MODELS OF ELLIPTICALS AND BULGES

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1. INTRODUCTION

During the last seven years we have become much more aware of the importance of velocity anisotropy in spheroidal components. There were never any sound arguments for assuming that the velocity ellipsoids in spheroidal components would be spherical, but the mathematical convenience of this assumption is such that velocity anisotropy was either absent from or unimportant in the models that seemed so promising at the last Besancon meeting in 1974. With the advent of accurate velocity information from absorption-line studies of early-type systems, it became apparent that the real world is a good deal more complex than it might have been, and the theoretical situation is now less satisfactory than it seemed in 1974. All I can do here is to report on our somewhat painful efforts to pick ourselves up from the floor to which the observers knocked us in 1975-7.

2. SPHERICAL SYSTEMS

As is well known, the most general distribution function for a system that is spherically symmetric in all its properties, is a function f(E,L) of the specific stellar energy $E=\frac{1}{2}v^2+\Phi$ and angular momentum $L=|\mathbf{r}\times\mathbf{v}|$, and the velocity ellipsoids in the galaxy are everywhere spherical if and only if $(\partial f/\partial L)\equiv 0$. Numerical simulations of the relaxation of spherical star clusters from Gott (1973) to van Albada (1982) have tended to show that while the velocity ellipsoids at the centres of the final equilibrium systems are spherical, those near the periphery are elongated along the local radial direction. Thus in these systems $(\partial f/\partial L)\neq 0$, and we should be wary of assuming that the distribution functions of spherical galaxies have f(E).

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However, until recently we have tended to think in terms of models whose distribution functions depend on E only. A nice illustration of how misleading this can be is furnished by M87. Young et al (1978) and Sargent et al (1978) obtained CCD photometry and long-slit spectroscopy of this galaxy and interpreted their observations on the assumption that the velocity dispersion is everywhere isotropic. They concluded that the ratio M(r)/L(r) of the mass contained interior to radius r to the light inside r, rises steeply from $M/L_V<9$ at $r>600$ pc to $M/L_V>60$ at $r<200$ pc. However, if one drops the assumption of velocity isotropy, these same observations are consistent with a constant mass-to-light ratio $M/L_V=7.6$ (Binney and Mamon 1982). Furthermore, it can be argued that the radial variation of the anisotropy parameter $\beta \equiv (1-\sigma_\theta^2/\sigma_r^2)$ that is implied by the assumption of constant mass-to-light ratio in M87, is of the same type as one would have predicted from the theory of Tremaine et al (1975) that galactic nuclei are formed as a result of massive globular clusters becoming trapped in galactic centres through the action of dynamical friction.

Mamon and I only showed that the first moment of the Vlasov equation can be satisfied by a constant M/L model of M87. We did not prove that the Vlasov equation can itself be satisfied. Unfortunately we do not yet know how to find a distribution function f(E,L) that generates given surface density and velocity dispersion profiles $\Sigma(R)$ and $\sigma_V(R)$. The different information contents of one function of two variables f(E,J), and two functions $\Sigma(R)$ and $\sigma_V(R)$ of one variable, suggests that if any non-negative f(E,L) generates the given profiles, many other distribution functions will also be possible. But it is not clear under what circumstances no non-negative distribution function is compatible with a set of data. Duncan and Wheeler (1980) and Tremaine and Ostriker (1982) have tackled the problem of interpreting the brightness and velocity dispersion profiles of M87 and M31 from this more demanding point of view.

3. AXISYMMETRIC SYSTEMS

3.1 Systems with $f(E,L_z)$

The classical model of an axisymmetric galaxy (e.g. Wilson 1975) has a distribution function $f(E,L_z)$ that depends on energy and the component L_z of angular momentum along the symmetry axis. These models are often referred to as "isotropic" because the velocity ellipsoids cut meridional planes in circles (see Fig.

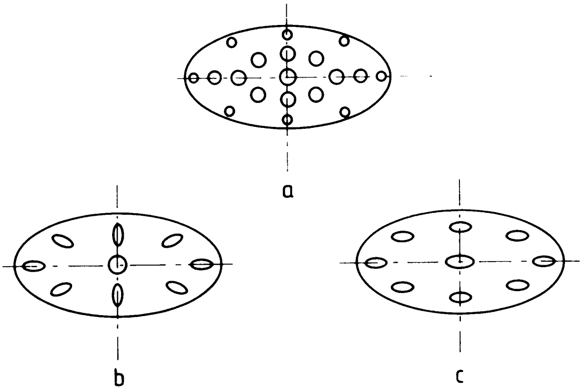

Figure 1. Possible arrangements of the velocity ellipsoids in the meridional plane of an axisymmetric galaxy.

1a). Models of this type have been at a discount since Bertola and Capaccioli (1975) and Illingworth (1977) demonstrated that most giant elliptical galaxies are not flattened by rotation. Actually there has never been any hard evidence that giant elliptical galaxies cannot be modelled in this way: Any model based on $f(E,L_z)$ immediately gives rise to a family of models in which the distribution functions $f(E,L_z)$ differ from each other only in the parts that are odd in L_z. Since the rotation speed of a model is proportional to the part of the distribution function that is odd in L_z and does not contribute to the density distribution, all these models have the same density, but among them are models that have very small or even zero rotation rates.

Personally I have always been strongly prejudiced against models of this type since I can see no obvious way of ensuring that $f=f(E,L_z)$, while flattened but non-rotating ellipticals with $f\neq f(E,L_z)$ are a natural consequence of either the Zel'dovich (1970, 1978) pancake theory of galaxy formation (Binney 1976), or the merger picture of the formation of ellipticals (White, this symposium). However two recent developments give pause for thought:

(1) Frenk and White (1980) have shown that the velocities of the galactic globular clusters are nearly isotropically distributed, rather than being strongly biased around the radial direction as are the velocities of the RR-Lyrae stars (Woolley 1978). Freeman (this symposium) cautions us against assuming blindly that globular clusters are necessarily typical of the spheroidal component, but the conclusion of Frenk and White shows that velocity anisotropy does not always play an important role in spheroidal systems.

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(2) The observations of early-type disk galaxies and of low luminosity ellipticals that have been reviewed by Illingworth (this symposium), show that if the velocity ellipsoids in these galaxies are not spherical, they must be elongated along the radial directions (see Fig. 1b) so that anisotropy does not make an appreciable contribution to the flattenings of these system.

Jarvis (1981) has recently fitted models based on $f(E,L_z)$ to observations of the disk galaxies NGCs 4594, 7123 and 7814. He models the bulges of these galaxies as systems with $f=f_o[\exp(-E/\sigma^2)-1]\exp(\Omega L_z/\sigma^2)$ that are placed in the disk-like gravitational potential $\Phi_d(R,z)=-GM_d\{R^2+[a+(z^2+b^2)^{1/2}]^2\}^{-1/2}$ that was introduced by Miyamoto and Nagai (1975). Figure 2 illustrates the effect of a disk with a quarter of the spheroid's mass on the position of the spheroid in the usual v_m/σ_o diagram.

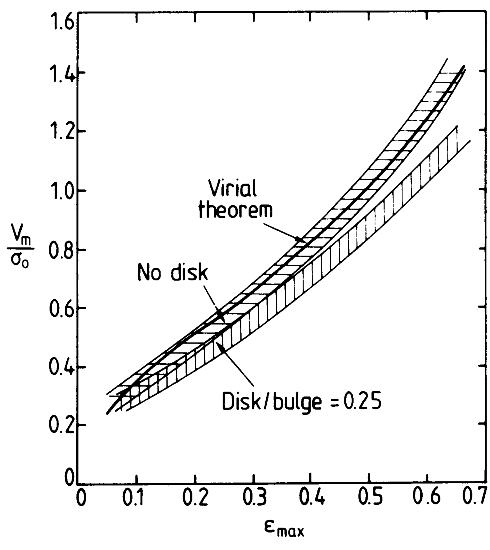

Figure 2. Maximum rotation speed over central velocity dispersion for Jarvis' models. The models with and without a disk lie in the shaded areas, while the full curve is the ratio of global parameters that one obtains from the virial theorem.

By adjusting the five parameters of his models, Jarvis is able to fit extensive photometric and kinematic data for the earliest of his galaxies, NGC 7814, extremely well. Frankly I find the quality of Jarvis' fits disconcerting since, as Hunter (1977) has

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pointed out, any model with a distribution function such as that of Jarvis, of the form $f(E,L_z)=g(E)h(L_z)$, will tend to become spherical near the centre. Furthermore, it is straightforward to show that near the centre of Jarvis' models, the spheroid must rotate as a solid body as regards the variation of $\langle v_\phi \rangle$ with both radius and height above the disk. My guess is that neither the ellipticity profiles nor the velocity fields of bulges have these characteristics. Unfortunately, the disks of Jarvis' galaxies effectively obliterate the central regions. It will be interesting to see whether rotationally-flattened low-luminosity ellipticals are consistent with distribution functions of the form $g(E)h(L_z)$.

3.2 Systems with $f(E,L_z,I_3)$

The great majority of orbits in potentials like those of disk galaxies admit a third integral of motion in addition to L_z and E (eg. Martinet and Mayer 1975). There is no unique form for the third integral since, given any third integral I_3, any non-trivial function $I_3'(E,L_z,I_3)$ yields a new third integral I_3'. However, the potentials of spheroidal components are usually fairly spherical, and in this case it is natural to consider I_3 to be a generalization of the magnitude $L \equiv |\mathbf{r} \times \mathbf{v}|$ of the angular momentum vector (Saaf 1968, Innanen and Papp 1977, Richstone 1982). Adopting the convention that I_3 is the natural generalization of L, consider the general structure of models based on the following simple distribution functions.

(i) $f = f_K(E)\exp\left[\frac{\Omega L_z}{\sigma^2} - \left(\frac{I_3}{r_a\sigma}\right)^2\right]$

(ii) $f = f_K(E)\exp\left[-(I_3^2-L_z^2)/(r_a\sigma)^2\right]$

(iii) $f = f_K(E)\exp\left[\frac{\Omega L_z}{\sigma^2} - (I_3-L_z^2)/(r_a\sigma)^2\right]$

where $f_K=f_o[\exp(-E/\sigma^2)-1]$ is King's (1966) distribution function, and Ω and r_a are parameters.

A model built around the first of these distribution functions will rotate with central angular speed Ω like the models of Prendergast and Tomer (1970), Wilson (1975) and Jarvis (1981), while having radially elongated velocity ellipsoids as in the models of Michie and Bodenheimer (1963). Figure 1b is a caricature of a model of this type. Such models may account for observations of rotationally-flattened spheroidal components and globular clusters. Lupton and Gunn (in preparation) have constructed models of this type and fitted them to observations of globular clusters, by assuming I_3=L. Petrou (1982 and this symposium) has constructed models in which a more sophisticated approximation to I_3 is employed.

Since the distribution function (ii) is even in L_z, it leads to models which do not rotate. However, these models will be flattened because the argument of the exponential is approximately equal to $[-(L_x^2+L_y^2)]$, and so orbits that carry stars far from the equatorial plane will be depopulated. Clearly we can set a model of this type rotating by adding a component to f that is odd in L_z as in (iii). Petrou (1982) has used a distribution function akin to (iii) to build models which are flattened by anisotropy rather than rotation.

Two problems that should not be difficult to solve, but have been outstanding for some years, are (a) to find the distribution function that generates a realistic spheroidal system with a flat rotation curve, and (b) to find the distribution function that generates a box shaped bulge like that of NGC 128 (p.7 of the Hubble Atlas).

3.3 Schwarzschild's Method

Schwarzschild (1979) introduced a technique into galactic dynamics which enables one to construct a model with a predetermined density distribution, without assuming anything about non-classical integrals such as I_3. In Schwarzschild's technique, one chooses a convenient potential and then uses the computer to calculate a library of orbits in this potential. Linear programming techniques are then used to determine whether these orbits can be populated in such a way as to generate the initially assumed potential.

This technique has so far only been applied by Schwarzschild to the construction of triaxial systems, and by Richstone (1980 and this symposium) and Meys et al (this symposium) to the construction of rather special scale-free models. It is a pity that nobody has yet used Schwarzschild's method to construct a realistic axisymmetric model, since the technique is very well suited to this problem, and the labour involved, though considerable, is much less than that involved in the construction of Schwarzschild's triaxial models.

4. TRIAXIAL SYSTEMS

Over the last four years a wide range of triaxial equilibrium stellar models have been published. Aarseth and Binney (1978) and

Wilkinson and James (1982) have described n-body models in which the triaxial figure is stationary in space, while Schwarzschild (1979) has used his technique to construct a model of this type around a predetermined Hubble-like density profile. Wilkinson and James (1982) and Schwarzschild (1982) have shown that anisotropy-supported bars of this type can be generalized to include figure rotation. Hohl and Zang (1979) and Miller and Smith (1979) have shown that rapidly rotating clouds of stars invariably relax to tumbling bars. The product of a galaxy merger (White, this symposium) is oblate if the galaxies spiral together from a large impact parameter encounter, and prolate if the galaxies collide head-on and their spins are dynamically unimportant. A head-on collision between galaxies with suitably aligned spins can generate a system which is part prolate and part oblate (Gerhard 1982a) and may not even settle to a state that is steady in a suitable rotating frame of reference (Gerhard 1982b).

Thus numerical work indicates that triaxial equilibria are the outcome of a wide variety of initial conditions. Attempts to determine whether elliptical galaxies are more often prolate or oblate (Marchant and Olson 1979, Richstone 1979, Lake 1979, Merritt 1982) have yet to produce a definite result for want of sufficient photometric and spectroscopic data. At the moment the best hope of pinning down the shapes and figure rotation speeds of ellipticals seems to lie in gas in and around these galaxies. Westerbork observations (Knapp, this symposium) indicate that the velocity fields of many of these gas features are remarkably regular. This suggests that each element of gas is moving on a closed orbit. The spatial and velocity structure of these orbits should betray the figure and rotation speed of the underlying potential (Binney 1978, 1981, van Albada _et al_ 1981, Heisler _et al_ 1982, Magnenat 1982, Tohline and Durisen 1982).

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DISCUSSION

RICHSTONE : In the work with Mamon on M87 you used a hydrostatic approach. Don't you think considerations of the collisionless Boltzmann equation make the large jumps in your anisotropy parameter (as a function of radius) somewhat unreasonable ?

BINNEY : At present Jean's moment equations provide the only flexible framework within which to analyse observational data. Of course, one would prefer to think about observations in terms of an algorithm that generates distribution functions f(E,L) that are compatible with given observations, but we don't have such a treasure. So Mamon and I thought we would probe the limits of what can be achieved with Jean's equations, and emphasize the danger of assuming with Sargent et al that β can be simply set to zero.
The following simple argument shows that very rapid changes in β are possible in principle : in the portion of an unconfined Michie model in which $\beta \simeq 1$, the density $\rho(r) \simeq \rho_1 \, (r_1/r)^{3.5}$. If we set this system in the low-density core of a giant elliptical we will have

$$\beta(r) \approx 1 - \rho_2\sigma_\Theta^2 \left[\rho_1 s^2 \left(\frac{r_1}{r}\right)^{3.5} + \rho_2\sigma_r^2 \right]^{-1}$$

where ρ_2, σ_Θ and σ_r are the radius-independent parameters of the elliptical envelope, and s is the radially-directed velocity dispersion towards the outside of the Michie model. If one now sets $\rho_1=\rho_2$, s= σ_r and $\beta(\infty) = 0.4$, one finds that $\beta(0.75r_1) = 0.84$ and $\beta(1.5r_1) = 0.52$. When account is taken of the potential generated by the elliptical envelope, the density of the anisotropic core will fall more steeply than $\rho \sim r^{-3.5}$ and so β will decrease even more rapidly than in this naive model. In our model of M87, β changes from 0.85 at 100 pc to 0.4 at 200 pc.

INAGAKI : Is it possible,or even easy to construct dynamically stable models with velocity dispersion decreasing inwards ?

BINNEY : An example of a system of this type is the spherical galaxy with f(E) that obeys the $R^{1/4}$ law in projection (Mon. Not. R. Astr. Soc. 200, 951). Antonov's work (Vestnik Leningrad Univ. No 19 : 96) shows that this system is stable to all types of perturbation, and σ decreases interior to $0.07R_e$. It is easy to construct systems of this sort by inserting cold quasi-isothermal models into hot models of the same type (Mon. Not. R. Astr. Soc. 190, 873).

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