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Does Frege Have Aristotle's Number?

ABSTRACT: *Frege argues that number is so unlike the things we accept as properties of external objects that it cannot be such a property. In particular, (1) number is arbitrary in a way that qualities are not, and (2) number is not predicated of its subjects in the way that qualities are. Most Aristotle scholars suppose either that Frege has refuted Aristotle's number theory or that Aristotle avoids Frege's objections by not making numbers properties of external objects. This has led some to conclude that Aristotle's accounts of arithmetical and geometrical objects differ substantially. I close this supposed gap by showing that Aristotle's arithmetical objects, like geometrical objects, are just certain sensible things qua certain properties they in fact possess. Specifically, numbers are pluralities qua quantitative or relational properties like ten units or ten. I show that this view is resistant to the Fregean concerns about arbitrariness and numerical predication.*

KEYWORDS: Aristotle, Frege, arithmetic, number, relative

Introduction

Frege looms large in discussions of Aristotle's philosophy of arithmetic. According to Frege, number is so unlike the things we accept as properties of external objects that it cannot be such a property (1980: 28–33). Two of the principal dissimilarities he highlights are:

(1) Arbitrariness: while a color belongs to an object 'independently of any choice of ours', the number varies depending on how we choose to think of it.
(2) Numerical predication: while the foliage's color belongs to each leaf and to their totality, its number belongs to neither (28).

Frege concludes that number is not a property of external objects.

Aristotle scholars broadly endorse Frege's conclusion and suppose it has implications for interpretations of Aristotle's number theory (e.g., Annas [1976: 30–38]; Lear [1982: 183, 183 n. 28, 185]; Mignucci [1987: 188]; Halper [1989: 251–52, 257, 263]). Most argue either that Frege refutes Aristotle or that Aristotle avoids Frege's objections by not making numbers properties of objects (e.g., Annas [1976: 33]; Lear [1982: 185]; Mignucci [1987: 198–201]; Gaukroger [1982: 320–21] is an exception; however, his reply on Aristotle's behalf is

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incomplete). Consequently, some conclude that Aristotle's philosophies of geometry and arithmetic differ substantially. For example, while Jonathan Lear finds Aristotle's philosophy of geometry plausible, he claims that 'the main obstacle preventing Aristotle from giving a successful account of arithmetic is that number is not a property of an object' (1982: 183). On Lear's interpretation, geometry and arithmetic deal with completely different things: geometry considers an object's property separated in thought (165–75), while arithmetic considers the object itself *qua* indivisible (183–84). Further, only geometrical objects are, strictly speaking, isolated via abstraction or subtraction (*aphairesis*; 1982: 184). Like Lear, Julia Annas finds significant differences between Aristotle's geometry and arithmetic (1976: 31); on her view, Aristotle's *Metaphysics* M.3 account of mathematical objects applies well enough to geometrical magnitudes but poorly to numbers (33).

Certainly, arithmetical and geometrical objects have different natures for Aristotle: numbers (*hoi arithmoi*) are discrete, while magnitudes (*ta megethē*) are continuous ('magnitude' translates *megethos*, which in a mathematical context means a geometrical quantity). But in M.3, where Aristotle gives his most detailed discussion of mathematical objects, he gives a single account that is clearly supposed to cover both magnitude and number.

Annas's and Lear's works have been deservedly influential. But perhaps one reason Aristotle's number theory receives little interpretative attention is that scholars concerned about the Fregean objections suspect that Aristotle's position is ultimately incoherent.

My aim is to show that for Aristotle both geometrical and arithmetical objects are isolated in the same way, namely, by considering certain sensible things *qua* properties they in fact have. (In Katz [2019] I offer an interpretation of Aristotle's philosophy of geometry along these lines.) This is not to say there are no important differences between Aristotle's accounts of magnitude and number. For example, a shape belongs to an individual while a number belongs only to a plurality. Further, Aristotle's account of number is more complex than his account of magnitude. He distinguishes between two kinds of number: countable number and counting number (*Ph.* Δ.11 219b6–7).

Nevertheless, on my interpretation, arithmetical objects, like geometrical objects, are just certain sensible things *qua* certain quantitative properties. I argue that countable numbers are such things as seven dogs or ten units and that these are, like magnitudes, definite quantities. By contrast, seven or ten *tout court* are counting numbers, and these are relatives. This interpretation requires that certain external objects (pluralities) have quantitative properties like seven dogs and relational properties like seven and that each of these is a kind of number—something Frege is thought to have ruled out.[1] I develop the interpretation (section 1) and then show how it handles the Fregean concerns (section 2).

[1] While many Aristotle scholars suppose that Frege has refuted Aristotle, some philosophers of mathematics endorse key elements of Aristotle's view; see, for example, Franklin (2014) and Hossack (2020). Both adopt several Aristotelian commitments and deflect Frege's objections. Unfortunately, their useful insights have not been taken up by Aristotle scholars.

1. Aristotelian Numbers

1.1 Number as a Quantity

We should begin by clarifying the key terms: quantity (*poson*), plurality (*plēthos*), and number (*arithmos*). For Aristotle, a quantity is a nonsubstantial being (*Cat.* 6): it is a property of some underlying subject. For example, a heap of rocks, which is a plurality, has the property seventeen rocks—a quantity. (Halper [1989: 252] insists the subject must be a substance, but Aristotle allows that many nonsubstantial substrata have attributes: fire is hot, odors are sharp, mountains are large, etc.) Aristotle's examples of pluralities that have quantities are groups of things like dogs or horses (e.g., *Ph.* Δ.14 224a14–15; *Metaph.* N.1 1088a10–11). Thus, a plurality *has* a quantity. Yet, Aristotle elsewhere suggests that a plurality *just is* a kind of quantity (*Metaph.* Δ.13 1020a8–11).

This is not inconsistent. In one sense a plurality has a quantity and in another it is a quantity. Consider Frege's example of a tree's foliage. The foliage is a plurality since it is divisible into discrete parts (e.g., leaves). But this plurality is not identical with the quantity 1,000 leaves, as the foliage has other properties besides this quantity: it is green, an oxygen-producer, etc. The plurality *has* the quantity 1,000 leaves as one of many properties. But Aristotle also maintains that each science isolates the properties belonging to its domain via subtraction of irrelevant properties (*aphairesis*) and studies sensible things *qua* the isolated properties. As he explains in *Metaphysics* M.3, each science treats its object *qua* properties relevant for that science and studies just those other properties it has in virtue of these (1077b17–1078a26; see also E.2 1026b3–11; for arguments connecting *aphairesis* with Aristotle's use of the *qua* locution, see Cleary [1985] and Katz [2019]).

Aristotle calls both accidental properties like musicality and accidental beings like 'the musical *thing*' (*to mousikon*, i.e., something *qua* its musicality) 'accidents' (*symbebēkota*; e.g., *Metaph.* Δ.9 1017b30–2, Z.6 1031b22–6; *Soph. el.* 5 166b29–30, 24 179a28, 24 179a36–7). I follow Gareth Matthews in calling accidental beings 'kooky objects' (1982: 224). The arithmetician's object is a kooky object like 'the 1,000 leaved thing': a sensible plurality *qua* a quantitative property (1,000 leaves). Likewise, a geometrical object is a sensible figure *qua* one of its quantitative properties, e.g., triangular shape. (In Katz [forthcoming], I discuss kooky objects and their role in Aristotle's philosophy of geometry.)

Thus, the arithmetician can consider the foliage *qua* its quantity of 1,000 leaves, while the ecologist can consider the same foliage *qua* its property of oxygen-producer. When its nonquantitative properties are subtracted and the plurality is considered just *qua* 1,000 leaves, the plurality just is the number 1,000 leaves. This kooky object, 'the 1,000 leaved thing,' is the foliage just insofar as it has the quantity 1,000 leaves.

Note that while 'plurality' suggests a discrete multiplicity, for Aristotle whatever is not one and indivisible is a plurality (*plēthos*); accordingly, even continuous things are pluralities. Laura Castelli's (2018) observation about *plēthos*, which has no exact English equivalent but which everyone translates as 'plurality', is pertinent: *plēthos*

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means not just discrete multiplicity but more generally quantity or mass—and Aristotle's use of *plēthos* covers both meanings. A *plēthos* is not just what *is* divided into noncontinuous parts but also what *can* be so divided (*Metaph.* Δ.13 1020a10–11, I.3 1054a22–3). Hence, even a continuous quantity is a plurality (rather than a one) just because it is divisible (Castelli 2018: 97). However, a continuous plurality (e.g., water), unlike a discrete plurality, is not a number as such; it is a number only when related to an imposed measure and so treated as a discrete plurality (e.g., of milliliters; see section 1.6.1 below)

This brings us to number (*arithmos*). Since for Aristotle numbers are pluralities, the *arithmoi* include only the natural numbers two and greater. (Ancient Greek mathematicians broadly agree that the first number is the first plurality [two]; see Euclid's *Elements,* book VII, definition 2.) Going forward, the word 'number' should be understood to cover only the *arithmoi*. Aristotle maintains that plurality is 'like a genus of number' (*Metaph.* I.6 1057a2–3). Only measured pluralities are numbers—and specifically, pluralities measured by the one (*Metaph.* Δ.13 1020a13, I.6 1057a3–4, N.1 1088a5–6). Thus, number is not a plurality of anything whatsoever; it is a plurality of ones, indivisibles, or units. And because units are measures, number is a plurality of measures (*Ph.* 3.7 207b7–8; *Metaph.* Δ.13 1020a13, I.1 1053a30, M.9 1085b22, N.1 1088a5–6).

Aristotle asserts both that certain pluralities *are* numbers and that certain pluralities *have* numbers (*Metaph.* N.1 1088a10–11; *Ph.* Δ.11 220a23–4, Δ.14 224a13–15). Again, this is not inconsistent. The foliage has as a property the number 1,000 leaves (Hossack [1980: 3] takes up this Aristotelian idea that number is a property of pluralities). And the foliage can be considered just insofar as it has this number. So considered, the plurality just is the number 1,000 leaves. (In Katz [2021], I argue that for Aristotle a plurality is a heap [*sōros*]. When the heap is measured, it has a number and, *qua* its quantity, *is* a number.)

To be clear, the property 1,000 leaves and the plurality itself *qua* this property ('the 1,000 leaved thing', a kooky object) are both rightly called 'number'—just as an ornithologist rightly calls 'male' both a bird's property of maleness and a bird *qua* its maleness ('the male', a kooky object). However, the kooky objects—the sensible things *qua* the relevant properties—are for Aristotle the arithmetician's and the ornithologist's proper objects (he commits himself to this most clearly in *Metaph.* M.3).

Another clarification: Aristotle allows that no number is *the* number of a given plurality (see also Katz 2021: 212, n. 74). Each plurality has as many numbers as it has measures, and a plurality can have a variety of measures (e.g., a tree's foliage might have the numbers 1,000 leaves, 8,000 lobes, and 10,000 veins.) This is the source of many commentators' assessment that Aristotle cannot really think numbers are properties of external objects (or objects considered *qua* those properties), for on the interpretation I have sketched out, number seems quite arbitrary, and Frege has argued that given this apparent arbitrariness, number cannot be a property of external objects. In section 2.1, I show that Aristotle's view that a given plurality has many numbers is compatible with number's being a property of objects. But before we can evaluate Aristotle's view, we must get it all on the table.

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1.2 Number as a Relative

The numbers I have so far described are quantities like 1,000 leaves. This looks incompatible with another Aristotelian commitment, namely, that 'a number, whatever it is, is always a number of something, of fire or earth or units' (*Metaph.* N.5 1092b19–20).[2] Aristotle makes the same point in *Metaphysics* I.2: 'in all things the number is of things [*arithmos tinōn*] of a certain kind' (1054a7). (The statement begins 'If indeed . . .' [*eiper*, 1054a5], but it is clear from the context that Aristotle endorses the claim [*eiper* is factive]). If number is always *of* something else, then it belongs to the category of relative (*pros ti*). But a quantitative number is not a relative; 1,000 leaves is not *of* something else.

I argue that quantitative number for Aristotle is one of two kinds of number. The other kind is a relative; it is a property the plurality has relative to a measure. More precisely, it is the plurality just insofar as it has this relational property.[3] For example, the foliage has the property 1,000 relative to the measure leaf. That is, 1,000 is a relational property of the plurality, and the plurality *qua* this property is a number: 1,000.

There is considerable textual evidence that Aristotle considers number a relative and more specifically the measurable relative to a measure. (That the measurable and its measure are relatives is clear from, e.g., *Metaph.* Δ.15 1021a29–30, I.6 1056b21–2, b32–4, 1057a4–6, a16–17.) We know from his treatment of relatives in *Categories* 7 that 'we call *relatives* all such things as are said to be just what they are, *of* or *than* other things, or in some other way *in relation to* something else' (6a36–7). And we have seen that this is just what Aristotle claims about number in *Metaphysics* N.5 and I.2.

Further, numbers, like relatives, are subject to what we now call Cambridge change: I have the property tall relative to my son, and without undergoing any decrease in height, I have the property short relative to my brother. The same subject is tall and short without itself changing, but rather just because its correlative has been changed (see *Metaph.* N.1 1088a33–5). Similarly, the same foliage is 1,000 and 8,000 just because its correlative, the measure, has been changed from leaf to lobe. (Aristotle denies that such accidental (*kata symbebēkos*) 'change', which is a special feature of relatives, is *really* change [*Ph.* 5.2 225b11–13, 7.3 246b11–12; *Metaph.* N.1 1088a33–5].)

Aristotle is clearest that number is a relative and specifically the measurable or measured in *Metaphysics* I.6. He writes:

> Plurality is like a genus of number: for number is plurality measured by one, and in a way one and number are opposed to each other, not as

[2] Translations of Aristotle are from Barnes (1984), except for *Metaphysics* I (Castelli 2018) and *Metaphysics* M–N (Annas 1976). 'Human being' replaces 'man' for *anthrōpos* throughout; other modifications noted.

[3] The idea that number is relational is taken seriously in modern philosophy of mathematics. For example, for Kessler (1980: 69) and Franklin (2014: 37, 104), number is a relation holding between a plurality and a special sort of property. Note that this not quite Aristotle's view since, as I will argue, for Aristotle relational number is a plurality *qua* one of its relational properties (rather than a relation holding *between* the plurality and one of its properties).

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> contraries but as we have said that some of the relatives are. They are opposed in that one is measure and the other is measurable. (1057a2–6)

Aristotle then explains that the one and a plurality will be correlatives if the one is a measure and the plurality a number (I.6 1057a15–17). He writes: 'The one is opposed to the many in numbers as measure to measurable; and these are opposed as those relatives which are not relative in their own right' (I.6 1056b32–4).[4] Since number is measurable plurality, Aristotle is identifying number with the measurable and the one with the measure. Number, then, is one of the *relata* in the measurable-measure relation. (Number is not an 'in its own right' (*kath' hauto*) relative because measurable things such as pluralities 'are what they are independently and are relative only in so far as the relative relates to them' [Duncombe 2020: 151]; for example, the foliage is a measurable thing and thus relative only because a measure is related to it.)

Some scholars have recognized that for Aristotle number is somehow a relative (see Alexander of Aphrodisias *in Metaph.* 86.5–6; Klein [1968: 48]; Halper [1989: 273]; and Annas [1976: 35]). What has not been worked out is *how* number is both a quantity and a relative and whether an account placing number in both categories is coherent. In what follows, I align quantitative number with what Aristotle elsewhere calls countable number and relational number with what he elsewhere calls counting number (*Ph.* Δ.11 219b6–7).

1.3 Different Pluralities, Same Number

In *Physics* Δ.11, Aristotle claims that a number belongs to what it numbers and 'belongs also elsewhere'. Ten is 'the number of these horses' but also the number of other things (220a22–4). At Δ.12 220b10–12 he explains that 'the number of [*ho arithmos* + genitive] a hundred horses and a hundred men is the same'. Thus, not only does the same plurality have different numbers (section 1.1); different pluralities have the same number. As Aristotle writes at Δ.14 223b4–6, 'if there were dogs, and horses, and seven of each, it would be the same number' (*ho autos arithmos*).

Aristotle is clear that it is only the number *of* each plurality that is the same; the plurality itself is different in each case. In the Δ.12 passage, he continues: 'but the things numbered [*hōn arithmos*] are different—the horses or the men' (220b12). And in Δ.14, he writes: 'It is said rightly, too, that the number of the sheep and of the dogs is the same *number* if the two numbers are equal, but not the same *decad* or the same *ten*' (224a2–4). It is not the same ten because 'the things of which it is asserted differ; one group are dogs, and the other horses' (224a14–15).

Since Aristotle insists that the number is the same but that it is 'not the same ten', he must be working with two kinds of number. These are 'the things numbered' (*hōn arithmos*), for example, ten horses, and 'the number of' those things (*ho arithmos* + genitive), that is, ten. The first is what Aristotle calls counted or countable number

[4] I add 'in numbers' to Castelli's translation (she reads *tois pollois* with EJϒ rather than Ross's *kai ta polla ta hen arithmois*). Castelli accepts that 'in numbers' explains how 'the many' should be understood (2018: 260).

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(*to arithmoumenon kai to arithmēton*). The second is what he calls number by which we count or counting number (*hō arithmoumen; Ph.* Δ.11 219b6–7). While both are numbers, 'these are different kinds of thing' (219b8–9).

Notice that Aristotle's examples of the first kind of number are definite quantities of specific things, for example, ten horses. (This is the quantitative number discussed in section 1.1.) We can say of the plurality in the field that it is a ten of horses, and we can say of the plurality in the kennel that it is a ten of dogs. Ten horses and ten dogs are different numbers because they are pluralities of different measures (horse, dog). Yet, as Aristotle argues, they are nevertheless the same *number*: ten (*Ph.* Δ.12 224a2–4). That is just to say that each plurality has the same property—ten—relative to a given unit (horse, dog). (This is the relational number discussed in section 1.2.)

This is not something special about number; it applies even to non-numerical relatives. For example, tall is a relational attribute my brother has relative to me and also a relational attribute this tree has relative to that shrub. While the property-bearers differ in each case (my brother, the tree) and while each has that property relative to a different *relatum* (me, the shrub), the property (tall) is the same in both cases. In the same way, while the bearer of the relational property ten differs in each case (plurality of horses, plurality of dogs) and while each has the property ten relative to a different *relatum* (horse, dog), the property (ten) is the same in both cases. This relational property, shared by all countable tens, is a number *by which* we count. For example, when I count up dogs, I go through the dog-units one by one, each time counting off the next in the series of counting numbers two, three, etc. I count the countable dog-number *by* the counting numbers.

While I have aligned counting and relational number, scholars who do not recognize relational number typically align counting number with quantitative number of pure units, that is, mathematical number (e.g., Ross 1936: 598, 611; Gaukroger 1982: 312–13; Coope 2005: 89–90). But such an alignment is implausible because it fails to account for Aristotle's claims that (a) number is also a relative and (b) counting number is the number *of* the plurality (*ho arithmos* + genitive)—which, as we have seen, invokes the category of relatives.

1.4 How Countable and Counting Number are Related

Although countable and counting numbers belong to different categories, they are closely related. In *Physics* Δ.14, Aristotle argues that just as equilateral and scalene triangles are different kinds of triangle but the same kind of figure (triangle), so ten of horses and ten of dogs are different kinds of ten but the same kind of number (ten) (see also *Metaph.* Δ.6 1016a31–2). What makes ten of horses a different ten from ten of dogs is just that ten is said of different things. But since neither dog nor horse is a differentia (*diaphora*) of number, ten of horses and ten of dogs are different countable numbers (different tens) but the same counting number (ten) (224a2–15).

On this analogy, the counting and countable numbers are related as genera to species: just as scalene is a species of triangle, so ten of dogs is a species of ten:

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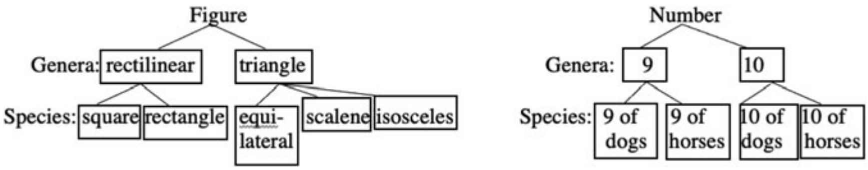

One might wonder how a genus can be in the category of relative and its particular kinds or species in the category of quantity. But Aristotle argues in *Topics* 4.4 that if a genus is a relative, the species need not be—'for knowledge is a relative, but not so grammar' (124b15–19). We find more details in *Categories* 8: the genus knowledge is a relative, while species of knowledge, like knowledge of grammar or knowledge of music, are qualities (11a32, 35–6). Knowledge is relative because it is 'called just what it is, of something else (it is called knowledge of something)' (11a24–6): it is knowledge *of* grammar or *of* music. But a particular kind of knowledge is not relative because it is not *of* anything else: knowledge of grammar is 'not grammar of something' (11a27–30; see also Duncombe 2020: 137).

To put it in Fregean terms, knowledge is unsaturated: it is knowledge of ___. Such unsaturated entities are relatives. By contrast, knowledge of grammar is saturated: knowledge of ___ has been filled in with a *relatum*: grammar. Such saturated entities are qualities. On my reading of *Physics* Δ.14 224a2–15, a counting number is to countable numbers as knowledge is to particular knowledges: ten is to ten of dogs and ten of horses as knowledge is to knowledge of grammar and knowledge of music. The counting number ten, like knowledge, is unsaturated: it is always ten of ___ and belongs to the category of relative. By contrast, the countable number ten of dogs, like knowledge of grammar, is saturated: ten of ___ has been filled in with the *relatum* dog. Such saturated entities belong to the category of quantity.

Someone may yet worry that Aristotle should not recognize relational 'number' at all because a relative is not a plurality and Aristotle defines number as a kind of plurality. But while relational number is a relative, namely, the measured thing, the measured thing (a kooky object) is ultimately just the plurality *qua* its relational property of being measured. For example, 1,000 is a plurality (the foliage) insofar as it is measured as 1,000 relative to the measure leaf. Similarly, as we have seen (section 1.1), a quantitative countable number like 1,000 leaves is really just the plurality *qua* its quantitative property 1,000 leaves. In short, the relational number 1,000, like the quantitative number 1,000 leaves, is ultimately the plurality—in the former case *qua* a relational property and in the latter *qua* a quantitative property.

1.5 The Objects of Arithmetic

But which are the objects of arithmetic: countable or counting numbers? It turns out that each is the object of a different kind of arithmetical activity and statement.

Counting up sheep is the kind of arithmetical activity that produces true statements like 'There are ten sheep in the field'. For Aristotle this statement is true

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of the plurality in the field *qua* its quantitative property ten sheep. Since ten sheep is a countable number, arithmetical statements that result from counting up a plurality are about countable numbers.

Now consider a different arithmetical activity: comparing the numbers of two pluralities. Such activity produces true statements like 'The number of sheep in the field is equal to the number of dogs in the kennel'. For Aristotle, what makes such a statement true is that the pluralities in the field and kennel have the same relational property, ten, relative to the units sheep and dog, respectively. He maintains that it is rightly said 'that the number of the sheep and of the dogs is the same number if the two numbers are equal' (*Ph.* Δ.14 224a2–3). Hence arithmetical statements comparing the numbers of different pluralities are about pluralities *qua* their counting numbers. The pluralities in the field and kennel are arithmetically the same, that is, equal, when each is considered just *qua* their counting number.

Finally, which numbers should we say are the objects of purely arithmetical statements like '4 plus 6 equals 10'? At first glance, it seems they should be the relational counting numbers 4, 6, and 10 *tout court*. (Thus, e.g., Mignucci identifies counting number with mathematical number [1987: 198].) But for Aristotle, the 10 of a purely arithmetical statement is saturated: it is a ten *of* mathematical units. A ten consisting of such pure units is what Aristotle calls mathematical, arithmetical, or *monadikon* number (*Metaph.* M.8 1083b16–17; M.6 1080b30; M.7 1081a5–6, 1081a19–20, 1082b6–7). It is distinctive of Aristotle's view that a number of pure units is the same *kind* of being as a sensible plurality like ten of sheep: both are quantities. And a pure unit is really just a sensible thing insofar as it is indivisible (*Metaph.* M.3 1077b30). Pure arithmetic considers sensible things apart from or without (*aneu*) the sensible things' movement, magnitude, or any other feature besides their indivisibility in the respect in which they are one (*Metaph.* M.3 1078a10–12 with K.3 1061a28–35). This is because whatever is one is indivisible insofar as it is one (*Metaph.* I.1 1053b7–8). A human being is one insofar as she is human and so indivisible *qua* human; accordingly, the arithmetician treats her as 'one indivisible' (M.3 1078a23–4). This does not mean that arithmetic considers a human being *qua* human; arithmetic is not concerned with human beings as living organisms. Rather, it considers human beings *qua* indivisible *qua* human being—that is, just in the respect in which each is indivisible, which is the respect in which each is one (*Metaph.* Δ.6 1016b3–6; *Ph.* Γ.7 207b5–7; see also Maher 2011: 37).

Thus '4 plus 6 equals 10' is about quantitative countable numbers: pluralities of entities from which the arithmetician has subtracted all attributes such as weight, color, etc., leaving only their indivisibility in some respect. The same flock is a ten of sheep *qua* its quantitative property ten sheep and a ten of pure units *qua* its quantitative property ten indivisibles.

1.6 Q&A

Before I show how this interpretation avoids the Fregean worries (section 2), I address two lingering questions.

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1.6.1 Why are there numbers of continuous quantities?. As we have seen (section 1.1), for Aristotle only pluralities that are discrete multiplicities are *already* measured, i.e., numbers. But because a continuous quantity is divisible, it is a plurality (rather than a one). And when a measure is imposed on it, it is (or is treated as) a divided plurality of discrete parts; as such, it is a number. As Castelli emphasizes, this just means that the continuum's parts are either not continuous with one another (2018: 167) or not treated as such. (See *Cat.* 6 4b25–6 with Katz [2021: 211 n. 71], Alexander *in Metaph.* 396.27–9, in Hayduck [1891] and Dooley [1993: 156 n. 197].)[5] Thus, for example, because a length is divisible, it is a plurality. And when placed in relation to the measure foot, this length is treated as a divided and measured plurality: a number of the measure foot (e.g., 5 feet).

This explains why Aristotle insists on the one hand that a continuous body is *much* but not *many* (*Metaph.* I.6 1056b16) and on the other that continua are numbers (*Ph.* Δ.11 220a24–6). There is no number of water as such; yet, a mass of water is a multiplicity in relation to a measure (e.g., milliliter) that marks off internal limits at equal intervals. In other words, something continuous can be treated either as one and continuous, or it can have a measure imposed on it and so be treated as a number. Aristotle makes this point about time in *Physics* Δ.11: 'as continuous [time] is long or short and as a number [time is] many or few' (220b2–3). In the same way, water is much or little as continuous and many or few as a number (of milliliters). While there is no number of water as such, there is a number of measures or units of water (e.g., milliliters).

1.6.2 In what sense is number measured *plurality?.* Aristotle defines number as measured plurality. The failure to disambiguate the word 'measured' in this definition has been the source of much confusion. What exactly does it mean for number to be *measured* plurality?

(i) In one sense, we say that a plurality is 'measured' when someone has reckoned its quantity. Let us call this counted plurality. But Aristotle's view cannot be that only counted pluralities are numbers because he is (rightly) clear that 'number is either what has been, or what can be, counted' (*Ph.* Δ.14 223a24–5; see also *Ph.* Γ.5 204b8). Counting a plurality is not what makes it a number; counting is just the act of ascertaining the number (of a specific measure) it already is. Yet, many scholars suppose that this is Aristotle's view (e.g., Maher 2011: 33 n. 9, 34, 38–39; Hussey 1983: 173; Ross 1936: 68, 391). One suggestive passage is *Ph.* Δ.14 223a21–8:

> Whether if soul did not exist time would exist or not, is a question that may fairly be asked; for if there cannot be [*adunaton*] some one to count there cannot be [*adunaton*] anything that can be counted either, so that evidently there cannot be number; for number is either what has been, or what can be, counted. But if nothing but soul, or in soul reason, is qualified to count, it is impossible for there to be time unless there is

[5] See also *Ph.* Θ.8 263a30–b3: a divided line remains continuous, but when we count its halves, we do not treat them as continuous with each other, since their joining at a point is irrelevant for counting.

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> soul, but only that of which time is an attribute. (Barnes 1984, insertions mine)

But as Mignucci observes, Aristotle does not claim that if there *happened* to be no-one to count, then countable things (numbers) would not exist; he claims that if it were *impossible* (*adunaton*) for there to be someone to count, then it would be *impossible* for there to be countable things (1987: 184–86). And while Aristotle claims at a25–8 that time, a kind of number, is dependent on counters, this is only because time is the number of *continuous* movement (223a33–4); as we have seen, a continuum is countable only when someone imposes a measure on it (see also Coope 2005: 169–70). In short, there is no reason to suppose that for Aristotle only counted pluralities are numbers.

(ii) For some scholars, a plurality is potentially many different numbers and only actually a specific number—a *measured* plurality—once a measurer divides it into actual units by imposing a measure. For example, the foliage is potentially 1,000 or 8,000 and becomes actually 8,000 when I impose the measure 'lobe' (Castelli offers such a suggestion and two ways to interpret it [2018: 167]). Let us call this divided-by-a-measurer plurality.

But divided-by-a-measurer plurality cannot be Aristotle's definition of number. The foliage is already 8,000 lobes before I think of it this way. There is no potentiality in the foliage that is actualized by my choosing a measure, such that the foliage somehow becomes a number. Nor have I physically divided the foliage.

For their part, continuous pluralities do not have parts that measure them until a measure is imposed. And this does sometimes involve dividing the continuous plurality into actual parts. We might actualize parts by physically separating them —for example, by pouring a mass of water into 5 vessels holding 100 ml each. We can also just physically mark off parts—for example, by marking off the middle point on a line. Although parts resulting from being marked off are not physically separated, for Aristotle they are nevertheless physically actualized (*Metaph.* Z.13 1039a6–7; *Ph.* Θ.8 262a21–6, 262b30–263a1).

However, more often than not, when imposing a measure, we only *treat* a continuum as divided—for example, we pour water into a marked 500-ml vessel and think of it as divided by planes at the 100-ml marks on the vessel, even though we have not physically marked off those planes; or we weigh some produce on a scale and think of it as divided into ounces. While I have actualized a thought—for example, that there are 17 ounces of this produce—I have not physically actualized those ounces in the produce itself. (As Alexander of Aphrodisias observes, we need not actually divide a piece of wood for it to be 3 cubits long (*On Time* 94.23, in Sharples 1982: 62.) Nor am I actualizing divisions between the 17 ounces even in my mind. Yet, the produce does have a number of ounces: 17. Hence, 17 ounces is a number even though it is not divided by a measurer into 17 actual parts.

A proponent of (ii) might suggest extending the notion of divided-by-a-measurer plurality so that it includes pluralities that are simply treated by a measurer as divided into parts. But while a continuous plurality must be measured in at least this extended sense to be a number, this cannot be the sense of 'measured' at play in the definition of number. This is because to be divided (or treated as such) by a measurer is not what it is to be a number. It is only a necessary condition of a continuous plurality's being a number and not even a necessary condition of a discrete plurality's being a number. Hence, while some numbers are indeed divided-by-a-measurer pluralities, this is not what it is to *be* a number.

(iii) Number must then be 'measured plurality' in a third sense: number is a plurality insofar as it *is related to a measure*, regardless of whether it (a) has given discrete parts that measure it (e.g., a herd), (b) has been divided into such parts (e.g., water poured into 5 vessels of 100 ml each), or (c) is only being treated as divided into such parts (e.g., water in a single 500-ml vessel marked at 100-ml intervals).

On sense (iii), some activity may have been done on the plurality so that it is related to a measure that either (b) in fact divides it or (c) is treated as dividing it. But the word 'measured' does not signify that very act. Instead, it signifies a property of the plurality: the property of being related to a measure.

A plurality that is 'measured' in this sense is of course also measur*able* because what is measured must be measurable. Thus, Aristotle rightly calls number either measured or measurable plurality. (In *Metaph.* I.6, number is *metrēton* plurality; *metrēton* can mean either 'measurable' or 'measured'. In N.1, Aristotle defines number as *memetrēmenon* plurality, with the perfect participle indicating that he means 'measured'.) On this understanding of 'measured plurality', all discrete pluralities are already numbers, as are all continuous pluralities that have had a measure imposed on them. And an unmeasured plurality is just a continuous plurality that is neither related to actual parts that measure it nor being treated as such.

2. How this Addresses Frege's Objections

We can now consider how the view I attribute to Aristotle handles the Fregean concerns about (1) number's arbitrariness and (2) numerical predication.

2.1 The Arbitrariness of Numbers

Recall that one of Frege's reasons for denying that number is a property of external objects is that if we can equally legitimately ascribe conflicting properties to the same object, then we have failed correctly to identify the bearer of those properties. Since I call the same foliage 1,000 or 8,000 depending on whether I am thinking of leaves or lobes, neither number belongs to the foliage 'in its own right'. Hence number is not a property of external objects (1980: 28–29).

But Frege has only shown that 1,000 is *different* from the foliage's other properties; he has not shown that 1,000 is not a property of the foliage (see also Irvine 2010: 242–45). Indeed, Aristotle would agree with Frege that we can point to the foliage's different colors but not to the foliage's different relational numbers (1980: 29). Aristotle would find this unproblematic because it is consistent with his position that while qualities like green and relational numbers like 1,000 are both properties, they are fundamentally different *kinds* of being. To paraphrase

Categories 6, the foliage is called green all by itself (*auto kath' hauto*), but it is not called 1,000 all by itself. As with all relatives, something is 1,000 always 'by reference to something else' (5b16–18). Like Frege, Aristotle does not class relational numbers like 1,000 'along with color and solidity' (1980: 30).

Aristotle would also agree with Frege that the question 'What is the number of this foliage?' is unanswerable. But for Aristotle, the problem is not that we have failed correctly to identify the bearer of the number; it is rather that we have asked an elliptical question (see Barnes 1985: 115 and Annas 1976: 36). The question treats number as a property like green, but a counting number (e.g., 1,000) is a relative. And since relative terms are elliptical (*Soph. el.* 1.31 181b26–8), the question really means 'What is the number of ___ of this foliage?'. (See Harari 2011: 532–34 and Duncombe 2020: 114–16 for discussion of *Soph. el.* 1.31 passage.) That is, asking 'What is the number of this foliage?' is like asking 'Is Alice taller than __?'. If the question is to be answerable, the asker must fill in the correlative—e.g., her brother. (Kessler [1980: 69] makes much the same point while defending Mill against this objection.)

Thus, Aristotle recognizes an important difference between relatives and other properties yet does not conclude with Frege that relatives are not properties of objects. In fact, he clearly identifies external objects as the bearers of relational properties. He maintains that touching 'is relative to something' and 'an attribute of some one of the things which are limited' (*Ph.* Γ.8 208a11–13). And in *Categories* 7 he writes that 'a mountain is said to be large in relation to something else' (6b8): the mountain is the bearer of the relational property large (see also *Metaph.* Δ.15 1021b6–11). Just as the mountain's having the property large relative to a hill is for Aristotle a fact about the mountain, so the foliage's having the property 1,000 relative to the unit leaf is a fact about the foliage—an external object (see also Franklin 2014: 101–103). And we have already seen (section 1.1) that he identifies pluralities as the bearers of numbers (e.g., at *Metaph.* N.1 1088a10–11; *Ph.* Δ.11 220a23–4, Δ.14 224a13–15).

Since any genuine interpretation of Aristotle's number theory ought to be consistent with his fundamental ontological commitments, it is not just that recognizing these commitments allows us to show that he avoids Frege's concern. It is that it would be a mistake to treat relatives and qualities as having the same ontological status for Aristotle and a further mistake to infer that he considers relatives anything other than properties.

We must also resist thinking of Aristotle's relatives as second-order properties. Aristotle rejects the idea that, strictly, properties can have properties (*Metaph.* Γ.4 1007b2–5, *An. post.* 1.22 83a36–9). While he occasionally loosely describes certain relatives as properties of quantities (*Metaph.* N.1 1088a24–5, Δ.13 1020a23–5), his ultimate view is that relatives belong to objects—though they belong to them in virtue of other properties (e.g., a mountain is large in virtue of its height).

So much for relational or counting number. Countable number is also unaffected by the arbitrariness objection, but for a different reason. Frege's objection appears forceful because it compares an unsaturated entity (relational number) to a saturated entity (a quality like green) and finds that they are different. As I have

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argued, Aristotle accepts this difference while maintaining that relational numbers are properties of external things. For their part, countable numbers are saturated entities. Such numbers are pluralities *of measures*, for example, 1,000 *of leaves*, so that the unit is always already specified. A quantitative property such as 1,000 of leaves *is* like a qualitative property such as green inasmuch as we can say which is the number of leaves of the foliage just as we can say which is the color of the foliage. The number of leaves is not something that we may alter 'simply by thinking of it differently' (Frege 1980: 28). We may of course choose a different unit and count that instead of leaves. But as Barbara Sattler notes, if we change units 'then we are counting something else' (2017: 268)—not a different plurality, but a plurality of different measures (a different quantitative number).

2.1.1 The arbitrariness of measures. Have I simply kicked the can down the road here? After all, a quantitative number is a plurality of units or measures, and Frege argues that 'any and every thing is a unit or can be regarded as one' (1980: 44). Since we can vary the number by varying the unit (33), one might reasonably ask: If anything at all in a plurality can be regarded as a unit or measure, so that the plurality has unlimitedly many quantitative numbers, does it really have any of those quantitative numbers as properties?

Aristotle rejects the antecedent. On his view, while we may choose our unit when counting a given plurality, our choice is constrained by more than just what we can regard as one. I find in Aristotle's treatment two relevant constraints on measures:

(I) A measure of a given plurality must divide the plurality (a) into equal items that (b) are not further divisible into things of the same kind; and (II) a measure of a given plurality must also be a way in which that plurality *is*. Let me unpack this a bit.

(Ia) First, 'equal' here means only that each instance of the measure must not differ with respect to the measure. As Aristotle puts it, 'the measure must always be some one and the same thing [*to auto ti*] applying to all cases; for example, if there are horses the measure is horse' (*Metaph.* N.1 1088a8–9). Thus, while Balius may be taller than Xanthus, these two token units must not differ with respect to the measure horse; each must be exactly one horse. This equality of the units allows everyone to get the same result when counting up the horses in this plurality. We can already see here that there is more to being a unit than the capacity to be regarded as one. Two can be regarded as one (we can count by multiples of two) and is a measure of even numbers, but two is not a measure of odd numbers because they are not divided into *equal* items by twos (*Metaph.* Δ.25 1023b15–17, Z.10 1034b32–3; *An. post.* 2.13 96a36–7).

(Ib) Second, if horse is to be a measure, then a horse must be indivisible *qua* horse; it cannot be divisible into further horses. As Aristotle explains, 'those things that do not admit of division are one insofar as they do not admit of it, e.g. if something *qua* human being does not admit of division, it is one human being' (*Metaph.* Δ.6 1016b4–5). It is because a human being is indivisible and one *qua* human being that 'the arithmetician posits him as one indivisible' (M.3 1078a23–4). This is why water cannot be a measure of water: water is divisible *qua* water (any part of water is water), so that any water-part will be divisible into further water-parts (*Metaph.* Δ.3 1014a31; *Gen. corr.* 1.10 328a11).

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(II) A measure of a given plurality must be a way in which that plurality *is*. That is, a measure of a given plurality must be such that there is at least one instance of the measure in that plurality. For example, dog, ear, tail, etc. are all ways in which the plurality in the kennel is; hence, each is a measure of that plurality. But beak is not a way in which the plurality in the kennel is. Because there are no instances of beaks in this plurality, it cannot be divided into beaks and is not a number of beaks. (It is zero beaks, but zero is not an *arithmos*.) It is only because tails in fact belong to the plurality in the kennel that tail measures this plurality. In short, while a beak can be regarded as one and is indivisible *qua* beak, this alone does not make it a measure of the plurality in the kennel. Nor am I free to measure the ounces of vocal sounds or the hertz of a line, since vocal sounds have no weight and lines have no frequency.

Thus, there is more to being a unit or measure than just being somehow one. Indeed, Aristotle's account of measure has a further complexity: he distinguishes between two kinds of oneness for measures: (A) oneness in quality or form and (B) oneness in quantity.

(A) Oneness in quality or form: Some pluralities consist of already somehow discrete elements. In *Sense and Sensibilia* 3, Aristotle describes such pluralities as 'divisible into minimal parts [*ta elachista*] as human beings, horses, or seeds' (440b5–6). Such pluralities have sorts or kinds like human being or seed already dividing them into parts that measure them. That is, they are divisible into parts that are themselves indivisible *qua* what they are—for example, a human being is indivisible *qua* human being (*Metaph.* M.3 1078a23–4). This is the kind of measure Aristotle describes as 'simple in quality' (I.1 1052b35); this measure is the indivisible we arrive at when we divide 'on the basis of form' or kind (1053a19–20), and it is indivisible in form (N.1 1088a2–3; see also Δ.6 1016b23–4).

The most straightforward example of this kind of measure, for Aristotle, is substantial form: natural forms like horse and forms of artifacts like shoe. Each sensible substance has such a form, which is the cause of the substance's oneness, and each substance is indivisible *qua* its form. (Aristotle's forms are beings belonging to and embedded in the physical world though they are not themselves physical entities.) For example, Lassie is one dog because she has dogform; *qua* dog, Lassie is indivisible—a dog-unit. For Aristotle, Lassie's being a dog-unit is importantly not arbitrary. I am not free to decide either that Lassie is a dog or what constitutes one dog. These are determined by dog form. Neither am I free to decide that something is a shoe or what constitutes one shoe. That is determined by shoe form (which is determined by the craft of cobbling). Aristotle also takes certain organized parts of living things to have forms of a sort (internal organizing principles; *De an.* 2.12 424a25–8). Sense-organs as well as body parts such as hands or beaks each exist for the sake of certain functions, and those functions determine both the matter and the form or structure of the part (*Part. an.* 2.1 646b12–35, 2.9 655b2–11). Because such body parts are indivisible in form (e.g., an eye is indivisible *qua* eye), they, too, are sorts and measures. (A common genus can also measure a plurality [*Metaph.* N.1 1088a10–11].)

Aristotle likely also takes certain individuating attributes to function as 'simple in quality' measures. For example, 'sitting thing' might plausibly be such a measure

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because something is indivisible *qua* sitting thing (sitting being a specific arrangement of specific body parts).

Thus, the measures of a discrete plurality are constrained by the sorts or kinds in fact dividing that plurality into equal token units. While the plurality may have several measures and while we may vary the number by varying the measure, there are not unlimitedly many measures of a discrete plurality as such.[6] Hence, it does not have unlimitedly many quantitative numbers, and there is no reason to doubt that it has those numbers as properties.

We must say something different about the measures of continuous quantities. We have seen that Aristotle considers even continuous quantities pluralities. While a volume of water is unmeasured as such because nothing in a mass of water already divides it into parts that measure it, it can be measured relative to a conventional measure like a milliliter. Yet, a milliliter is not present in the water in the same way that a horse is present in a herd. The herd *comes to us* in horses, but water does not come to us in milliliters. We must put the water into relation with the measure milliliter before we can consider it *qua* milliliters. So one might reasonably wonder whether milliliter is really a way in which a mass of water *is*.

Aristotle recognizes that measures of continua are importantly different from measures of discrete multiplicities. Such measures are one not in form but in quantity.

(B) Oneness in quantity: A measure of volume like milliliter is simple only in quantity (*Metaph*. I.1 1052b35; see also Δ.6 1016b23–4); this kind of measure is the indivisible we arrive at when we divide 'on the basis of quantity' (1053a19–20), and it is indivisible only relative to perception (N.1 1088a2–3, I.1 1053a23).

That is, the measures of continuous pluralities are matters of convention determined by human perception (see Sattler 2017: 267). But while measures of continua are arbitrary in this way, they are also constrained. I cannot use seconds to measure volume. A volume can only be measured by a measure that is itself a volume; a measure 'always belongs to the same kind [*syngenes*] as the things it is a measure of' (*Metaph*. I.1 1053a24–5). It is in this sense that milliliter is a way in which a volume of water is: while milliliters are not given features of water, water *can* be related to the measure milliliter, and there *are* a certain number of milliliters of that water, and they are indivisible *qua* milliliter. The statement 'There are 286 milliliters of water in this vessel' is true because there *are* 286 milliliters of water in the vessel. By contrast, since there are no seconds in the water, nothing can make the statement 'There are 286 seconds of water in this vessel' true.

Thus, the measures of discrete and continuous pluralities must both be ways in which the pluralities are. But since the latter measures are arbitrary to a degree the former are not, one might worry that perhaps each continuous plurality has unlimitedly many quantitative numbers. If so, then one might doubt that a continuous plurality has any of these numbers as properties.

[6] The 'as such' is important because a discrete multiplicity can be treated apart from its given divisions, i.e., not as a discrete multiplicity. The produce on the scale as a discrete multiplicity is 85 grapes or 4 bunches, but we can also consider it just insofar as it has a weight, e.g., 17 ounces. When we weigh it, we treat it apart from its given divisions and instead consider it as a continuous mass divided by the measure ounce.

In fact, however, a continuous plurality has *no* quantitative numbers as such—that is, as a continuum. A continuous plurality is not a *measured* plurality, that is, a number, unless it is considered in relation to an imposed conventional measure—in which case it is not being considered *as* a continuum. Unlike a discrete plurality, a continuum has no measures of its own; and when a measure has been imposed, the continuum has *just one* number of that particular measure. I can vary the measure (milliliter, deciliter), but I cannot vary the *number of* that measure that in fact belongs to the water in the vessel. Similarly, I can decide to whom I compare Alice (her brother, her dad), but I cannot decide that she is taller than her brother and shorter than her dad. And as we have seen, for Aristotle 'taller than her brother' really *belongs to* Alice, as 286 milliliters really belongs to the water in the vessel.

In sum, Aristotle accepts an attenuated version of Frege's claim that we may alter the unit 'simply by thinking of it differently' (1980: 28). Pluralities have a variety of measures, so that we may choose either leaves, lobes, or veins as measures of the foliage and centimeters or inches as measures of a length. Yet, Aristotle rejects the stronger claim that 'any and every thing is a unit' (1980: 44). A unit for Aristotle is fundamentally relational: it is a measure *of* a plurality. Thus, even if Frege is right that any object can be regarded as one, being a unit for Aristotle is more than just being one. Being a unit is being a measure, and we have seen that there are constraints on what can measure a given plurality. Most crucially, a measure of a given plurality must be a way in which that plurality *is*. Hence, the number a given plurality has relative to such a measure is also a way in which that plurality is; it is a property of that plurality.

2.2 Numerical Predication

We can now consider Frege's concern about numerical predication. He asks: 'To what does the property 1,000 really belong?' and argues that 1,000 belongs 'neither to any single one of the leaves nor to the totality of them all'. He concludes that 1,000 'does not really belong to things in the external world at all' (1980: 28).

It is true that neither 1,000 nor 1,000 leaves are properties of the individual leaves. But according to Frege's reasoning, neither can they be properties of the agglomeration of leaves because the individual items' being agglomerated is irrelevant to their having a number (30). Yet unless we equivocate, this point about agglomerations does not count against Aristotle. While an agglomeration is by definition stuff gathered together, a plurality is just an amount of stuff. For Aristotle, a number is a plurality and heap (*sōros*)—but the items forming a *sōros* need not be gathered together. (For the argument that number for Aristotle is a heap rather than a whole, see Katz 2021.) *Sōros* can mean a mound (hence it is typically translated as 'heap' when Aristotle contrasts it with an organized whole (*holon*)) but more generally it just means an indeterminate quantity—some amount of stuff. So Frege's point about agglomerations does not undermine Aristotle's view that 1,000 and 1,000 leaves are properties of a *plurality*. (Irvine [2010: 252] shows that Mill, too, can avoid Frege's objection.)

In short, like Frege, Aristotle does not class relational numbers like 1,000 'along with color and solidity' (1980: 30): 1,000 is a relational property and so always *of* something else, while properties like green are not of anything else. But while for Aristotle 1,000 does not belong to external objects in exactly the way that green does, 1,000 belongs to external objects nevertheless. It belongs to a specific kind of external object—pluralities—and it belongs to them in a special way—namely, relative to a unit. For its part, a quantitative number like 1,000 leaves is, like a quality such as green, a property that is not *of* anything else, and like 1,000, it is a property of pluralities.

3. Conclusion

Many scholars have supposed that, given Frege's objections to the view that numbers are properties of external objects, we should avoid attributing this view to Aristotle. I have tried to free the discussion of Aristotle's number theory from Frege's specter. To this end, I have developed an interpretation according to which Aristotle's numbers are properties of external objects and then showed that Frege's arbitrariness and numerical predication concerns do not count against this view. This interpretation also closes the supposed gap between Aristotle's philosophies of arithmetic and geometry. Since quantitative and relational numbers, like geometrical magnitudes, are properties of external objects, Aristotle's broader account of mathematical objects applies to numbers just as well as it does to magnitudes. The objects of arithmetic, like the objects of geometry, are 'kooky objects', that is, certain sensible things just insofar as they have certain relevant properties.

I have addressed the Fregean concerns because they have for some time constrained interpretations of Aristotle's philosophy of arithmetic. Much remains to be explored, and I hope the discussion can now more easily move forward.

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