isomorphism relation for a class of compact metrizable structures. This provides a more direct proof of the theorem above and allows one to view the earlier results of Sabok and of Clemens, Gao, and Kechris as consequences of it.

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Anton Bobkov, ***Computations of Vapnik–Chervonenkis Density in Various Model-Theoretic Structures***, University of California, Los Angeles, 2017. Supervised by Matthias Aschenbrenner. MSC: Primary 03C, Secondary 05C. Keywords: VC-density, trees, Shelah–Spencer graphs, superflat graphs, p-adic numbers.

Abstract

Aschenbrenner et al. have studied Vapnik–Chervonenkis density (VC-density) in the model-theoretic context. We investigate it further by computing it in some common structures: trees, Shelah–Spencer graphs, and an additive reduct of the field of p-adic numbers. In the theory of infinite trees we establish an optimal bound on the VC-density function. This generalizes a result of Simon showing that trees are dp-minimal. In Shelah–Spencer graphs we provide an upper bound on a formula-by-formula basis and show that there isn't a uniform lower bound, forcing the VC-density function to be infinite. In addition we show that Shelah–Spencer graphs do not have a finite dp-rank, so they are not dp-minimal. There is a linear bound for the VC-density function in the field of p-adic numbers, but it is not known to be optimal. We investigate a certain P-minimal additive reduct of the field of p-adic numbers and use a cell decomposition result of Leenknegt to compute an optimal bound for that structure. Finally, following the results of Podewski and Ziegler we show that superflat graphs are dp-minimal.

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Athipat Thamrongthanyalak, ***Extensions and Smooth Approximations of Definable Functions in O-minimal Structures***, University of California, Los Angeles, 2013. Supervised by Matthias Aschenbrenner. MSC: Primary 03C64, Secondary 14P10, 32B20. Keywords: o-minimal structures, Whitney Extension Theorem.

Abstract

A jet of order m on a closed set $E \subseteq \mathbb{R}^n$ is an indexed family $(f_\alpha)_{\alpha\in\Lambda}$, where $\Lambda = \{(\alpha_1,\dots,\alpha_n) \in \mathbb{N}^n : \sum_{i=1}^n \alpha_i \leq m\}$. In 1934, H. Whitney proved Whitney's Extension Theorem, which gives a necessary and sufficient condition on the existence of C^m-extensions of a jet of order m on a closed subset of $\mathbb{R}^n$. In the same year, he asked how one can determine whether a real-valued function on a closed subset of $\mathbb{R}^n$ is the restriction of a C^m-function on $\mathbb{R}^n$ and gave an answer to the case $n = 1$. Later, the case $m = 1$ was proved by G. Glaeser using the concept of "iterated paratangent bundles". A complete answer to Whitney's Extension Problem was provided much later in early 2000s by C. Fefferman.

In the first part of this thesis, we study the above questions in an o-minimal expansion of a real closed field. We prove a definable version of Whitney's Extension Theorem. In addition, we solve the C^1 case of Whitney's Extension Problem in o-minimal context.

In the rest of this thesis, we discuss the following question: Suppose R is a real closed field and U is an open subset of R^n. If $f: U \to R$ is continuous, definable in an o-minimal expansion of R, and $\varepsilon \in R^{>0}$, is there a definable C^m-function $g: U \to R$ such that $|g(x) - f(x)| < \varepsilon$ for all $x \in U$? We gave a positive answer to this question. This result was inspired by a series of articles by A. Fischer.

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