34 1	6

	y			
Plate no.	(*d*)	(*e*)	(*f*)	P.E. of U.W.
	″	″	″	″
5	−1·37 ± 0·03	+0·03 ± 0·03	+0·23 ± 0·02	± 0·09
6	−1·47 2	+0·09 2	+0·10 1	7

Reduction from one plate to another one of the same field

	x			
Objects	(*a*)	(*b*)	(*c*)	P.E. of U.W
	″	″	″	″
Nebulae	+1·10 ± 0·05	+0·09 ± 0·05	−11·83 ± 0·03	± 0·17
Stars	+1·11 3	+0·03 3	−11·82 2	9

	y			
Objects	(*d*)	(*e*)	(*f*)	P.E. of U.W.
	″	″	″	″
Nebulae	−0·24 ± 0·03	+1·04 ± 0·03	+12·36 ± 0·02	± 0·09
Stars	−0·31 2	+1·04 2	+12·39 1	7

The results confirm, on the whole, the conclusions drawn from results of Tables 1 and 2. A smaller probable error in measurements of nebulae may be due to a more careful selection of nebulae to be measured.

Prof. Zverev commented on the letters which were received by him in the process of organizing the symposium. He proposed to read communications by Profs. Hertzsprung and Lundmark.

Letter from Prof. Hertzsprung

For this purpose I suggest, as it has already been done by W H. Wright, to use coarse gratings of fine wires placed in front of the telescope, thus producing faint diffraction images of the stars. Wright has planned to make the connexion in *two* steps, owing to the large difference in magnitude between the nebulae and the stars in question. I think it would be possible to do it in *one*, provided photovisual plates behind a yellow screen are used. The part of the prismatic spectrum, active photovisually, is so narrow, that the haze, caused by the secondary spectrum of the central image of the star, will not affect the faint diffraction images seriously. The photovisual images are clear cut in comparison with ordinary photographic images. They may even be used in connexion with a photographic refractor! The extra-galactic nebulae are rather yellow and the modern colour-plates are so sensitive, that the exposure time will not be excessive. Refraction troubles are less photovisually than photographically especially concerning the difference in effective wave-length between objects of different colour.

I should therefore much prefer the photovisual way of working to the old photographic one.

Letter from Prof. K. Lundmark

With regard to the letter from Prof. M. S. Zverev to the members of the I.A.U., I would like to present a few remarks concerning the problem of referring star to anagalactic nebulae (or, rather, certain parts of such objects, be it the central nucleus or some apted secondary nuclei).

The problem was considered by me in 1919 as soon as my work concerning the Andromeda nebula and some other nearby galaxies, with use of novae and the brightest dissolved stars as two independent distance indicators, made it very likely that the nebulae in question were distant galaxies. At that time all existing accurate nebular positions were collected and discussed for 83 objects, and 'quasi-proper motions' derived in the usual way. In 1927 the results were published in my paper: 'Studies on Anagalactic Nebulae' Together with an apex solution, the general rotation of the Galaxy and the improvement to the precession constant, were sought.

After that time much work has been put in in order to reduce all the existing accurately determined positions of nebulae to a final catalogue.

From around 1860 to 1910 a number of catalogues giving positions of nebulae (or nebular parts) were published. As a rule, those catalogues gave positions rendered by micrometric means. Sometimes meridian circles or photographic means were used for deriving nebular positions.

The foremost among these contributions is the *Catalogue of Nebulae* by G. Bigourdan (six volumes). That wonderful source has, in fact, been published as a journal of observation.

Now that some 5600 objects have been observed by Bigourdan during some 30 years on different occasions, it is easily understood that the reduction of such a collection of observations will involve considerable work.

Most of that work has now been performed at the observatory at Lund under my supervision. I have also enjoyed considerable help from Dr Nils Hansson and M. Thorild Dahlgren, M.A.

The reduction of the other 'nebular catalogues' is near its completion.

Thus we shall in the near future possess a catalogue of accurate positions of anagalactic objects, maybe some 2000, reduced to a system. I regret that I am not in the position right now to give more accurate data.

A considerable number of these objects—maybe 500 or even more—have been observed on different occasions and by different observers and methods; their positions will be used in the same way as when determining actual proper motions. The proper motions so found will show a reflex of the p.m. for the comparison stars. But they will also contain corrections to the assumed values of galactic rotation as well as a possible improvement to the constant of precession.

Going over to the determination of positions of anagalactic objects by photography, the need of positions of intermediate stars of 11^m–14^m should be emphasized. Prof. Zverev mentions the need of such determinations for 12^m–13^m stars. For many reasons, not to be given here, I move that such a catalogue of intermediate stars should embrace the magnitudes (photographic) from 11^m, or rather $10^m{\cdot}5$, down to 14^m.

Letter from A. N Vyssotsky

On the Reference of the Motions of Faint Stars to a Fundamental System

In the past the proper motions of faint stars have been referred to a fundamental system of absolute motion by differential measurements with respect to stars with known absolute motions. It is now proposed to follow a very different procedure: namely to measure their motions with respect to extra-galactic nebulae.

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Our work on proper motions at McCormick has emphasized the very great importance of an adequate separation of unknowns in the final solutions. With this in mind, I have formulated a sample set of observation equations uniformly distributed over the sky from the north pole to declination $-20°$, with the exception of areas which fall in Hubble's 'zone of avoidance' This is the approximate coverage of the sky contemplated in the current Lick programme.* Each of my observation equations represents an area about 20 degrees square and altogether there are sixty such regions.

An examination of the consequent normal equations is enlightening.

Table 1

Coefficients of normal equations for observations in right ascension

$+36.00\,X$	$-5.32\,Y$	...	$+1.34\,P$	$+3.64\,Q$
	$+59.37\,Y$	...	$+2.31$	-6.69
		...	...	...
			$+7.09\,P$	-1.58
				$+11.57\,Q$

Coefficients of normal equations for observations in declination

$+13.09\,X$	$+1.30\,Y$	$+6.57\,Z$	$+2.68\,P$	$+0.86\,Q$
	$+9.16\,Y$	$+1.14$	-0.42	-7.02
		$+73.70\,Z$	$+9.33$	$+3.97$
			$+9.61\,P$	$+3.16$
				$+15.41\,Q$

Coefficients of combined normal equations for all latitude zones

$+49.09\,X$	$-4.02\,Y$	$+6.57\,Z$	$+4.02\,P$	$+4.50\,Q$
	$+68.53\,Y$	$+1.14$	$+1.89$	-13.71
		$+73.70\,Z$	$+9.33$	$+3.97$
			$+16.70\,P$	$+1.58$
				$+26.98\,Q$

Naturally, the right-hand sides of the equations in Table 1 are left blank. The impossibility of observing extra-galactic nebulae in low latitudes is responsible for the relatively small coefficients of Oort's terms of galactic rotation, P and Q. We see that in both right ascension and declination the coefficients of Q are correlated with the coefficients of Y

Furthermore, the weight of P from the right ascension equations is low and in the declination equations the coefficients of this term are strongly correlated with the coefficients of Z. In other words, in respect to both P and Q, the equations are somewhat indeterminate. Of course, if we were sure of the secular parallax this would be of small consequence. But the secular parallax of faint stars in various magnitude and latitude groups is certainly not sufficiently well known to warrant any assumption which would be devoid of doubt.

It should be noted that the normal equations in Table 1 are suitable for stars of the eleventh magnitude. They will differ to some extent for stars of other magnitudes because of the necessity of using weights which make allowance for the increasing dispersion in the motions with increasing latitude, and also because the coefficients of X, Y and Z must be multiplied by factors, called β-factors, to allow for the variation in secular parallax with latitude. The precise values of the β-factors adopted here are based on the proper-motion investigations of the McCormick observers. This reliance on the results of previous proper-motion work is, of course, inadmissible in a strictly fundamental investigation.

Uncertainty in the appropriate values of the β-factors really introduces several additional unknowns into the normal equations, although they do not appear explicitly. Incidentally, even if there were no variation in secular parallax with latitude so that no

* W. H. Wright, *Proc. Amer. Phil. Soc.* **94**, 1, 1950.

https://doi.org/10.1017/S0251107X00032909 Published online by Cambridge University Press

β-factors would be required, the correlations between the coefficients of P and Z and those of Q and Y would still be very serious indeed. Furthermore, an examination of the computations indicates that for the determination of P and Q there is very little of value contributed by observations in latitudes higher than 40°; this may be seen from the normal equations in Table 2.

Table 2

Coefficients of combined normal equations for low-latitude zones

$+28.18\,X$	$+0.68\,Y$	$-3.27\,Z$	$+2.31\,P$	$+4.16\,Q$
	$+23.42\,Y$	-1.23	$+0.66$	$+10.64$
		$+28.49\,Z$	$+6.91$	$+4.31$
			$+12.85\,P$	$+1.62$
				$+22.04\,Q$

Of course, if the eleventh magnitude stars are to be used purely as an intermediate step in the determination of the motions of seventh magnitude stars, then the indeterminateness will disappear, since the secular parallaxes of the brighter stars are well known. However, in this case, the limited number of these brighter stars together with their much greater 'cosmical errors' will impose very great limitations on the precision of the determinations of P and Q. Thus, we could not possibly obtain probable errors for P and Q smaller than $\pm 0\overset{''}{.}0006$ and $\pm 0\overset{''}{.}0005$ respectively. Furthermore, it would be impossible, without complete coverage of the sky in my opinion, to disentangle the systematic errors of the GC from the precession constants.

To avoid the indeterminateness just pointed out, observations with a similar instrument in the southern hemisphere are vitally needed. These would eliminate the interlocking terms and consequently would yield reliable values of P and Q, provided that systematic errors caused by guiding errors, changes in collimation between epochs, etc., can be avoided. Moreover, a much better check concerning the possible presence of such errors is afforded if the entire sky is covered.

The components of the secular parallax of the faint stars in the various zones of galactic latitude would in this case also be well determined by a combination of the corresponding zones in the north and south. However, we should still be unable to derive the corresponding mean distances unless the variation in the solar motion with increasing distance of reference stars from the galactic plane is better established than it now is. At present our chief information on this point comes from the radial velocities and absolute magnitudes of the eleventh magnitude K stars obtained by Prof. Edmondson of Indiana University. Obviously, it is desirable that this work be extended to include other spectral classes. In this way the motions of faint stars referred to the extra-galactic nebulae can finally be used for a derivation of the parallaxes of the stars.

Dr Nemiro's final remarks:

There exists a general opinion that astrometry is a routine branch of astronomy. Our symposium, as it seems to me, proved that in astrometry we meet with deep ideas and with tasks of great present interest.

Our symposium is over. We have discussed many problems and received interesting information on the important investigations which are in progress.

I am very glad that we all are unanimous on what is our main aim—to carry on investigations of positions and proper motions of faint stars. The disaccordances concern only details. It is especially important, that besides the work which is of interest for our time, we take into consideration the future development of our science.

I would like to express thanks to all the participants of our symposium and especially to the reporters for their pertinent and interesting communications and to the speakers too.

It is true that the symposium initiated the scientific contact between us and no doubt will stimulate the success of our science and peaceful collaboration between peoples all over the world.

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