ON THE ADVANTAGES OF THE MODERN METHODS OF COMPUTATION IN LIFE ASSURANCE CALCULATIONS.

To the Editor of the Assurance Magazine.

SIR,—I observe Mr. Laundy's note and problems in the October Number, attempting to show the greater advantages of the "modern" methods of computation over the "ancient" methods, and referring to a former Number for more striking illustrations to the same effect.

I, however, cannot perceive so great advantages in the columnar system over the other to a person who possesses a set of temporary and deferred annuities for the first 20 or 30 years following each year of age. Possessing these tables, my own practice inclines to the "ancient" method. By the more modern method, I found myself continually hunting up the logarithms of the sums or differences of the D's, A's, and M's, and might, perhaps, turn up the same logarithms many, many times, in the course of a year, without advancing one step; whereas, by the ancient system, the component parts of the immediate annuity, the deferred and temporary annuities in advance and not in advance, and the endowments and their logarithms, once tabulated, are there for every future occasion. Of course, for occasional calculations at a different rate of interest, the modern or columnar system is the best. I think, however, that the Carlisle columnar system has been greatly enhanced by the few pages of the logs. of the single lives D's, A's, and M's, in Mr. Chisholm's invaluable volumes, and that it might be still further improved by the tabulation of the logs. of the *difference* of the A's.

The problems, solved according to Milne's notation, are as follow:—

1. Annual premium, P, payable $n+1$ times, for annuity of £m deferred n years.

$$P=\frac{m\cdot{}_{\overline{n}|}a_x}{{}_{\overline{n+1}|}a_{x+1}-{}_{n+1}av^{n+1}}=\frac{m\cdot{}_{\overline{n}|}a_x}{1+{}_{n|}a_x}.$$

The addition of three logs., of which two are tabulated, solves this question. Its columnar form is* $=\dfrac{mN_{x+n}}{N_{x-1}-N_{x+n}}$.

2. Annual premium, payable t times, for assurance of £m for n years.

$$P=m.\frac{v\{{}_{n|}a_{x+1}-{}_{n}av^{n}\}-{}_{n|}a_x}{{}_{t|}a_{x+1}-{}_{t}av^{t}}.$$

In this case, v and its log., as well as the log. of $1-v$, are as familiar as my own name. Its columnar form $=\dfrac{m\{M_x-M_{x+n}\}}{N_{x-1}-N_{x+t-1}}$.

* Although the N s are here stated according to Jones' arrangement of the column, I greatly prefer to use those of Mr. Chisholm, by which

$N_x=D_x+D_{x+1}+$ &c.; and, therefore,

${}_{n|}a_{x+1}-{}_{n}av^{n}=N_x-N_{x+n}$ in place $N_{x-1}-N_{x+n-1}$,

and ${}_{\overline{n+1}|}a_{x+1}-{}_{n+1}av^{n+1}=N_x-N_{x+n+1}$,, $N_{x-1}-N_{x+n}$.

https://doi.org/10.1017/S2046165800000988 Published online by Cambridge University Press

3. Annual premium, payable t times, for assurance of £m payable at death or at the end of n years if then alive.

$$P=m\,.\,\frac{1-d\left\{{}_{\overline{n}|}a_{x+1}-{}_{n}av^{n}\right\}}{{}_{\overline{t}|}a_{x+1}-{}_{t}av^{t}}\,.$$

The working out of this formula does not exceed that of the

$$\text{columnar form}=m\,.\,\frac{\left\{D_{x+n}+M_{x}-M_{x+n}\right\}}{N_{x-1}-N_{x+t-1}}=m\,.\,\frac{D_{x}-d\left\{N_{x-1}-N_{x+n-1}\right\}}{N_{x-1}-N_{x+t-1}}.$$

4. Conversion of policy for £m payable at death of x, at a premium P, into another policy for £m payable at death, or in n years if then alive, at a premium P′. Required the premium P′ to be payable for n years.

x receives $\quad m\left\{1-d\left({}_{\overline{n}|}a_{x+1}-{}_{n}av^{n}\right)\right\}+P(1+a_{x})$

and gives up $\quad m\left\{1-d(1+a_{x})\right\}+P'\left\{{}_{\overline{n}|}a_{x+1}-{}_{n}av^{n}\right\}$

$$\therefore\ P'=\frac{m\left\{1-d\left({}_{\overline{n}|}a_{x+1}-{}_{n}av^{n}\right)\right\}+P(1+a_{x})-m\left\{1-d(1+a_{x})\right\}}{{}_{\overline{n}|}a_{x+1}-{}_{n}av^{n}}$$

$$P'=\frac{P(1+a_{x})+md\left\{{}_{|n}a_{x}+{}_{n}av^{n}\right\}}{{}_{\overline{n}|}a_{x+1}-{}_{n}av^{n}}=\frac{(P+md)\,(1+a_{x})}{{}_{\overline{n}|}a_{x+1}-{}_{n}av^{n}}-md\,;$$

its columnar form being $\quad =\dfrac{PN_{x-1}+m\left\{D_{x+n}-M_{x+n}\right\}}{N_{x-1}-N_{x+n-1}},$

$$P'=\frac{PN_{x-1}+md\left\{N_{x+n-1}\right\}}{N_{x-1}-N_{x+n-1}}=\frac{(P+md)N_{x-1}}{N_{x-1}-N_{x+n-1}}-md.$$

5. Conversion of policy for £m on age x, payable at death, or at age $x+n$ if alive, for an annual premium of P until age $x+n-1$, into a policy for £m payable at death. Required the premium P′ to be payable during life.

$$x\text{ receives}=m\left\{1-d(1+a_{x})\right\}+P\left({}_{\overline{n}|}a_{x+1}-{}_{n}av^{n}\right),$$

$$\text{gives up}=m\left\{1-d\left({}_{\overline{n}|}a_{x+1}-{}_{n}av^{n}\right)\right\}+P'(1+a_{x}),$$

$$\therefore\ P'=\frac{P\left({}_{\overline{n}|}a_{x+1}-{}_{n}av^{n}\right)-md\left\{{}_{|n}a_{x}+{}_{n}av^{n}\right\}}{1+a_{x}}=P-\frac{(P+md)\left({}_{|n}a_{x}+{}_{n}av^{n}\right)}{(1+a_{x})}$$

$$=\frac{(P+md)\left\{{}_{\overline{n}|}a_{x+1}-{}_{n}av^{n}\right\}}{(1+a_{x})}-md;$$

its columnar form being $=\dfrac{P\left\{N_{x-1}-N_{x+n-1}\right\}-m\left\{D_{x+n}-M_{x+n}\right\}}{N_{x-1}}$

$$=P-\frac{(P+md)\,(N_{x+n-1})}{N_{x-1}}.$$

6. Conversion of policy for £m at a premium P into policy for £m free of premiums. Age now $x+n$.

https://doi.org/10.1017/S2046165800000988 Published online by Cambridge University Press

$$x+n \text{ receives} = m'(v-da_{x+n})+\text{P}(1+a_{x+n}),$$

$$\text{gives up} = m(v-da_{x+n}),$$

$$\therefore\ m' = \frac{m(v-da_{x+n})-\text{P}(1+a_{x+n})}{v-da_{x+n}}$$

$$= m - \frac{\text{P}(1+a_{x+n})}{v-da_{x+n}}$$

$$= m - \frac{\text{P}^x}{\text{P}_{x+n}}.$$

Upon the whole, I think many of your readers will admit, that, with a complete set of temporary annuities, the foregoing problems, as expressed and solved by the columnar notation, do not show any material advantage over the notation adopted by

"JOSHUA MILNE."

Edinburgh, November, 1858.