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# RESOLVABILITY PROPERTIES OF SIMILAR TOPOLOGIES

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**Abstract**

We demonstrate that many properties of topological spaces connected with the notion of resolvability are preserved by the relation of similarity between topologies. Moreover, many of them can be characterised by the properties of the algebra of sets with nowhere dense boundary and the ideal of nowhere dense sets. We use these results to investigate whether a given pair of an algebra and an ideal is topological.

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## 1. Introduction

The notion of similarity between two topological spaces was introduced in [5], although the same relation was mentioned earlier in [16] as a $\pi$-relation. Among other results, [5] shows that every topology is similar to some abstract density topology.

The notion of resolvable space was introduced by Hewitt [12] in 1943. Since then many authors have examined properties connected with resolvability. The definitions of maximal resolvability [8] and extraresolvability [10] were introduced. In particular, maximal resolvability and extraresolvability of some density topologies were demonstrated in [7, 13, 15, 17] and topologies with very 'bad' resolvability properties were examined, in [1, 2, 18].

In the present work we show that many properties connected with resolvability are preserved by the relation of similarity. We characterise some of these properties in terms of the algebra of sets with nowhere dense boundary and nowhere dense sets. We also give the reason why the analysis of resolvability of abstract density topologies in some sense exhausts all the possibilities.

In the last part of the paper we use the results to give a partial characterisation of pairs $(\mathcal{A}, \mathcal{I})$ of an algebra and an ideal of sets that can be obtained for some topology as families of sets with nowhere dense boundary and nowhere dense sets, respectively. This problem has been investigated previously in [3, 4, 6].

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## 2. Preliminary facts

**2.1. Similarity.** For an arbitrary topological space $(X, \mathcal{T})$ let us denote by $\mathcal{NI}(X, \mathcal{T})$, $\mathcal{ND}(X, \mathcal{T})$, $\mathcal{NB}(X, \mathcal{T})$, $\mathcal{D}(X, \mathcal{T})$ the families of sets with nonempty interior, nowhere dense sets, sets of nowhere dense boundary and dense sets, respectively.

Recall that the set $A$ is *semi-open* in $(X, \mathcal{T})$ if $A \subset \mathrm{Cl}(\mathrm{Int}(A))$. We will denote the family of semi-open sets of the space $(X, \mathcal{T})$ by $\mathcal{SO}(X, \mathcal{T})$.

Let $\mathcal{F}, \mathcal{G} \subset 2^X \setminus \{\emptyset\}$. We say that $\mathcal{F}$ is coinitial to $\mathcal{G}$ if for every $G \in \mathcal{G}$ there exists $F \in \mathcal{F}$ such that $F \subset G$.

Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be two topologies on the space $X$. We say that they are *similar* when $\mathcal{NI}(X, \mathcal{T}_1) = \mathcal{NI}(X, \mathcal{T}_2)$. We shall denote the relation of similarity by $\simeq_s$.

The relation of similarity was investigated in [5]. In that paper one can find the following useful characterisation.

THEOREM 2.1. *The following statements are equivalent:*

- $\mathcal{T}_1 \simeq_s \mathcal{T}_2$;
- $\mathcal{D}(X, \mathcal{T}_1) = \mathcal{D}(X, \mathcal{T}_2)$;
- $(\mathcal{NB}(X, \mathcal{T}_1), \mathcal{ND}(X, \mathcal{T}_1)) = (\mathcal{NB}(X, \mathcal{T}_2), \mathcal{ND}(X, \mathcal{T}_2))$;
- $\mathcal{T}_1 \setminus \{\emptyset\}$ *and* $\mathcal{T}_2 \setminus \{\emptyset\}$ *are mutually coinitial.*

It is a simple observation that for every topological space $(X, \mathcal{T})$ the family $\mathcal{NB}(X, \mathcal{T})$ forms an algebra of sets and $\mathcal{ND}(X, \mathcal{T})$ is an ideal contained in $\mathcal{NB}(X, \mathcal{T})$.

DEFINITION 2.2. Let $\Phi$ be an arbitrary property of topological spaces. We will say that $\Phi$ is $\simeq_s$*-invariant* if and only if, for every space $(X, \mathcal{T}_1)$ having the property $\Phi$, every similar topology $\mathcal{T}_2$ also has the property $\Phi$.

In [14] there are examples of similar topologies that differ significantly; they may have no nontrivial common elements, or different cardinality.

**2.2. Resolvability.** Let $(X, \mathcal{T})$ be an arbitrary topological space. For any cardinal $\kappa$ we say that $(X, \mathcal{T})$ is $\kappa$*-resolvable* if and only if there exists a family of cardinality $\kappa$ of pairwise disjoint dense subsets of $X$. A 2-resolvable space we call simply *resolvable*. If the space is not resolvable, it is called *irresolvable*.

The cardinal number $\Delta(X, \mathcal{T}) = \min\{|G| : G \in \mathcal{T}, G \neq \emptyset\}$ is called the dispersion character of the space $(X, \mathcal{T})$.

A dense-in-itself $\Delta(X, \mathcal{T})$-resolvable space is called *maximally resolvable*. It is obvious that no space can be $\kappa$-resolvable for $\kappa > \Delta$.

The space $(X, \mathcal{T})$ is called *extraresolvable* if there exists a family $\mathcal{D}$ of dense subsets of $X$ such that $|\mathcal{D}| > \Delta(X, \mathcal{T})$ and for every $A, B \in \mathcal{D}$ we have $A = B$ or $A \cap B \in \mathcal{ND}(X, \mathcal{T})$.

The space is called:

- *hereditary irresolvable* (HI) if none of its subspaces is resolvable;
- *open hereditary irresolvable* (OHI) if none of its open subspaces is resolvable;
- *submaximal*, if all its dense subsets are open;
- *NODEC*, if all its nowhere dense subsets are closed.

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Hewitt [12] proved the following theorem.

THEOREM 2.3. *Every topological space $(X, \mathcal{T})$ can be uniquely represented as the disjoint union $X = F_X \cup G_X$ where $F_X$ is closed and resolvable and $G_X$ is open and hereditary irresolvable. This pair of sets is called the Hewitt decomposition of the space $X$.*

**2.3. Abstract density topologies.** Let $\mathcal{A}$ be an algebra and $\mathcal{I} \subset \mathcal{A}$ be an ideal of subsets of the given set $X$. We write $B \sim C$ if and only if $B \triangle C \in \mathcal{I}$. We say that an operator $\Phi : \mathcal{A} \to 2^X$ is a *lower density operator* if:

(a) $\Phi(\emptyset) = \emptyset, \Phi(X) = X$;
(b) for all $A, B \in \mathcal{A}, (\Phi(A \cap B) = \Phi(A) \cap \Phi(B))$;
(c) for all $A, B \in \mathcal{A}, (A \sim B \Rightarrow \Phi(A) = \Phi(B))$;
(d) for all $A \in \mathcal{A}, (A \sim \Phi(A))$ (the analogue of the Lebesgue density theorem).

We say that the pair $(\mathcal{A}, \mathcal{I})$ has the *hull* property if for every set $A \subset X$ there exists a set $H \in \mathcal{A}, H \supset A$, such that for every $P \subset H \backslash A$ if $P \in \mathcal{A}$ then $P \in \mathcal{I}$.

Hejduk and Loranty [11] proved that for every pair $(\mathcal{A}, \mathcal{I})$ having the *hull* property and a lower density operator $\Phi$ on $(\mathcal{A}, \mathcal{I})$, the family

$$\mathcal{T}_\Phi = \{G \in \mathcal{A} : G \subset \Phi(G)\}$$

is a topology. Moreover, $\mathcal{NB}(X, \mathcal{T}_\Phi) = \mathcal{A}$ and $\mathcal{ND}(X, \mathcal{T}_\Phi) = \mathcal{I}$. Every topology that can be constructed in this way is called an *abstract density topology.*

Crossley [9] introduced the operator $\mathcal{F}$. For every topology $\mathcal{T}$ on $X$, the family

$$\mathcal{F}(\mathcal{T}) = \{G \backslash N : G \in \mathcal{T}, N \in \mathcal{ND}\}$$

forms a topology finer than $\mathcal{T}$, similar to $\mathcal{T}$ and NODEC.

By virtue of [14, Corollaries 3.4 and 3.5] we can state the following proposition.

PROPOSITION 2.4. *Let $(X, \mathcal{T})$ be an arbitrary topological space. The following statements are equivalent:*

- *$\mathcal{T}$ is an abstract density topology;*
- *$\mathcal{T} = \mathcal{F}(\tau)$ for some topology $\tau$ on $X$;*
- *$\mathcal{T} = \mathcal{F}(\mathcal{T})$;*
- *$\mathcal{T}$ is NODEC.*

*Moreover for any topology $\tau$ on $X$ the topologies $\tau$ and $\mathcal{F}(\tau)$ are similar.*

## 3. Results

Since two similar topologies have the same families of dense sets and nowhere dense sets and are mutually coinitial, the following notions are $\simeq_s$-invariant:

- $\kappa$-resolvability;
- $\Delta(X, \mathcal{T})$;
- maximal resolvability;

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- extraresolvability;
- being dense-in-itself.

Let $\mathcal{A}$ be an arbitrary algebra. By $\mathcal{H}(\mathcal{A})$ we shall denote the maximal hereditary subfamily of $\mathcal{A}$:

$$\mathcal{H}(\mathcal{A}) = \{A \in \mathcal{A} : \forall_{B \subset A} B \in \mathcal{A}\}.$$

It is easy to see that $\mathcal{H}(\mathcal{A})$ is the greatest ideal contained in $\mathcal{A}$.

Theorem 3.1. *Let $(X, \mathcal{T})$ be an arbitrary topological space and let $(F_X, G_X)$ form the Hewitt decomposition of the space $X$. Then $\mathcal{H}(\mathcal{NB}) = \{A \cup N : A \subset G_X \wedge N \in \mathcal{ND}\}$.*

Proof. '$\supset$'. Let $A \subset G_X$. Suppose that $A \notin \mathcal{NB}$. Then there exists an open set $H \neq \emptyset$, $H \subset \mathrm{Fr}(A)$. Since $A \subset G_X$ and $\mathrm{Fr}(G_X) \in \mathcal{ND}$ then $H \cap G_X \neq \emptyset$. Both sets $A \cap H$ and $A' \cap H \cap G_X$ are dense in $H \cap G_X$, in contradiction to the fact that $G_X$ is HI.

'$\subset$'. Let $B \in \mathcal{H}(\mathcal{NB})$ and $N = B \backslash G_X$. We shall show that $N \in \mathcal{ND}$. Since $N \in \mathcal{NB}$, it is sufficient to show that $\mathrm{Int}(N) = \emptyset$. Suppose that $\mathrm{Int}(N) \neq \emptyset$. Then $\mathrm{Int}(N) \subset F_X$, so $\mathrm{Int}(N)$ is resolvable. Let $A$ and $A'$ be dense in $\mathrm{Int}(N)$. Then $\mathrm{Int}(N) \subset \mathrm{Fr}(A \cap \mathrm{Int}(N))$, hence $A \cap \mathrm{Int}(N) \notin \mathcal{NB}$. This contradicts the assumption that $B \in \mathcal{H}(\mathcal{NB})$. □

Corollary 3.2. *$(X, \mathcal{T})$ is resolvable if and only if $\mathcal{H}(\mathcal{NB}) = \mathcal{ND}$.*

Proof. By virtue of the last theorem, the condition $\mathcal{H}(\mathcal{NB}) = \mathcal{ND}$ implies that $G_X$ is nowhere dense. But $G_X$ is open, hence it is empty. Hence $X = F_X$ is resolvable. The converse implication is obvious. □

Half of this equivalence can be found in [5].

Theorem 3.3. *The following statements are equivalent:*

(1) *$(X, \mathcal{T})$ is OHI;*
(2) *$\mathcal{NB} = 2^X$;*
(3) *$(\mathcal{H}(\mathcal{NB}) \backslash \mathcal{ND})$ is coinitial to $(\mathcal{NB} \backslash \mathcal{ND})$.*

Proof. (1) $\Rightarrow$ (2). Let $(X, \mathcal{T})$ be an OHI space. Then $F_X$ has empty interior, hence it is a nowhere dense set. Thus, for every $A \in 2^X$, $A = (A \cap G_X) \cup (A \cap F_X) \in \mathcal{H}(\mathcal{NB}) \subset \mathcal{NB}$.

(2) $\Rightarrow$ (3). Obvious, since $\mathcal{H}(2^X) = 2^X$.

(3) $\Rightarrow$ (1). Let $G$ be open, $G \neq \emptyset$. Then $G \in \mathcal{NB}(X, \mathcal{T}) \backslash \mathcal{ND}(X, \mathcal{T})$. Hence there exists a set $P \subset G$ such that $P \in \mathcal{H}(\mathcal{NB}(X, \mathcal{T})) \backslash \mathcal{ND}(X, \mathcal{T})$. As $G$ is open, $P \in \mathcal{H}(\mathcal{NB}(G, \mathcal{T}|_G)) \backslash \mathcal{ND}(G, \mathcal{T}|_G)$. Hence $(G, \mathcal{T}|_G)$ is irresolvable. □

Corollary 3.4. *OHI is $\simeq_s$-invariant.*

In [2] one can find the easy observation that a topological space is submaximal if and only if it is OHI and NODEC. In [18] it was proved that every submaximal space is HI, and every HI space is OHI. Taking these facts together with Proposition 2.4 yields the following corollary.

Corollary 3.5. *For every OHI topology $\mathcal{T}$, the topology $\mathcal{F}(\mathcal{T})$ is similar to $\mathcal{T}$ and submaximal (and hence HI).*

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Example 3.6. Let $\mathcal{T}_{\text{nat}}$ denote the natural topology on $\mathbb{R}^2$. Let $Q = \{(x, 0) : x \in \mathbb{R}\}$. Let $\mathcal{T} = \mathcal{T}_{\text{nat}} \cup 2^{\mathbb{R}^2 \setminus Q}$. Then $\mathcal{T}$ is a topology, $F_X = Q$ and $(\mathbb{R}^2, \mathcal{T})$ is OHI. At the same time, $Q$ is a resolvable subspace of $(\mathbb{R}^2, \mathcal{T})$. Hence $(\mathbb{R}^2, \mathcal{T})$ is neither HI nor submaximal.

By virtue of Corollary 3.5 and Example 3.6 we obtain the following results.

Corollary 3.7. *Submaximality and HI are not $\simeq_s$-invariant.*

Corollary 3.8. *The similarity $(X, \mathcal{T}_1) \simeq_s (X, \mathcal{T}_2)$ does not imply the relation $(A, \mathcal{T}_1|_A) \simeq_s (A, \mathcal{T}_2|_A)$.*

The last corollary implies that the relation of similarity is not hereditary. However, similarity of subspaces does occur under additional assumptions:

Proposition 3.9. *Let $(X, \mathcal{T}_1) \simeq_s (X, \mathcal{T}_2)$. If $A \in \mathcal{SO}(\mathcal{T}_1) \cap \mathcal{SO}(\mathcal{T}_2)$ then $(A, \mathcal{T}_1|_A) \simeq_s (A, \mathcal{T}_2|_A)$.*

Proof. Let $H \subset A$, $H \in \mathcal{T}_1|_A$, $H \neq \emptyset$. Then there exists $G \in T_1$ such that $H = A \cap G$. Since $A \in \mathcal{SO}(X, \mathcal{T}_1)$, $\text{Int}_{\mathcal{T}_1}(H) \neq \emptyset$. From the definition of $\simeq_s$, we have $\text{Int}_{\mathcal{T}_2}(H) \neq \emptyset$. Hence $\text{Int}_{\mathcal{T}_2|_A}(H) \neq \emptyset$. □

Remark 3.10. The bijection $f : X \to Y$ between two topological spaces is called a *faint homeomorphism* if both $f$ and $f^{-1}$ preserve the sets of nonempty interior. Corollary 3.4 can be also obtained from [18, Theorem 6.9] and the fact that for similar topologies the identity function is a faint homeomorphism.

## 4. When is the pair $(\mathcal{A}, \mathcal{I})$ topological?

Let $\mathcal{A}$ be the algebra and $\mathcal{I} \subset \mathcal{A}$ the ideal of subsets of a given set $X$. We say that the pair $(\mathcal{A}, \mathcal{I})$:

(1) is *topological* (abbreviated as *top)* (compare [6]) when $(\mathcal{A}, \mathcal{I}) = (\mathcal{NB}, \mathcal{ND})$ for some topology $\mathcal{T}$ on $X$;

(2) has the *LDO property* if there exists a lower density operator on $(X, \mathcal{A}, \mathcal{I})$.

The following implications are known (compare [11]):

$$LDO \wedge hull \quad \Rightarrow \quad top \quad \Rightarrow \quad hull.$$

By Proposition 2.4, every topology is similar to some LDO topology. Hence,

$$top \quad \Rightarrow \quad LDO$$

and consequently

$$LDO \wedge hull \quad \Longleftrightarrow \quad top.$$

We make a further definition: the pair $(\mathcal{A}, \mathcal{I})$ has the *LDO+ property* if there exists a lower density operator $\Phi$ on $(X, \mathcal{A}, \mathcal{I})$ such that, for every $\mathcal{B} \subset \mathcal{A}$,

$$\bigcup_{B \in \mathcal{B}} B \cap \Phi(B) \in \mathcal{A}.$$

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PROPOSITION 4.1.

$$LDO \wedge hull \iff LDO+.$$

PROOF. First observe that the *hull* property is equivalent to the following *kernel* property: for every set $A \subset X$ there exists a set $K \in \mathcal{A}$ (a kernel of $A$) with $K \subset A$, such that for every $P \subset A \setminus K$, if $P \in \mathcal{A}$ then $P \in \mathcal{I}$. Suppose that $(\mathcal{A}, \mathcal{I})$ has the $LDO+$ property. We have to show that the kernel property is also satisfied. To do this, let $P \subset X$ and let $\mathcal{B} = \{B \in \mathcal{A} : B \subset P\}$. The set $C = \bigcup_{B \in \mathcal{B}} B \cap \Phi(B)$ is the desired kernel of $P$. In fact, if $A \in \mathcal{A}$, $A \subset P \setminus C$, then $A \cap \Phi(A) = \emptyset$, hence $A \in \mathcal{I}$. Conversely, suppose that $(\mathcal{A}, \mathcal{I})$ has the property $LDO \wedge hull$, $\mathcal{B} \subset \mathcal{A}$, $C = \bigcup_{B \in \mathcal{B}} B \cap \Phi(B)$ and $K$ be the kernel of $C$. Then for every $B \in \mathcal{B}$ we have $B \sim B \cap K$. Hence

$$K \subset \bigcup_{B \in \mathcal{B}} B \cap \Phi(B) = \bigcup_{B \in \mathcal{B}} B \cap \Phi(B \cap K) = \Phi(K) \cap \bigcup_{B \in \mathcal{B}} B \cap \Phi(B) \subset \Phi(K). \qquad \square$$

By Theorem 3.3, Corollary 3.5 and the fact that, for the algebra $2^X$ and an arbitrary ideal, the *hull* property is trivial, we obtain the following characterisation.

PROPOSITION 4.2. *Let $\mathcal{I} \subset 2^X$ be an arbitrary ideal. The following statements are equivalent:*

- $(2^X, \mathcal{I})$ *is* top*;*
- *there exists a submaximal topology $\mathcal{T}$ on $X$ such that $(\mathcal{A}, \mathcal{I}) = (\mathcal{NB}, \mathcal{ND})$;*
- $(2^X, \mathcal{I})$ *is* LDO.

Let us consider the third condition from Theorem 3.3. We say that an ideal $\mathcal{I}$ is *small* in the algebra $\mathcal{A}$ if and only if $\mathcal{H}(\mathcal{A}) \setminus \mathcal{I}$ is coinitial to $\mathcal{A} \setminus \mathcal{H}(\mathcal{A})$. The name for this property can be justified by the following simple observation.

PROPOSITION 4.3. *If $\mathcal{I}$ and $\mathcal{J}$ are ideals contained in the algebra $\mathcal{A}$, $\mathcal{I}$ is small in $\mathcal{A}$ and $\mathcal{J} \subset \mathcal{I}$, then $\mathcal{J}$ is small in $\mathcal{A}$.*

PROPOSITION 4.4. *Let $(\mathcal{A}, \mathcal{I})$ be such that $\mathcal{I}$ is small in $\mathcal{A}$. Then*

$$\text{top} \Rightarrow \mathcal{A} = 2^X.$$

PROOF. Assume that $\mathcal{T}$ is a topology on $X$ such that $\mathcal{NB} = \mathcal{A}$ and $\mathcal{ND} = \mathcal{I}$. The fact that $\mathcal{I}$ is small in $\mathcal{A}$ implies the third condition of Theorem 3.3. The second condition of that theorem gives the conclusion of the proposition. $\square$

Let $\mathcal{L}$ be the $\sigma$-algebra of Lebesgue measurable sets, so that $\mathcal{H}(\mathcal{L}) = \mathcal{N}$ is the $\sigma$-ideal of null sets. For $\alpha \in [0, 1)$ let $\mathcal{I}_\alpha = \{E \in \mathcal{L} : \dim_H(E) \leq \alpha\}$ where $\dim_H(E)$ denotes the Hausdorff dimension of the set $E$. For every $\alpha$, the family $I_\alpha$ forms a $\sigma$-ideal contained in $\mathcal{N}$.

In [19, Theorem 7.6], Zindulka proved that every analytic set $B \subset \mathbb{R}$ contains a universal measure-zero set $E$ such that $\dim_H E = \dim_H B$.

Since every set of positive Lebesgue measure contains a Borel (hence, analytic) set of positive measure and every universal measure zero set is in particular a null set, we obtain the following corollaries.

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Corollary 4.5. *For* $\alpha \in [0,1)$, *the ideal* $\mathcal{I}_\alpha$ *is small in* $\mathcal{L}$.

Corollary 4.6. *For* $\alpha \in [0,1)$, *the pair* $(\mathcal{L}, \mathcal{I}_\alpha)$ *is not* top.

The following example shows that there exists a pair $(\mathcal{A}, \mathcal{I})$ such that $\mathcal{I}$ is not small in $\mathcal{A}$ although $\mathcal{I} \subsetneq \mathcal{H}(\mathcal{A})$.

Example 4.7. Let $\mathcal{M}$ be the $\sigma$-ideal of meagre sets on $\mathbb{R}$. Then the $\sigma$-ideal $\mathcal{N} \cap \mathcal{M}$ is not small in $\mathcal{L}$.

In fact, let $\mathbb{R} = M \cup N$ be a decomposition of the real line such that $M \in \mathcal{M}$ and $N \in \mathcal{N}$. Then $M \in \mathcal{L} \setminus \mathcal{N}$ but $M$ contains no set from $\mathcal{N} \setminus (\mathcal{N} \cap \mathcal{M})$.

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