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ARTICLE

An Epistemic Augmentation to the Math-First Approach to Physical Theories

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Abstract

Echoing semanticists' view on scientific theories, David Wallace has recently argued that physical theories are best understood if we conceive of them as mathematical structures. He supplements this idea by suggesting that they attach to the world by structural relations, e.g. isomorphism. This view, which he calls the *math-first approach*, contrasts with the *language-first approach*, according to which physical theories are collections of sentences latching onto the world by linguistic relations, e.g. truth. He then submits the structural realist stance is the appropriate metaphysics for this semantic framework. While agreeing that Wallace's proposal is semantically and metaphysically tenable, I will argue that it is epistemically incomplete, since it leaves untouched the question "what are cognitive attitudes directed toward a physical theory?". This issue becomes crucial especially when we notice that physical theories so construed cannot be the vehicle of propositional cognitive attitudes, e.g. belief and knowledge. Drawing on Elgin's revisionary epistemology, I will suggest an augmentation to the math-first approach by certain non-propositional cognitive attitudes in such a way that both realist and anti-realist stances can be expressed within the resulting augmented math-first approach.

Keywords: The math-first approach; Elgin; acceptance; understanding; representation

1. Introduction

In the very same year that W.V.O. Quine (1951) published his seminal paper to critique a crucial *linguistic* distinction lying at the core of logical empiricism, Patrick Suppes (1951), a pioneer of the semantic view, tackled a philosophical problem related to quantities with *non-linguistic* constructions, i.e. set-theoretic structures. Addressing the question "what is the structure of a scientific theory?", the early semanticists (Beth 1949; McKinsey *et al.* 1953; Suppe 1967; van Fraassen 1970) diverged from the established view of the logical empiricists, later dubbed as "the received view," on which scientific theories are the collections of sentences axiomatized in first-order logic (Suppe 1974).[1] They argued that scientific theories are (characterized by) collections of models, and a model, whatever it might be, was not a linguistic entity. The semantic view gradually

[1]For a critique of such a characterization of the received view, see Lutz (2012).

gained momentum and became the paradigmatic format for philosophers of science to construe scientific theories. In the 1970s, the non-linguistic conception of physical theories was utilized to show how they are related synchronically (Sneed 1971) and diachronically (Stegmüller 1976). In the next two decades, one of the long-standing dispute in the history of philosophy, i.e. the realist-empiricist debate, was couched in terms of the semantic view in a way that both realists (Giere 1985; Pereira and French 1990; Ladyman 1998) and empiricists (van Fraassen 1980; Bueno 1997) could express their views. Besides these developments, the first decade of the twenty-first century witnessed a *semantic augmentation* to the semantic view, according to which scientific theories are not only characterized by mathematical structures but also connected to the world by structural relations, e.g. isomorphism, partial isomorphism, and homomorphism (French 2003; Bartels 2006; van Fraassen 2008). Having employed these two new approaches to scientific theories and their semantic status – that they are mathematical entities instead of linguistic ones and that they attach to the world by mathematical relations instead of linguistic ones – in recent years philosophers of physics have been able to deal with specific philosophical problems about physical theories without explicitly mentioning the semantic view being adopted. Whether the different formulations of the general theory of relativity are equivalent (Rosenstock *et al.* 2015), the whole argument confronts us with a dilemma concerning substantivalism-relationalism debate about spacetime (Weatherall 2018; Halvorson and Manchak 2022), the theory change from classical mechanics to quantum mechanics weakens the pessimistic meta-induction argument against scientific realism (Thébault 2014), the world depicted by our best scientific theories consists of individuals and their intrinsic properties or of worldly structures (Ladyman *et al.* 2007; Roberts 2011) are among such problems which have been addressed within non-linguistic frameworks, e.g. set, category, and group theory. Having said that, some philosophers have criticized the semantic view, particularly objecting that the view cannot identify scientific theories rightly unless it takes some linguistic constructions into consideration (Halvorson 2012; Lutz 2017).

Echoing semanticists' view on scientific theories, however, David Wallace has recently claimed that:

> the content of a physical theory is to be understood primarily in terms of its mathematical structure and the representational relations it bears to physical systems, rather than as a collection of sentences that attempt to make true claims about those systems (Wallace 2022, p. 345).

He refers to the approaches conceiving of physical theories as the former and latter conceptions as "the math-first" and "the language-first," respectively, while refraining from using traditional names, i.e. "the semantic view" and "the received view" ("the syntactic view").[2] More precisely, the math-first approach may be seen as *the semantic view semantically augmented*, as described above, though his reason for such abstaining is different. In the next step, he provides also a *metaphysical augmentation* to the math-first view, calling it "math-first OSR" which more or less realizes the ontic structural realist's conception of the ontology of the world, according to which "fundamentally speaking there are no objects" (Wallace 2022, p. 367). Thus understood, Wallace's proposal

[2]For him, the traditional names mirror the conventional syntax/semantics of a formal language. As such, the distinction between the received and semantic views would be superficial, since "any recursively-enumerable set of sentences in first-order logic closed under logical consequence will determine a class of models, and conversely, from that class of models the original set of sentences can be recovered" (Wallace 2022, p. 349).

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comprises three components pertaining to the source of representation (physical theory), the source-target representation relation (structural relations), and the target (the world). That being said, even if someone endorses Wallace's proposal regarding the manner in which physical theories are characterized and linked to the world, as well as the way the world is, there still exists a notable *epistemic gap* in his philosophical framework that requires attention: *what are subject's cognitive attitudes directed toward physical theories?* After all, physical theories couched in terms of the math-first framework (or the semantic view) cannot be the vehicle of propositional epistemic attitudes such as belief and knowledge, given that they lack propositional characteristics. Consequently, the math-first view must be supplemented with an epistemic component as well if it is to be regarded as a coherent philosophical framework for understanding physical theories. Drawing on Elgin's (2017) revisionary epistemology, which accommodates non-propositional epistemic attitudes, I will propose an *epistemic augmentation* to the math-first approach by specific non-propositional attitudes. Based on her non-veritistic epistemology, the conventional epistemological concepts of truth, belief, and knowledge are replaced with three novel concepts: exemplification/representation, acceptance, and understanding. Suggesting certain adjustments to her epistemological system, I will contend that physical theories could serve as the vehicle of non-propositional cognitive attitudes in such a way that both realists and anti-realists can articulate their stances regarding these theories.

The paper is structured as follows. Section 2 explores the math-first view, specifically addressing its semantic and metaphysical dimensions. Additionally, it examines the reasons presented by Wallace to support the understanding of physical theories through this lens. This section concludes by noting that the math-first approach does not ascertain the nature of epistemic attitudes directed towards physical theories. Moving on to Section 3, I will demonstrate that the perspectives upheld by the proponents of the semantic approach, in their endeavor to recognize and surmount this epistemic shortfall, are doomed to failure. In Section 4, I introduce Elgin's revisionary epistemological system which substitutes some non-propositional epistemic attitudes with propositional ones within the traditional epistemology. This section presents a solution aimed at remedying the epistemic shortcoming of the math-first view. Proceeding to Section 5, I will introduce the epistemically augmented math-first approach, ensuring that it maintains equilibrium without leaning towards either realism or anti-realism.

2. The math-first view, its semantics, and metaphysics

David Wallace, in his recent paper "Stating Structural Realism: Mathematics-First Approaches to Physics and Metaphysics" (2022), puts forward a straightforward argument for the structuralist form of scientific realism against its standard version. Upon the former, we know just the structural aspects of the world (its epistemic version), or the world is nothing but structures which are knowable by our best scientific theories (its ontic version). The argument may be outlined as follows:[3]

1. We have strong reasons to consider physical theories as collections of mathematical structures, rather than as collections of sentences;
2. The relationship between these mathematical structures and the world is representation – a structural relation (like partial isomorphism) – instead of truth;

[3]This argument, on its own, does not support scientific realism *per se*. Instead, it serves to justify a form of structural realism as opposed to its standard version, relying on the assumption of scientific realism.

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3. Therefore, if our physical theories are successful and hence determine the way the world is, it is due to their representation of the (both observable and unobservable) world rather than their truth.

Indeed, the two first premises together constitute the math-first view, as I discussed in the introduction. I refer to the first premise, which expresses the idea of the semanticists regarding the characterization of the source of representation, as the "vehicle thesis" and the second one, which deals with the source-target relation, as the "representation thesis." For Wallace (2022, p. 361), the combination of these two theses implies a form of scientific realism which he calls "math-first scientific realism." In metaphorical terms, the world, according this sort of realism, is not *described* through physical theories as if they are the "Book of the World," but rather it is *represented* through the framework of physical theories forming a "Model of the World." He labels this sort of doing metaphysics "math-first metaphysics" which gives rise to "math-first OSR,' when combined with "math-first realism" (Wallace 2022, p. 365). My intention here is not to evaluate the validity of the aforementioned argument, although I find myself aligned with its perspective. The crux of the matter lies in the fact that the math-first view and the resulting scientific realism need to be supplemented with another thesis having an epistemic nature. Ultimately, whether engaging in descriptive or representative metaphysics, there should be someone that holds specific epistemic attitudes towards physical theories. In the case of descriptive metaphysics, this is not a concern, as the cognitive subject adopts typical propositional attitudes, such as belief and knowledge, toward these theories. However, concerning representative metaphysics, the question arises: *what are the attitudes of the subject in relation to physical theories which are non-propositional?* In metaphorical terms once again, what is our standpoint regarding the Model of the World? This issue is not new, and several semanticists have endeavored to propose resolutions for it. As I will argue in the next section, however, these attempts are ill-fated. Therefore, a more revolutionary change in our conception of cognitive attitudes should be pursued, a change that Elgin's revisionary epistemology provides. Before delving into these resolutions and their shortcomings, let us explore the virtues that underpin the math-first approach.

Wallace (2022, pp. 350–56) aims to demonstrate that the math-first perspective on physical theories surpasses the language-first approach in effectively capturing three pivotal concepts within the realms of philosophy of science and physics: the theory-world relationship, equivalence between physical theories, and intertheoretic reduction. These advantageous facets unfold as follows:

1. Predictions stemming from physical theories do not align perfectly with the data garnered from measurements. Consequently, from the perspective of the language-first approach, empirically adequate physical theories are deemed to be only *approximately true*, rather than totally true. As such, this approach should involve sentences such as "it is approximately true that the trajectory of projectiles follows a parabolic path," which can be understood through sentences akin to "it is true that projectiles move approximately in a parabolic trajectory." Nevertheless, the latter notion merely serves as a linguistic representation of how a mathematical model captures a data model with a certain degree of accuracy. Thus, the linguistic concept of approximate truth becomes practically redundant, while the mathematical concept of *accuracy fit*, characterizing the alignment between theoretical and data models, assumes fundamental importance.

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2. For every physical theory, there exists another theory, in principle, that provides a more accurate description of the world. Utilizing the concept of approximate truth, we assert, for instance, that "it is approximately true that spacetime (as posited by the special theory of relativity) is flat." Dismissing this *universal* perspective, physicists in practical scenarios express "it is true that spacetime is *locally* flat." Consequently, guided by the concept of *domain specificity*, which finds comprehensive representation within the math-first framework, there is no place for approximate truth suggested by the language-first view.
3. Upon the language-first view, two theories are equivalent if they posit identical entities and attribute the same properties and relations to them. Concepts like intertranslability or interdefinability establish linguistic criteria for this notion of equivalence. On the math-first view, two theories are equivalent if they share the same mathematical structure. Structural relations, e.g. set or category-theoretic isomorphism, reify this concept of equivalence, when contextual issues are duly considered. Nevertheless, there exist numerous cases in which physicists regard theories as equivalent, despite not meeting linguistic criteria primarily because these linguistically construed theories depict radically different realms of entities and their properties/relations. Criteria defined within the math-first framework, however, capture their equivalence.
4. Within the language-first view, theory *A* (e.g. thermodynamics) is reduced to theory *B* (e.g. statistical mechanics) just in case the laws of *A* can be derived from those of *B*, provided that appropriate translational means are used. Yet, in instances where physicists endorse a form of reduction, the objects/properties/relations delineated by theory *A* cannot be piecemeal, but just in a holistic manner, constructed through those theory *B* proposes. Through a functionalist discourse, the math-first approach asserts that theory *A* reduces to theory *B* if a certain structure attributed to theory *B* realizes or instantiates a specific structure of theory *B*, which these functionalist notions themselves are mathematically reified within the math-first framework. This suggestion aptly accommodates scenarios of reduction that remain beyond the grasp of the language-first stance, as exemplified by the reduction of the classical theory of solids to its quantum counterpart. Beyond this advantage, physicists' intuition guides us to recognize reductions as pertinent only within localized contexts, not universally. The notion of approximate truth, the inadequacy of which was discussed earlier, constitutes the exclusive means through which the language-first view addresses this concept of locality. In contrast, the domain specificity inherent in the mathematics-first perspective effectively elucidates this notion.

Having considered the preceding explanations, we find strong reasons to view physical theories as collections of mathematical models connected to the world through structural relations. As Wallace (2022, pp. 361–63) has argued, this perspective concurs with both positive and negative arguments for scientific realism. Moreover, it is essential to recognize that successful physical theories serve as *representations* of the world rather than (approximate) *truths* about it. Thus, when confronted with empirically adequate physical theory *A*, a math-first scientific realist would assert her cognitive position regarding *A* by stating "I   *A*" Nevertheless, due to the non-propositional nature of *A*, conventional propositional attitudes like belief and knowledge prove unfit to fill in this assertion. As such, the advocate of the math-first view faces a dilemma: *she must either adopt non-propositional attitudes toward A or recast A within a linguistic framework, thereby enabling conventional propositional attitudes to be taken*; a soon-to-be-seen conundrum which may be called the

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"attitude dilemma." As I will argue in the ensuing section, the latter option undermines the foundations of the math-first perspective, which compels not only the math-first scientific realist but also anyone who regards physical theories as mathematical models to opt for the former alternative. However, is there any epistemic framework allowing cognitive attitudes to be defined non-propositionally, making some possibility for considering the first horn of the dilemma? Elgin's revisionary epistemology, which has been primarily developed to explain the cognitive acceptability of scientific theories/models, gives us a novel understanding of cognitive attitudes with two distinctive features fitting well with the math-first approach. First, there are certain cognitive attitudes, her framework suggests, whose vehicles are possibly *non-propositional*, e.g. understanding and acceptance. Secondly, these attitudes are linked to their objects by having their vehicles tethered to the world *non-linguistically*. Indeed, the former is what the math-first approach demands and the latter aligns well with the representation thesis of the approach. In Sections 4 and 5, the details of her epistemic and semantic proposal will be introduced and the way in which it can be closely intertwined with the math-first approach will be explored, but first the existing resolution of the semanticists in facing the dilemma.

3. A Trojan horse of propositions in our midst

Over the span of three decades, Steven French has meticulously developed an elaborate form of *scientific structuralism* characterized by the coalescence of three distinct theses: the vehicle thesis, the representation thesis, and the target thesis upon which the entities that are structurally represented by physical theories as mathematical models are themselves structures.[4] French's scientific structuralism, indeed, bears some resemblance to the recently proposed math-first view, coupled with the math-first OSR, as presented by Wallace.[5] He stands as one of the select proponents of the semantic view who has astutely identified the attitude dilemma, endeavoring to bolster the semantic view by opting for the second horn of the dilemma as an epistemic augmentation. Following this, I will delve into a discussion of his proposal and then proceed to offer a critique, evaluating its alignment with the merits of the math-first perspective.

Analyzing van Fraassen's account of the nature of theories and the epistemic attitude towards them, French with da Costa identify an apparent perplexity in his claims, which resembles the attitude dilemma introduced above. On one hand, van Fraassen asserts that "if the theory as such is to be identified with anything at all – if theories are to be reified – then a theory should be identified with its class of models" (quoted in da Costa and French 2003, p. 30). On the other hand, he claims that "a theory is the sort of thing that we may believe or disbelieve" (quoted in da Costa and French 2003, p. 32). Given a propositional account of belief, therefore, van Fraassen would encounter a perplexity:

[4]French has articulated this project across numerous papers and books, with his work *The Structure of the World: Metaphysics and Representation* (2014) standing out as a particularly well-organized embodiment of the three theses.

[5]Naturally, substantial differences exist between these two projects, with a notable disparity being that, from my perspective, French's approach necessitates the re-presentation of physical theories within a meta-language framework to address the associated philosophical quandaries, whereas Wallace's math-first project relies primarily on the language employed by physicists in their practical contexts. For a more comprehensive examination of the distinctions, see Wallace (2022).

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> But if the content of a theory is to be taken to be true or false and expressed as a set of propositions, what are we to make of the dismissal of linguistic formulations ? (da Costa and French 2003, p. 32)

To address this *apparent* perplexity from both their and van Fraassen's perspectives, they maintain that a theory *per se* is distinct from its formulation or characterization. Within their view, one can describe a theory either in a logico-linguistic manner or in a model-theoretic manner, depending on the context in which she engages with the theory. If someone intends to take an epistemic attitude towards theories, the logico-linguistic description becomes the exclusive option:

> When we turn from a discussion of the structure of theories to a consideration of our epistemic attitudes toward them, we have *no choice* but to resort to a linguistic formulation of some form or other (da Costa and French 2003, p. 33, emphasis added).

Between linguistically recasting theories or adopting non-propositional epistemic attitudes, they and (their) van Fraassen opt for the former, and this choice appears to be reasonable under the assumption that non-propositional epistemic attitudes are lacking. However, recent works in epistemology have recognized the existence and importance of non-propositional epistemic attitudes, suggesting that their choice is ill-founded. I shall explore these proposals in the next section, but for now let us examine how da Costa and French elaborate the idea of different formulations of the same theory.

Borrowing Suppes's (1967) idea regarding the dichotomy between *extrinsic* and *intrinsic* characterizations of a theory, they argue that when we treat a theory as mathematical models or structures – for example, when we define "an isometry between metric spaces" or prove "a representation theorem in the theory of groups" (da Costa and French 2003, p. 34) – we are, in fact, presenting or characterizing the theory from a model-theoretical perspective, which means adopting an extrinsic viewpoint on the theory. However, if our aim is to incorporate "the doxastic attitudes of scientists themselves, we must shift to the intrinsic characterization" (da Costa and French 2003, p. 35), which involves a linguistic presentation of the theory. They are, however, well aware that our currently best scientific theories will one day come to be known as false or that their validity is domain-specific. Building on these observations, da Costa and French proceed to enhance the linguistic or intrinsic characterization with a novel concept of belief, termed "representational belief," on which the report "*S* believes that *p*" does not identify with "*S* believes that *p* is true," but rather with "*S* believes that *p* is *partially* true." Here, "*p* is partially true" signifies *p* is true in a partial structure that expresses the (semi-propositional) content of the representational belief. Several studies, particularly in the philosophy of physics, have attempted to demonstrate that this partial conception of truth and structure is a suitable mechanism for capturing notions such as accuracy fit, domain specificity, and the intertheoretic reduction of physical theories (see e.g. Manero 2022). That said, in the following discussion, I will present two specific criticisms against this proposal. First, it merely defers the attitude dilemma, pushing it one step back. Second, it is comparable to the proverbial act of putting the cart before the horse. These criticisms remain valid even if the attempts at reconstructing the practices of physicists turn out to be successful.

Consider the scenario where a physicist, aiming to tackle a specific astronomical issue, engages with a mathematical model belonging to the theory of special relativity.

da Costa and French suggest that physicist's epistemic attitude towards the theory, extrinsically characterized by its models, is accommodated by considering her belief, which is semi-propositional in content, meaning its truth is partial. This attitude might initially be expressed through assertions such as "I believe that spacetime is flat" (the first doxastic assertion). According to their proposal, this equates to stating "I believe that spacetime is flat is partially true" (the second doxastic assertion). However, to understand how a semi-proposition can be partially true, and hence how such an attitude should be interpreted, we must resort to a partial structure. This structure is fundamentally a mathematical model. Therefore, understanding the second doxastic assertion requires understanding, or more broadly adopting another doxastic attitude towards, the mathematical model. However, this is merely the initial step we began with, i.e. determining the physicist's epistemic attitude towards a mathematical model. As such, their proposal does not provide a resolution to the dilemma but rather, at best, merely defers it.

In connection with this concern, another issue arises concerning the proper prioritization of intrinsic and extrinsic representations within a theoretical framework. da Costa and French, in their effort to address the attitude dilemma, give precedence, above all else, to the concept of representational belief. It is crucial to note, however, that this very notion is grounded in another concept, namely, that of partial structure which possesses a mathematical nature, or more precisely, a set-theoretic characterization. Put differently, the intrinsic approach implemented to resolve the attitude dilemma is, in fact, predicated upon the extrinsic characterization of the theory, akin to the adage of putting the cart before the horse.

Before concluding this section, I would like to address a question regarding the resurgence of propositionality in the semanticists' program. Why do da Costa and French not opt for the first horn of the dilemma, which would allow non-propositional attitudes to be taken towards scientific theories extrinsically characterized? It seems that their willingness to revive propositionality is not due to the belief that conventional propositional attitudes, like belief and knowledge, *per se* are in any way indispensable. Instead, as scientific realists who consider scientific theories to correspond to the world or as empiricists seeking to delineate the boundary between their stance and that of realists, they presuppose that *the notion of (partial) truth is indispensable*. On the other hand, propositions are the only viable candidates as bearers of truth. Given these considerations, the only way to preserve the notion of (partial) truth and make sense of the realism-empiricism debate is to accommodate propositional attitudes. However, the key issue here is that truth is just one concept among others that can reify the correspondence between a representation and the world. For example, a recorded sound may correspond auditorily, or a painting may correspond visually, with elements in the world. Yet, neither of these forms of correspondence involves truth, as neither representation is propositional. Therefore, depending on the type of representation under consideration, an appropriate concept of correspondence should be employed. Auditory and pictorial representations possess their distinct forms of correspondence separate from truth, which is specifically related to propositional representation. To reassure the semanticist that the notion of truth is dispensable for characterizing realist and empiricist stances, we can incorporate mathematical relations that embody correspondence, as already embedded in the math-first approach. With that said, the primary challenge lies in defining appropriate epistemic or cognitive attitudes towards mathematical entities presumed to be tethered to the world through mathematical relations. In the next section, I will endeavor to address this challenge, drawing upon Elgin's revisionary epistemology.

4. Elgin's revisionary epistemology

In her book *True Enough* (2017), Catherine Elgin presents a revisionary, if not revolutionary, epistemological framework to explain the cognitive achievements of science and account for its epistemic acceptability. She contends that the traditional, veritistic epistemology, anchored in the key concepts of truth, belief, justification, and knowledge, not only fails to account for the epistemic acceptability of science but also portrays scientific disciplines as epistemically defective systems. Within her proposed epistemology, Elgin advocates for the relaxation of the traditional notions, replacing truth with exemplification/representation, belief with acceptance, justification with responsibility, and knowledge with understanding. The relevance of her framework to our project lies in its emphasis on non-propositional epistemic attitudes towards scientific theories/models and non-linguistic relations between them and the world, specifically by substituting the propositional attitudes (belief and knowledge) with the non-propositional attitudes (acceptance and understanding). Moreover, Elgin abandons truth as a linguistic relation in favor of exemplification and representation as non-linguistic ones. As such, we can apply her strategy to address the attitude dilemma by asserting that scientific theories linguistically formulated are unnecessary as the vehicles of our propositional epistemic attitudes. Instead, we can effectively employ the non-propositional alternatives towards scientific theories/models, recognizing them as non-propositional entities.

Having said that, her revisionary epistemology is explicitly non-factive, not committing to representations "that carve the mind-independent world as its mind-independent joints" (Elgin 2017, p. 151). Consequently, the realist stance within the math-first approach cannot be maintained, leading the augmentation process to fall short. Despite this limitation, her revisionary, non-veritistic epistemology seems adaptable into a *more accommodating framework* that allows cognitive attitudes towards scientific theories/models to be interpreted either factively or non-factively. To achieve this, we might discard, following her lead, the linguistic tether between scientific theories/models and the world, i.e. truth, and then replace it with a non-linguistic relation of representation that can be interpreted in two different ways: either as a relation in which the source of representation mirrors nature structurally, or as a relation, constrained by pragmatic-contextual factors, in which it purports to exemplify the features nature instantiates. In the next step, the non-factively characterized non-propositional attitudes, i.e. acceptance and understanding, are substituted with new non-propositional ones that can be conceived as either factive or non-factive attitudes, depending on the interpretation of the non-linguistic relation between scientific theories/models and the world. Thus construed, this new, more accommodating epistemology is primarily revisionary, given that the involved cognitive attitudes are non-propositional. Also, it is non-veritistic, as the inherent aim of these attitudes is not true.

Regarding this latter feature, however, there is a crucial point that plays a pivotal role in moving towards the more accommodating framework. Indeed, being non-veritistic does not preclude any kind of correspondence or *adequation* between scientific theories/models and the world, nor does it imply that cognitive attitudes towards them are necessarily non-factive. As already noted, it is true that "Truth is the adequation of things and intellect" ("*Veritas est adaequatio rei et intellectus*"), but it is not the only form of adequation, conformity, or correspondence between scientific theories and the world. Indeed, we may discern, at least, three forms of adequation: *veritistic*, *structural,* and *exemplificative*. According to the first, which is needed in characterizing traditional scientific realism (see e.g. Sankey 2008, pp. 12–20), our best scientific theories, as linguistic entities comprised of sentences with subject-predicate parts, mirror what the

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world is made of, i.e. its objects, properties, relations, facts, etc. Structural adequation is the kind of conformity required to articulate math-first scientific realism, induced by mathematical relations such as isomorphism. In submitting that our best scientific theories/models mirror or resemble the world structurally, i.e. the vehicle thesis of math-first realism, this conception of adequation is in request. Exemplificative adequation, on which Elgin's epistemic-semantic system is based, is strong enough to provide an external criterion for the tenability of a theory/model,[6] but not too strong to assure us that science mirrors nature.[7] A scientific theory/model is exemplificatively adequational just in case it exemplifies features a part of the world as its target instantiates, though exemplification here has a technical meaning which will be explicated in the following. All in all, the more accommodating framework is, pro Elgin, non-veritistic but, contrary to her view, makes room for a certain form of mirroring to be embedded. More particularly, it has a schematic form which allows either anti-realist or realist stances to be expressed once the exemplificative or structural notion of correspondence, respectively, is deployed. In the next section, I will introduce this framework, but first let us consider Elgin's own view on the epistemology of science.

Before proceeding to that task, it is worth noting that Elgin's argument for adopting such a revisionary epistemology is multifaceted, which primarily revolves around two key rationales:

1. *Non-propositional holism*: What cognitive attitudes are directed towards, namely scientific theories, models, and more broadly what she refers to as "scientific accounts" – which include claims about a topic and the way these claims are supported by each other and evidence (Elgin 2017, p. 12) – possess a holistic and non-propositional nature. This perspective suggests that traditional epistemic attitudes (such as belief and knowledge), conventional notions of truth as the link between scientific theories/models and reality and the conventional concept of justification, which is closely tied to truth, are all inadequate. Instead, we should embrace non-propositional epistemic tools.
2. *Non-veritism*: Science often benefits from felicitous false theories/models. In such cases, attempting to make these theories/models more truthful can actually diminish their cognitive value. This observation leads to the conclusion that to account for the epistemic acceptability of science truth and related epistemic concepts, which are founded upon truth, ought to be abandoned.

In the following sections, I will introduce the core elements of this revisionary epistemology, focusing on three pivotal arguments that align with the first rationale. This focus is justified for two reasons. First, the primary aim of this paper is to enhance the math-first approach with non-propositional epistemic tools, addressing the attitude dilemma, which aligns well with Elgin's first rationale. Second, our augmented math-first approach should be versatile enough to accommodate realist perspectives, as well as non-realist ones. Therefore, our project cannot rely essentially on the second rationale, particularly if it is regarded as an impetus to exclude the structural sense of adequation between science and the world.

[6]" we also need a tether that connects the system to the facts it pertains to . [Exemplification] is the connection between a sample or example and whatever it is a sample or example of" (Elgin 2017, 184).

[7]"Science, we are told, is (or at least aspires to be) a mirror of nature . If so, a representation should resemble its referent as closely as possible . This stereotype, I have urged, is false and misleading . Many seemingly powerful and effective representations turn out on a mimetic account to be at best flawed, at worst unintelligible" (Elgin 2017, pp. 249–50).

4.1 From knowledge to understanding

For Elgin, any adequate epistemology must address an explanatory task: why are our best scientific theories and models cognitively acceptable? (Elgin 2017, p. 16). In much of *True Enough*, she argues that their acceptability does not stem from the fact that they provide knowledge about scientific facts, but rather because they confer understanding of them. To grasp how she justifies this stance, it is useful to explore her terminology regarding the characterization of understanding (Elgin 2017, p. 37). According to a more generalized treatment, every cognitive attitude relates to two elements: the vehicle and the object. The object of a cognitive attitude is what the attitude is *about*, encompassing facts, phenomena, topics, concrete systems, and domains. A vehicle is what the attitude, whether dispositional or representational, is *toward*, enabling a subject to cognize an object. Propositions, scientific theories, models and, more broadly, scientific accounts serve as different kinds of vehicles. By investigating the format of scientific accounts, Elgin aims to show that they inherently possess a holistic nature which cannot be adequately reified through propositional tools such as conjunction. Although her arguments for this perspective on the format of scientific accounts are quite nuanced and detailed (Elgin 2017, Ch. 3), a simplified outline of one of these arguments (Elgin 2017, pp. 33–37) can be presented as follows. Assume a scientific account is a conjunction of propositions. We observe that even our best scientific accounts face anomalies – the account implies an individual proposition that conflicts with evidence. Given this, the epistemic tenability of the account would be undermined, since a conjunction is justified only if each of its conjuncts is justified. Therefore, even our best scientific accounts are not epistemically tenable. Furthermore, discarding the conflicting proposition as an exception is not a viable strategy because, among other reasons, it "pulls so strongly against the ideal of systematicity" (Elgin 2017, p. 36).

Now, two questions arise: 1. what are the kinds of cognitive attitudes whose vehicles are accounts? and 2. what are the objects of these kinds of cognitive attitudes? Among the attitudes having accounts as their vehicle, discussed in detail by Elgin, is *objectual understanding*. This sort of understanding, along with the two others, namely *propositional* and *interrogative* understanding, is now well-known and has been further discussed (see e.g. Baumberger 2014). However, Elgin's holistic view of objectual understanding is somewhat unique. Within her framework, objectual understanding is not only irreducible to propositional understanding, whose vehicles are propositions and whose objects are particular facts, but also it is propositional understanding that essentially depends on objectual understanding. For instance, to understand a case of quantum tunneling as an individual fact obtained in a laboratory, one should grasp "a suitably unified, integrated, tenable body of information that bears on that fact" (Elgin 2017, p. 44), which in this case includes understanding how particles behave in the quantum realm, what a potential barrier is, the concept of probability, how the boundary conditions in question restrict the behavior of quantum objects and many other pieces of information. These together may be termed the "topic of quantum tunneling." Understanding such topics is conferred by the accounts a discipline posits, which in this case is quantum mechanics. Thus, *to understand a particular fact as an object through a proposition as a vehicle, we must first understand a topic as an object through an account as a vehicle*. This approach may be termed "understanding holism" in Elgin's revisionary epistemology.

Having specified the vehicle and object of objectual understanding, it is also necessary to delineate its nature. Elgin (2017, pp. 44–46) characterizes objectual understanding in terms of *grasping*. As previously stated, scientific accounts exhibit a holistic nature, encompassing various elements of different types – for instance, propositions expressing facts relevant to the topic, principles, and postulates, criteria for distinguishing theoretical from observational aspects, the logical-mathematical rules governing the formation of inferences within the account and criteria for determining what constitutes evidence for the account. However, merely knowing these elements in isolation does not suffice to understand the topic. For such a cognitive attitude to be considered an epistemic success, the cognitive subject must grasp how these elements interconnect in a manner that enables her to derive inferences, explanations, unification and achieve other cognitive virtues related to the topic. In the case of quantum tunneling, for example, she must grasp, among other things, how solving the Schrödinger equation as a differential equation with specific boundary conditions gives predictions about the probability of a particle penetrating the potential barrier, incorporating a certain interpretation of probability in line with Born's rule and the continuity equation.

Given these preliminaries, we are in a position to articulate an argument for *understanding holism* in favor of objectual understanding over knowledge:

1. Knowledge is characterized by propositional vehicles;
2. Objectual understanding is characterized by non-propositional vehicles, namely accounts;
3. A particular scientific fact is cognized only if the topic pertaining to the fact is cognized;
4. A topic is cognized through a cognitive attitude whose vehicle is an account;
5. Therefore, to cognize scientific facts, the appropriate cognitive attitude is objectual understanding rather than knowledge.

Thus, according to Elgin's holistic epistemology, we should embrace objectual understanding as a cognitive attitude that utilizes a non-propositional vehicle to account for the cognitive tenability of scientific theories and models. Armed with this epistemic framework, we can address the attitude dilemma without the need to recast non-propositional scientific theories in propositional terms. Rather than engaging with the world through the cognitive attitude of knowledge, which relies on propositional vehicles, we approach it through objectual understanding, which utilizes non-propositional vehicles. Next, we will discuss another epistemic transition from propositionality to non-propositionality, namely the shift from belief to acceptance.

4.2 From belief to acceptance

Elgin's primary argument for favoring acceptance over belief stems from non-veritism being essential in her epistemic architecture. Given that belief aims at truth, it cannot be integrated into a non-veritistic epistemology (Elgin 2017, Ch. 1). The holistic aspect of her framework, however, further justifies substituting belief with a cognitive attitude directed toward non-propositional vehicles. To elucidate this shift through holism, it is beneficial initially to clarify the concepts of belief and its alternative, acceptance. Drawing on L. Jonathan Cohen's (1989, 1992) proposal, Elgin (2017, pp. 19) defines belief and acceptance as follows:

> To be *convinced* [to believe] that p is to be disposed, when attending to issues raised or items referred to by p, normally to feel that it is true that p and false that $\sim p$. To *accept* that p involves being willing to take p as a premise, as a basis for action or, I add, as an epistemic norm or a rule of inference, when one's ends are cognitive.

As we see, although her non-veritistic epistemology, by embedding acceptance characterized as above, tries to avoid any association with the concept of truth and related epistemic notions, it is nonetheless constituted by including cognitive ends. After all, it purports to account for the epistemic acceptability of science and, therefore, should take into account how scientists cognitively deal with scientific theories/models. Aside from this dimension, a key aspect within the holistic framework is that the acceptance of p is predicated on the acceptance of the account encompassing p as a component. Elgin advocates for this prioritization on epistemic grounds, arguing that a proposition is acceptable only if the encompassing account is deemed acceptable. For example, the claim that quantum particles can penetrate potential barriers is tenable only if the quantum model of tunneling is validated. While the criteria for an account's acceptability, and consequently its components, is of significance (Elgin 1999; Elgin, 2017, Ch. 4), it is beyond the scope of this discussion. The critical insight here is a form of holism, *acceptance holism*, that supports the shift argument from belief to acceptance:

1. Belief is confined to propositional vehicles;
2. Acceptance employs non-propositional vehicles;
3. Scientific accounts are non-propositional;
4. Thus, for scientific accounts, acceptance supersedes belief.

Having equipped the epistemology of science with two non-propositional cognitive attitudes, namely understanding and acceptance, the math-first approach, already emancipated from the confines of language-based semantics and metaphysics, is now liberated from the language-based epistemology in which propositional attitudes are pivotal. In this framework, such attitudes are linked to the world by tethering the propositions involved in them to the world, which is reified by the notion of truth. Yet, with the absence of a language-based tether, we encounter a compelling question: how are non-propositional attitudes tethered to the world?

4.3 From truth to exemplification

The main motivation for Elgin to shift from truth to exemplification and representation lies in the non-veritism she endorses (Elgin 2017, Ch. 1). Moreover, the holistic aspect of her framework necessitates moving beyond truth to incorporate a holistic tether into her revisionary epistemology. In what follows, I explore the shift based on this latter rationale. Elgin's approach to connecting a scientific model with the world to understand an object involves various concepts, each carrying its own technical meaning. To probe her conceptual framework (Elgin 2017, Chs. 9 and 12), it is helpful to consider a simple example.

Suppose a commuter wishes to understand the subway system of the city where she is to work, meaning she wants to know its stations, how the stations are connected, the time it takes to travel from the starting station to the final station, among other things. A city resident familiar with graph theory might assist by suggesting that "to understand the actual subway system (Y), it is sufficient to consider a certain graph (X) as a public transport system (Z). To do this, take the edges of the graph as the lines of public transport system and its vertices as transfer stations." Given this *interpretation* (I), i.e. considering the graph *as* a public transport system, there would be some correspondence between X's and Z's features. For instance, the adjacency of vertices in the graph is interpreted as the direct connectivity of stations in the public transport system. Referring to X with I as a "Z-representation," we say it *instantiates* the feature of being directly connected, such as stations and b, if the graph instantiates the feature of adjacency of vertices, such as α and β. Furthermore, if the commuter, under a certain context, is interested in understanding which stations are directly connected, we say this Z-representation *exemplifies* the feature of being directly connected. Of course, there might be features the commuter does not concern herself with, e.g. Z's feature of a station being under construction which corresponds with X's feature of a vertex being isolated. If so, the Z-representation instantiates the feature of being under construction but does not exemplify it. So far, however, the commuter has not yet been afforded any understanding of the actual subway system of the city, i.e. Y. To achieve this, the assistant goes on to add that "she should *impute* the exemplified features to Y." Once she does this, she is given a Z-representation *of* Y, or equivalently she represents Y as Z by X. Subsequently, if the actual subway system possesses, in some sense, the imputed features, she is then afforded an understanding of it by having an *effective* Z-representation of Y. In sum, *epistemic access to the subway system as the object of understanding is achieved through the public transport system representation as its vehicle*.

Having considered the above example, some points about Elgin's account of representation are in order. Firstly, we might have interpreted X's elements and features differently, for instance by taking vertices as lines and edges as parallel alignments of lines. Which interpretation is appropriate, and therefore which features are instantiated and then exemplified by a representation as, and finally, how effective a representation is, all depend on the way in which the cognitive subject wants to understand her object, which is a contextual-pragmatic issue. After some months, for instance, the commuter may be willing to understand how she will reach a point by stations which will open soon. Now the graph exemplifies the stations under construction as well. Secondly, although Elgin's account gives a significant weight to contextual-pragmatic factors, the adequation between the source and the target of representation matters after fixing the context: a representation is *exemplificatively adequational* just in case it exemplifies features it shares with its target. This type of adequation acknowledges some sort of pragmatics-dependent correspondence between a representation and its target, though it is not too strong to be a similarity-based adequation, meaning that it does not imply that an effective model resembles its target. To have this latter, another notion of representation is needed, which will be introduced in the next subsection.

Among accounts affording epistemic access to the world, scientific models hold significant importance in understanding natural phenomena. According to Elgin's proposal, we understand the behavior of gases (Y) *as* an ideal gas (Z) through the billiard model (X), and the quantum realm (Y') *as* a world without gravity, constituted by particles and fields, (Z') via the standard model (X'). Of course, we could have understood the former object *as* a non-ideal gas through the van der Waals model, and the latter object *as* a world with gravity via a quantum gravity model. Thus, understanding is always *understanding as*, since representation is always *representation as*. Putting aside this aspect, however, what matters here is the non-propositional nature of exemplification and representation. The vehicle of understanding, i.e. the carrier of representation with its interpretation, and the object of understanding, all of which are construed non-linguistically, are tethered together non-propositionally through the tools of instantiation, exemplification, and imputation. That being so, the final shift argument, echoing some sort of *representation holism*, runs as follows:

1. Truth is a propositional tether;
2. Representation as is a non-propositional tether;
3. Scientific accounts are non-propositional;
4. Thus, representation as, rather than truth, tethers scientific accounts to the world.

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In developing her account of representation, Elgin (2017, pp. 261–2) sees no need for any mechanism, more fundamental than exemplification and representation, that might suggest a form of similarity-based adequation between a scientific account and its corresponding object. Consequently, the notion of math-first realism, which is committed to the representation thesis, finds no foothold within her revisionary epistemology, even as the attitude dilemma is addressed through a non-propositional strategy. To remedy this situation, it is crucial to modify her framework to make it more inclusive, thereby allowing for the articulation of both anti-realist and realist stances. In the following subsection, I will undertake this task.

4.4 A more accommodating revisionary epistemology

A scientific account contributes to our understanding of a target by representing it. Thus, if representation is regarded exemplificatively, independent of any resemblance between an account and its target, then understanding becomes a non-factive cognitive attitude. Conversely, if representation is defined in terms of a structural relation such as isomorphism, it allows an account to be representational in a structural sense, rendering understanding a factive notion. Consequently, whether understanding and other cognitive attitudes dependent on the concept of representation are classified as *factive* or *non-factive* hinges on how the notion of representation is articulated, signifying whether it is taken as a *structural* or an *exemplificative* relation. Fortunately, philosophers specializing in scientific representation have analyzed this concept in various ways, attributing opposing characteristics to it. In what follows, I will outline two accounts to illustrate how these contrasting attributes can be captured.

Drawing on Elgin's theory of representation and exemplification, Frigg and Nguyen (2017; 2020, Chs. 8 and 9) have proposed an account of scientific representation, named "DEKI," to address the question "in virtue of what does a scientific model represent its target?". To see how DEKI functions, let X be the carrier of a scientific model that aims to represent its target, denoted by T, as Z. This *as* is achieved through using an interpretation map, labeled I, which is a bijection from X's features to Z's features. The pair $M = \, < X \, I >$, referred to as a "model," *represents* T *as* Z if and only if the following conditions are met:

1. M denotes T;
2. Given $Z_1 \, Z_2 \ldots Z_m$ as the relevant features of Z, determined by contextual-pragmatic factors, M I-exemplifies $Z_1 \, Z_2 \ldots Z_m$, which means X instantiates $I^{-1}(Z_1) \, I^{-1}(Z_2) \ldots I^{-1}(Z_m)$;
3. Depending on contextual-pragmatic factors, the agent dealing with M alters $Z_1 \, Z_2 \ldots Z_m$ using a key map, K, to obtain new features $Q_1 \, Q_2 \ldots Q_l$;
4. The agent imputes at least one of the features $Q_1 \, Q_2 \ldots Q_l$ to T.

As we observe, DEKI is a reconstruction of Elgin's conception of representation, articulating clearly its different parts. Significantly, it brings into the spotlight the alteration of exemplified features, reified by a key map, though Elgin (2017, p. 260) herself mentions its necessity. The impetus for highlighting such a step arises particularly in contexts of idealization within scientific modeling, where the features a model exemplifies can significantly deviate from the actual properties of its target. Given pragmatic-contextual considerations regarding the degree of idealization and the specific features to be idealized, it becomes necessary for the agent to alter and then impute these characteristics. Another distinctive element of this approach is its departure from a

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structuralist notion of representation, which the math-first realist is committed to. The DEKI framework does not necessitate any form of structural adequation or correspondence between X and T, induced by mathematical relations such as (partial) isomorphism. In this light, it can be understood as embodying an *exemplificative conception of representation*.

The second account posits that representation rides on a form of structural similarity between scientific models and their targets. According to Bueno and French's (2011) structuralist view, particularly, scientific account X represents its target T only if there exists a partial isomorphism between the structures attributed to X and T. This approach suggests that representation necessitates a certain kind of correspondence, embodied by the structural similarity that partial isomorphism induces. Therefore, it articulates a *structuralist notion of representation*. My aim here is not to adjudicate between these two accounts but rather to highlight the distinct ways in which the concept of representation can be construed, thereby accommodating both factive and non-factive conceptions of cognitive attitudes. Prior to this exploration, it is pertinent to adopt an unqualified use of representation to broaden the applicability of Elgin's epistemic framework. By defining representation as a holistic, non-propositional relation between a scientific account and its object, we arrive at the following definitions of cognitive attitudes:

> *Acceptance*: To accept scientific account A is to be disposed, when attending to issues raised or items denoted by A, normally to treat it as a representation of its object O in a manner conducive to achieving one's cognitive goals.

> *Understanding:* To understand object O through scientific account A involves successfully representing O by A, facilitated by a grasp of A, to achieve cognitive ends.

These components constitute *the more accommodating framework*, which is non-veritistic, tethering cognitive attitudes to the world through a non-linguistic relation, namely representation. This is also holistic, since the vehicle of acceptance and understanding is non-propositional, emphasizing their holistic nature. The determination of whether this framework is factive or non-factive turns on the specific notion of representation that is to be adopted. Deploying the exemplificative conception results in Elgin's perspective, which is explicitly non-factive. Conversely, adopting the structuralist notion of representation, as articulated by Bueno and French, imbues acceptance and understanding with factive qualities.

Before concluding this section and proceeding to introduce *the math-first view epistemically augmented* in the next section, it is pertinent to address some concerns. First, one might argue that this reading of Elgin's epistemology – maintaining its holism and non-veritism while challenging its pragmatic spirit to create a more accommodating framework that allows for both factive and non-factive interpretations – undermines grounds she gives for revisionism in the epistemology of science. According to this objection, her epistemic framework precludes a representation from being *both* veritistically and structurally adequational. In other words, her arguments for non-veritism do not allow a representation to be structurally adequational, too. Two responses are conceivable. The first, which could be termed the "essential" response, might demonstrate that Elgin's concerns regarding the mirroring of nature by scientific accounts dissipate if they are linked to the world through a form of representation that admits structural adequation. For example, one could argue that partial isomorphism effectively captures the role of idealization in science (see e.g. Ladyman and French 1998;

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French and Ladyman 1999). The second, which might be called the "shortcut" response, suggests that identifying the inconsistency requires more than showing that non-veritism rebuts the existence of structural adequation between scientific accounts and their targets. Instead, it must be shown that *holism* does this job. Given that two sorts of rationale, namely non-veritism and holism, support the revisionary epistemology, and that our way of introducing Elgin's framework and its more accommodating version lean explicitly on holism, the objection fails unless it can be shown that holism necessitates a notion of representation that blocks the idea of mirroring of nature by science, thus placing the burden of proof on the objector.[8]

The second concern shifts focus to the transition from conceiving the vehicles of attitudes as non-propositional to viewing them as structural entities. It might be contended that Elgin's holism indicates their non-propositional nature but doesn't necessarily characterize them as structural. As such, an additional step should be taken to resolve the attitude dilemma in a way that conforms to our specific desires. To assuage this, we revisit da Costa and French's argument, which posits the necessity of linguistically formulating theories due to the perceived absence of non-propositional attitudes. Elgin's framework, as we have demonstrated, acknowledges the existence of such attitudes, thereby undercutting the justification for the indispensability of choosing the first horn of the dilemma. Furthermore, Morales Carbonell (2025) has recently introduced a graph-theoretic model for understanding, which characterizes this attitude as not only non-propositional but also structural in content. Thus, Elgin's framework supports rejecting the necessity of linguistic formulations of scientific theories (the negative aspect) and advocates for the viability of non-propositional attitudes with structurally conceived vehicles, as suggested by the graph-theoretic account (the positive aspect).

5. The math-first view epistemically augmented

In this section, I aim to detail the math-first approach, now epistemically augmented, drawing upon the epistemic tools furnished by the more inclusive framework. Subsequently, I will delineate two perspectives within this approach, namely realist and anti-realist positions. This division illustrates that the approach transcends mere allegiance to math-first realism, establishing itself as an overarching philosophical stance that accommodates diametrically opposed views.

As canvassed previously in Section 2, Wallace's initial formulation of the math-first approach comprises two fundamental components: the vehicle and the representation theses. With the epistemic enhancements introduced in Section 4, we are now in a position to further enrich this view with an epistemic component, aptly designated as the "attitude thesis." Taken together, these elements form *the epistemically augmented math-first approach*:

1. *Vehicle Thesis*: Physical theories are collections of structures;
2. *Representation Thesis*: Physical theories are tethered to the world by representation;
3. *Attitude Thesis*: Cognitive attitudes directed toward physical theories have structural vehicles.

Dismissing both truth and belief, however, how can a realist articulate her stance toward a physical theory? As previously argued, truth is not the only option to reify the

[8]This appears to be a complex task, particularly when encountering accounts that favor holistic yet factive conception of (objectual) understanding (see e.g. Kvanvig 2003, 2009).

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concept of correspondence; structural adequation equally serves this purpose. In this way, the main concern of French and da Costa regarding tethering physical theories to the world in the realist sense is addressed. On the other hand, the concept of belief is equally dispensable. Recalling the dichotomy between acceptance and belief portrayed by Cohen, which Elgin leverages to distinguish veritism from non-veritism, we now propose a dichotomy between structural and exemplificative senses of representation. This dichotomy gives rise to two notions of acceptance, factive and non-factive. Thus, one can eschew belief in favor of factive acceptance and still maintain the stance of a scientific realist:

> *Factive Acceptance:* To accept scientific theory *T* is to be disposed, when attending to issues raised or items denoted by *T*, normally to treat it as a representation of its target *O* in the *structural sense*, in a manner conducive to achieving one's cognitive goals.

This sort of realism, i.e. the realism within the math-first view epistemically augmented, may be supplemented by stronger cognitive attitudes, including:

> *Factive Understanding:* To understand object *O* through scientific theory *T* involves representing *O* by *T* in the *structurally adequational sense*, facilitated by a grasp of *T*, to achieve cognitive ends.

In this way, the scientific realist, without appealing to any linguistic markers, asserts her stance. Someone who advocates for an Elginian form of anti-realism also feels comfortable stating her view, according to which there is no similarity-based correspondence between science and the world. For her, our best physical theories represent the world but in the exemplificatively adequational sense. This form of anti-realism, for instance, can now be characterized completely non-linguistically:

> *Non-Factive Acceptance:* To accept scientific theory *T* is to be disposed, when attending to issues raised or items denoted by *T*, normally to treat it as a representation of its target *O* in the *exemplificative sense*, in a manner conducive to achieving one's cognitive goals.[9]

This position may be enriched with additional cognitive attitudes, for instance with:

> *Non-Factive Understanding:* To understand object *O* through scientific theory *T* involves representing *O* by *T* in the *exemplificatively adequational sense*, facilitated by a grasp of *T*, to achieve cognitive ends.

By adopting the dual meanings of representation, there is no need for someone who neither completely forsakes the idea of science mirroring nature nor subscribes to a perfect correspondence between all aspects of a scientific theory and the world to identify her stance with slogans such as "acceptance of a theory involves belief only that it is empirically adequate" (van Fraassen 1980, p. 12). Similar to constructive

[9] It is well to remember that taking something as a representation of the world in the exemplificative sense does not imply that the representation is effective, in that there exists some exemplificative adequation between them. To have an effective representation, the world should possess the features that are exemplified by the representation and then imputed to the world.

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empiricism,[10] it is possible to hold a view that encompasses both veritistic and pragmatic dimensions. This can be articulated by employing a combination of the structuralist and exemplificative notions of representation:[11]

> *Semi-Factive Acceptance:* To accept scientific theory *T* is to be disposed, when attending to issues raised or items denoted by *T*, normally to treat it as a representation of the observable aspects of its target *O* in the *structural sense* and as a representation of its unobservable aspects in the *exemplificative sense*, in a manner conducive to achieving one's cognitive goals.

In so doing, the advocate of the math-first view accomplishes her objective, positioning herself to philosophize on the theoretical aspects of science without linguistic constructions, but solely through structural ones.

6. Conclusion

Since the 1950s, the philosophy of science has witnessed a shift away from linguistic methodologies in formulating and tackling philosophical issues, veering instead towards the adoption of mathematical-structural approaches. This perspective posits that scientific theories and their interrelations are not confined within linguistic boundaries, even suggesting the existence of entities in the world without linguistic counterparts. This raises a pertinent question: How can we understand the world through non-linguistic theories riveted with the world structurally rather than linguistically? After all, traditional epistemologists submit that cognizing the world involves forming cognitive attitudes towards propositions. This presents a significant challenge for a philosophy of science striving to transcend linguistic limitations. In this article, we endeavored to overcome this challenge. Leveraging Elgin's revisionary epistemology, we initially argued for the holistic nature of cognitive attitudes, which can encompass structural entities. Contrary to the anti-realist aspects peculiar to Elgin's framework, we proposed a more versatile revisionary approach, capable of being interpreted in both factive and non-factive manners. We then explored how the math-first view, integrating select aspects of a language-free philosophy of science, could be further enriched through the epistemic tools provided by a more flexible framework. This enhancement facilitates a more effective articulation of positions of both realists and anti-realists concerning physical theories.

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[10]"If I accept a theory then I believe that it is empirically adequate, and I also commit myself to seeing nature through that theory's eyes. Thus, in addition to that belief in the theory's empirical adequacy, there is a pragmatic aspect to acceptance The accepted theory is thus the guide both to theoretical and practical life" (van Fraassen 2001, p. 164).

[11]I do not wish to claim here that the resources provided by the more accommodating framework can fully capture van Fraassen's philosophy of science and all its nuances. His system of thought on science comprises at least three central pillars, which may be interconnected to some degree (Okruhlik 2014). These are constructive empiricism (van Fraassen 1980), epistemic voluntarism (van Fraassen 1984), and a hybrid account of scientific representation (van Fraassen 2008). Probing how his system relates to Elgin's non-veritistic epistemology and her account of representation, and in turn to the more accommodating framework developed in this article, is an intriguing question but one that is beyond the scope of our discussion. My aim here is simply to offer an initial suggestion that can be enriched with more details in further investigations.

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