

# Contents

<i>List of inserts</i>	<i>xviii</i>
<i>Preface</i>	<i>xx</i>
<b>1 Overview and overture</b>	<b>1</b>
1.1 The classical dynamics of geometry	1
1.2 Gravitons and photons	7
1.3 Beyond classical gravity: perturbative strings	11
1.4 Beyond perturbative strings: branes	15
1.5 The quantum dynamics of geometry	19
1.6 Things to do in the meantime	20
1.7 On with the show	22
<b>2 Relativistic strings</b>	<b>24</b>
2.1 Motion of classical point particles	24
2.1.1 Two actions	24
2.1.2 Symmetries	26
2.2 Classical bosonic strings	27
2.2.1 Two actions	27
2.2.2 Symmetries	29
2.2.3 String equations of motion	30
2.2.4 Further aspects of the two dimensional perspective	31
2.2.5 The stress tensor	35
2.2.6 Gauge fixing	35
2.2.7 The mode decomposition	37
2.2.8 Conformal invariance as a residual symmetry	37
2.2.9 Some Hamiltonian dynamics	38

2.3	Quantised bosonic strings	40
2.3.1	The constraints and physical states	41
2.3.2	The intercept and critical dimensions	42
2.3.3	A glance at more sophisticated techniques	45
2.4	The sphere, the plane and the vertex operator	47
2.4.1	States and operators	48
2.5	Chan–Paton factors	51
2.6	Unoriented strings	52
2.6.1	Unoriented open strings	52
2.6.2	Unoriented closed strings	54
2.6.3	World-sheet diagrams	55
2.7	Strings in curved backgrounds	56
2.8	A quick look at geometry	61
2.8.1	Working with the local tangent frames	61
2.8.2	Differential forms	63
2.8.3	Coordinate vs. orthonormal bases	65
2.8.4	The Lorentz group as a gauge group	67
2.8.5	Fermions in curved spacetime	68
2.8.6	Comparison to differential geometry	68
<b>3</b>	<b>A closer look at the world-sheet</b>	<b>70</b>
3.1	Conformal invariance	70
3.1.1	Diverse dimensions	70
3.1.2	The special case of two dimensions	73
3.1.3	States and operators	74
3.1.4	The operator product expansion	75
3.1.5	The stress tensor and the Virasoro algebra	76
3.2	Revisiting the relativistic string	80
3.3	Fixing the conformal gauge	85
3.3.1	Conformal ghosts	85
3.3.2	The critical dimension	86
3.4	The closed string partition function	87
<b>4</b>	<b>Strings on circles and T-duality</b>	<b>94</b>
4.1	Fields and strings on a circle	94
4.1.1	The Kaluza–Klein reduction	95
4.1.2	Closed strings on a circle	96
4.2	T-duality for closed strings	99
4.3	A special radius: enhanced gauge symmetry	100
4.4	The circle partition function	103
4.5	Toroidal compactifications	104

4.6	More on enhanced gauge symmetry	108
4.6.1	Lie algebras and groups	108
4.6.2	The classical Lie algebras	111
4.6.3	Physical realisations with vertex operators	113
4.7	Another special radius: bosonisation	113
4.8	String theory on an orbifold	117
4.9	T-duality for open strings: D-branes	119
4.9.1	Chan–Paton factors and Wilson lines	121
4.10	D-brane collective coordinates	123
4.11	T-duality for unoriented strings: orientifolds	125
<b>5</b>	<b>Background fields and world-volume actions</b>	<b>129</b>
5.1	T-duality in background fields	129
5.2	A first look at the D-brane world-volume action	131
5.2.1	World-volume actions from tilted D-branes	133
5.3	The Dirac–Born–Infeld action	135
5.4	The action of T-duality	136
5.5	Non-Abelian extensions	136
5.6	D-branes and gauge theory	138
5.7	BPS lumps on the world-volume	138
<b>6</b>	<b>D-brane tension and boundary states</b>	<b>141</b>
6.1	The D-brane tension	142
6.1.1	An open string partition function	142
6.1.2	A background field computation	145
6.2	The orientifold tension	148
6.2.1	Another open string partition function	148
6.3	The boundary state formalism	150
<b>7</b>	<b>Supersymmetric strings</b>	<b>155</b>
7.1	The three basic superstring theories	155
7.1.1	Open superstrings: type I	155
7.1.2	Closed superstrings: type II	160
7.1.3	Type I from type IIB, the prototype orientifold	165
7.1.4	The Green–Schwarz mechanism	166
7.2	The two basic heterotic string theories	169
7.2.1	$SO(32)$ and $E_8 \times E_8$ from self-dual lattices	171
7.2.2	The massless spectrum	172
7.3	The ten dimensional supergravities	174
7.4	Heterotic toroidal compactifications	176
7.5	Superstring toroidal compactification	178
7.6	A superstring orbifold: discovering the K3 manifold	179

7.6.1	The orbifold spectrum	180
7.6.2	Another miraculous anomaly cancellation	183
7.6.3	The K3 manifold	184
7.6.4	Blowing up the orbifold	185
7.6.5	Some other K3 orbifolds	189
7.6.6	Anticipating D-manifolds	191
<b>8</b>	<b>Supersymmetric strings and T-duality</b>	<b>192</b>
8.1	T-duality of supersymmetric strings	192
8.1.1	T-duality of type II superstrings	192
8.1.2	T-duality of type I superstrings	193
8.1.3	T-duality for the heterotic strings	194
8.2	D-branes as BPS solitons	195
8.3	The D-brane charge and tension	197
8.4	The orientifold charge and tension	200
8.5	Type I from type IIB, revisited	201
8.6	Dirac charge quantisation	201
8.7	D-branes in type I	202
<b>9</b>	<b>World-volume curvature couplings</b>	<b>205</b>
9.1	Tilted D-branes and branes within branes	205
9.2	Anomalous gauge couplings	206
9.3	Characteristic classes and invariant polynomials	210
9.4	Anomalous curvature couplings	216
9.5	A relation to anomalies	218
9.6	D-branes and K-theory	220
9.7	Further non-Abelian extensions	221
9.8	Further curvature couplings	222
<b>10</b>	<b>The geometry of D-branes</b>	<b>224</b>
10.1	A look at black holes in four dimensions	224
10.1.1	A brief study of the Einstein–Maxwell system	224
10.1.2	Basic properties of Schwarzschild	225
10.1.3	Basic properties of Reissner–Nordstrom	228
10.1.4	Extremality, supersymmetry, and the BPS condition	228
10.1.5	Multiple black holes and multicentre solutions	232
10.1.6	Near horizon geometry and an infinite throat	233
10.1.7	Cosmological constant; de Sitter and anti-de Sitter	233
10.1.8	de-Sitter spacetime and the sphere	234
10.1.9	Anti-de Sitter in various coordinate systems	235

10.1.10	Anti-de Sitter as a hyperbolic slice	236
10.1.11	Revisiting the extremal solution	237
10.2	The geometry of D-branes	238
10.2.1	A family of ‘ $p$ -brane’ solutions	238
10.2.2	The boost form of solution	239
10.2.3	The extremal limit and coincident D-branes	240
10.3	Probing $p$ -brane geometry with $Dp$ -branes	243
10.3.1	Thought experiment: building $p$ with $Dp$	243
10.3.2	Effective Lagrangian from the world-volume action	244
10.3.3	A metric on moduli space	245
10.4	T-duality and supergravity solutions	246
10.4.1	$D(p + 1)$ from $Dp$	246
10.4.2	$D(p - 1)$ from $Dp$	248
<b>11</b>	<b>Multiple D-branes and bound states</b>	<b>249</b>
11.1	$Dp$ and $Dp'$ from boundary conditions	249
11.2	The BPS bound for the $Dp$ – $Dp'$ system	252
11.3	Bound states of fundamental strings and D-strings	254
11.4	The three-string junction	255
11.5	Aspects of D-brane bound states	258
11.5.1	0–0 bound states	258
11.5.2	0–2 bound states	258
11.5.3	0–4 bound states	259
11.5.4	0–6 bound states	260
11.5.5	0–8 bound states	260
<b>12</b>	<b>Strong coupling and string duality</b>	<b>261</b>
12.1	Type IIB/type IIB duality	261
12.1.1	D1-brane collective coordinates	261
12.1.2	S-duality and $SL(2, \mathbb{Z})$	263
12.2	$SO(32)$ Type I/heterotic duality	264
12.2.1	D1-brane collective coordinates	264
12.3	Dual branes from 10D string–string duality	265
12.3.1	The heterotic NS-fivebrane	267
12.3.2	The type IIA and type IIB NS5-brane	268
12.4	Type IIA/M-theory duality	271
12.4.1	A closer look at D0-branes	271
12.4.2	Eleven dimensional supergravity	271
12.5	$E_8 \times E_8$ heterotic string/M-theory duality	273
12.6	M2-branes and M5-branes	276
12.6.1	Supergravity solutions	276

12.6.2	From D-branes and NS5-branes to M-branes and back	277
12.7	U-duality	278
12.7.1	Type II strings on $T^5$ and $E_{6(6)}$	278
12.7.2	U-duality and bound states	279
<b>13</b>	<b>D-branes and geometry I</b>	<b>282</b>
13.1	D-branes as probes of ALE spaces	282
13.1.1	Basic setup and a quiver gauge theory	282
13.1.2	The moduli space of vacua	285
13.1.3	ALE space as metric on moduli space	286
13.1.4	D-branes and the hyper-Kähler quotient	289
13.2	Fractional D-branes and wrapped D-branes	291
13.2.1	Fractional branes	291
13.2.2	Wrapped branes	292
13.3	Wrapped, fractional and stretched branes	294
13.3.1	NS5-branes from ALE spaces	295
13.3.2	Dual realisations of quivers	296
13.4	D-branes as instantons	300
13.4.1	Seeing the instanton with a probe	301
13.4.2	Small instantons	305
13.5	D-branes as monopoles	306
13.5.1	Adjoint Higgs and monopoles	309
13.5.2	BPS monopole solution from Nahm data	311
13.6	The D-brane dielectric effect	314
13.6.1	Non-Abelian world-volume interactions	314
13.6.2	Stable fuzzy spherical D-branes	316
13.6.3	Stable smooth spherical D-branes	318
<b>14</b>	<b>K3 orientifolds and compactification</b>	<b>322</b>
14.1	$\mathbb{Z}_N$ orientifolds and Chan–Paton factors	322
14.2	Loops and tadpoles for ALE $\mathbb{Z}_M$ singularities	324
14.2.1	One-loop diagrams and tadpoles	324
14.2.2	Computing the one-loop diagrams	325
14.2.3	Extracting the tadpoles	330
14.3	Solving the tadpole equations	333
14.3.1	T-duality relations	333
14.3.2	Explicit solutions	334
14.4	Closed string spectra	336
14.5	Open string spectra	339
14.6	Anomalies for $\mathcal{N} = 1$ in six dimensions	341

<b>15</b>	<b>D-branes and geometry II</b>	<b>345</b>
15.1	Probing $p$ with $D(p-4)$	345
15.2	Probing six-branes: Kaluza–Klein monopoles and M-theory	346
15.3	The moduli space of 3D supersymmetric gauge theory	348
15.4	Wrapped branes and the enhançon mechanism	352
	15.4.1 Wrapping D6-branes	353
	15.4.2 The repulson geometry	354
	15.4.3 Probing with a wrapped D6-brane	356
15.5	The consistency of excision in supergravity	360
15.6	The moduli space of pure glue in 3D	362
	15.6.1 Multi-monopole moduli space	363
<b>16</b>	<b>Towards M- and F-theory</b>	<b>367</b>
16.1	The type IIB string and F-theory	367
	16.1.1 $SL(2, \mathbb{Z})$ duality	368
	16.1.2 The $(p, q)$ strings	369
	16.1.3 String networks	371
	16.1.4 The self-duality of D3-branes	373
	16.1.5 $(p, q)$ Fivebranes	375
	16.1.6 $SL(2, \mathbb{Z})$ and D7-branes	376
	16.1.7 Some algebraic geometry	379
	16.1.8 F-theory, and a dual heterotic description	383
	16.1.9 $(p, q)$ Sevenbranes	384
	16.1.10 Enhanced gauge symmetry and singularities of K3	386
	16.1.11 F-theory at constant coupling	387
	16.1.12 The moduli space of $\mathcal{N} = 2$ $SU(N)$ with $N_f = 4$	392
16.2	M-theory origins of F-theory	394
	16.2.1 M-branes and odd D-branes	396
	16.2.2 M-theory on K3 and heterotic on $T^3$	399
	16.2.3 Type IIA on K3 and heterotic on $T^4$	400
16.3	Matrix theory	400
	16.3.1 Another look at D0-branes	401
	16.3.2 The infinite momentum frame	402
	16.3.3 Matrix string theory	404
<b>17</b>	<b>D-branes and black holes</b>	<b>409</b>
17.1	Black hole thermodynamics	409
	17.1.1 The path integral and the Euclidean calculus	409
	17.1.2 The semiclassical approximation	411
	17.1.3 The temperature of black holes	412

17.2	The Euclidean action calculus	414
17.2.1	The action for Schwarzschild	414
17.2.2	The action for Reissner–Nordström	416
17.2.3	The laws of thermodynamics	417
17.3	$D = 5$ Reissner–Nordström black holes	418
17.3.1	Making the black hole	420
17.3.2	Microscopic entropy and a 2D field theory	425
17.3.3	Non-extremality and a 2D dilute gas limit	427
17.4	Near horizon geometry	429
17.5	Replacing $T^4$ with K3	432
17.5.1	The geometry	432
17.5.2	The microscopic entropy	433
17.5.3	Probing the black hole with branes	434
17.5.4	The enhançon and the second law	437
<b>18</b>	<b>D-branes, gravity and gauge theory</b>	<b>440</b>
18.1	The AdS/CFT correspondence	441
18.1.1	Branes and the decoupling limit	441
18.1.2	Sphere reduction and gauged supergravity	443
18.1.3	Extracts from the dictionary	446
18.1.4	The action, counterterms, and the stress tensor	449
18.2	The correspondence at finite temperature	452
18.2.1	Limits of the non-extremal D3-brane	452
18.2.2	The AdS–Schwarzschild black hole in global coordinates	453
18.3	The correspondence with a chemical potential	455
18.3.1	Spinning D3-branes and charged AdS black holes	455
18.3.2	The AdS–Reissner–Nordström black hole	459
18.3.3	Thermodynamic phase structure	459
18.4	The holographic principle	464
<b>19</b>	<b>The holographic renormalisation group</b>	<b>467</b>
19.1	Renormalisation group flows from gravity	467
19.1.1	A BPS domain wall and supersymmetry	469
19.2	Flowing on the Coulomb branch	472
19.2.1	A five dimensional solution	472
19.2.2	A ten dimensional solution	475
19.2.3	Probing the geometry	475
19.2.4	Brane distributions	478
19.3	An $\mathcal{N} = 1$ gauge dual RG flow	480
19.3.1	The five dimensional solution	482



19.3.2	The ten dimensional solution	486
19.3.3	Probing with a D3-brane	487
19.3.4	The Coulomb branch	488
19.3.5	Kähler structure of the Coulomb branch	489
19.4	An $\mathcal{N} = 2$ gauge dual RG flow and the enhançon	494
19.4.1	The five dimensional solution	494
19.4.2	The ten dimensional solution	498
19.4.3	Probing with a D3-brane	499
19.4.4	The moduli space	500
19.5	Beyond gravity duals	502
<b>20</b>	<b>Taking stock</b>	<b>504</b>
	<i>References</i>	510
	<i>Index</i>	529