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VERTICAL STRAIN-RATE MEASUREMENTS IN AN ARCTIC ICE CAP AND DEDUCTIONS FROM THEM

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Abstract. Closely spaced measurements of diameter of thermally drilled bore holes reveal a pattern of small variations. These patterns serve to identify points on the bore-hole wall; thus the change in length of sections of bore hole can be determined as a function of time. This method has been used to measure vertical strain-rate as a function of depth in two bore holes near the crest of the Devon Island ice cap. The measured strain-rate, corrected for firn compaction, varies significantly with depth. The vertical component of velocity at the surface was determined from the contraction rate of a bore hole that penetrated to the base of the ice. Comparison of this velocity with the present accumulation rate suggests that the ice cap, in the vicinity of the bore hole, is thickening slightly at present. The age of the ice at various depths, as calculated from the measured vertical velocities, is in broad agreement with radio-carbon dates covering the past 6 000 years. This suggests that the flow of the ice cap has not varied significantly over this period, and thus that the present accumulation rate, which is causing thickening, is slightly above the average for the period.

Résumé. *Mesures de vitesse de déformation verticale dans une calotte glaciaire arctique et déductions à en tirer.* Des mesures précises de diamètre de forages thermiques révèlent un système de petites variations. Ces systèmes servent à identifier des points sur la paroi du forage; on peut dès lors étudier le changement en fonction du temps de la longueur de sections entre deux points particuliers. Cette méthode a été utilisée pour mesurer la vitesse de déformation verticale en fonction de la profondeur dans deux forages près de la crête de la calotte du Devon Island. La vitesse de déformation mesurée, corrigée pour tenir compte du tassement du névé, varie de manière significative avec la profondeur. La composante verticale de la vitesse à la surface est determinée à partir de vitesse de contraction de la profondeur d'un forage qui parvenait à la base de la glace. La comparaison de cette vitesse avec le taux d'accumulation actuel suggère que la calotte, au voisinage du forage, est en ce moment en cours d'épaississement modéré. L'âge de la glace à des profondeurs variées, caculé à partir des vitesses verticales mesurées, confirme en gros les datations au radio-carbone couvrant les 6 000 dernières années. Ceci prouverait que l'écoulement de la calotte n'a pas beaucoup varié depuis cette période et que le taux d'accumulation actuel, qui entraîne un épaississement, est légèrement supérieur à la moyenne pour l'ensemble de la période.

Zusammenfassung. *Messung der vertikalen Verformungsgeschwindigkeit in einer arktischen Eiskappe und Folgerungen daraus.* Eng benachbarte Messungen des Durchmessers thermisch abgeteufter Bohrlöcher zeigen ein Muster kleiner Schwankungen. Diese Muster können zur Identifizierung von Punkten an der Wand des Bohrloches herangezogen werden. Auf diese Weise lässt sich die Längenänderung von Bohrlochabschnitten als Funktion der Zeit bestimmen. Dieses Verfahren wurde zur Messung der vertikalen Verformungsgeschwindigkeit von der Tiefe in zwei Bohrlöchern nahe am Scheitel der Eiskappe von Devon Island herangezogen. Die gemessene Verformungsgeschwindigkeit, die wegen der Firnsetzung zu korrigieren war, ändert sich mit der Tiefe beträchtlich. Die vertikale Geschwindigkeitskomponente an der Oberfläche wurde aus der Verkürzung eines Bohrloches bestimmt, das bis zum Untergrund des Eises reichte. Der Vergleich dieser Geschwindigkeit mit der derzeitigen Akkumulationsrate lässt darauf schliessen, dass die Eiskappe in der Umgebung des Bohrloches derzeit geringfügig dicker wird. Das Alter des Eises in verschiedenen Tiefen, berechnet aus den gemessenen Vertikalgeschwindigkeiten, stimmt im allgemeinen mit Radio-Karbondaten über die letzten 6 000 Jahre zusammen. Dies lässt vermuten, dass der Fluss der Eiskappe sich in dieser Periode nicht wesentlich geändert hat und dass hinwiederum die derzeitige Akkumulationsrate, die eine Zunahme der Dicke bewirkt, geringfügig über dem Mittel für diese Periode liegt.

Introduction

In a polar ice cap, measurements of the vertical velocity component, or its derivative the vertical strain-rate, are important because they help to answer two questions:

Is the ice cap becoming thicker or thinner?

How old is the ice at different depths?

To answer the first question, the vertical velocity at the surface can be compared with the mass balance. A time scale for a climatic history derived from oxygen isotope analysis of cores can be obtained by integrating the reciprocal of the vertical velocity with respect to depth. In addition, strain-rate measurements provide a test of the commonly made assumption that the direct strain-rates do not vary with depth. However, the standard method of determining velocity at depth in a glacier, namely, to measure the rate at which a bore hole tilts, does not

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determine the vertical component. The majority of such experiments have been made in cased bore holes and, because the length of the hole changes as the glacier flows, the casing must slip along the hole. As a result, the motion of the casing parallel to the hole does not correspond to the ice motion.

One way to determine vertical strain-rate is to measure how the lengths of sections of a bore hole change with time. This is not a direct measurement, however, because shear also changes the length. Such measurements have been made in rapidly deforming temperate glaciers. On Austerdalsbreen, Ward (1961) determined the vertical strain-rate averaged over the ice thickness by measuring the apparent sinking into the ice of a bore hole casing over periods of half a day. He also pointed out that the variation with depth could be measured by stopping the casing at various points during drilling. On Blue Glacier, Fletcher and Kamb (1968) measured total contraction rates of eight bore holes, and their variation with time. Shear dominated the flow pattern here and the contraction rate was interpreted in terms of the difference between the horizontal velocity at the surface and the sliding velocity at the base. Also in Blue Glacier, Harrison (1975) set nine cables to different depths in separate bore holes and allowed them to freeze in. They were removed a year later by melting them free with a small electric current. The measured stretching of each cable gave the strain-rate. The best method appears to be that of Rogers and LaChapelle (1974). Electrically conducting rings are implanted at intervals along the bore-hole walls. These can be located by lowering a resonant electrical circuit down the bore hole; the frequency changes as the circuit passes a ring. The time variation of distance between rings can thus be measured.

Bader (1964) analyzed the measurement problem in the Greenland and Antarctic ice sheets and pointed out the high precision needed to obtain results in a reasonable time. No one appears to have followed up his suggestions however, and no vertical strain-rate measurements in polar ice caps have hitherto been made. This paper deals with the method and results of such measurements in the Devon Island ice cap in the Canadian Arctic.

The ice cap and bore holes

The ice cap lies in the eastern part of Devon Island, has an area of about 15 600 km² and a maximum elevation of about 1 900 m. Measured ice thicknesses are in the range 200 to 1 000 m (personal communication from G. de Q. Robin). The ice cap has an east–west summit ridge and is drained by a series of valley glaciers on its northern, eastern, and southern sides. Much of the ice edge lies at an elevation of about 600 m but it reaches sea-level in the south-east; the major valley glaciers also end in the sea. Annual precipitation decreases from about 0.4 m water equivalent in the south-east to 0.1 m in the north-west (Koerner, 1966).

The two bore holes used in this study lie on the same flow line about 600 and 900 m north of the summit ridge and about 7.5 km west of its highest point. The surface slope is 1.4°. Hole 71 is 230 m deep; drilling ceased when the drill became frozen in. Hole 72 reaches bedrock at 299 m. Both holes were dry when the strain-rate measurements were made and only the top 3.2 m of each was cased. Measured temperatures in hole 72 are -23°C at 12 m depth and -18.5°C at the bottom. The horizontal strain-rates measured at the surface in the down-slope and cross-slope directions are $+0.72\times10^{-3}$ and $+0.06\times10^{-3}$ a^{-1} at hole 71 and $+0.67\times10^{-3}$ and $+0.02\times10^{-3}$ a^{-1} at hole 72. The standard error of each measurement is 0.02×10^{-3} a^{-1}.

Method of measurement

To measure the change in length of sections of a bore hole, the walls must be marked at certain places; the small variations in hole diameter that occur during drilling provide such marks. With the CRREL intermediate-depth thermal drill (Ueda and Garfield, 1969) that we used, the hole is drilled in 1.5 m sections; at the end of each, the drill is raised for extraction of core and melt water. Part of the drilling procedure is to stop the drill feed for 2 min at a certain point in each run, to melt a "neck" in the core where it should break at the end of the run. This melting also enlarges the hole at that point and so provides a mark every 1.5 m. Other diameter changes result from small, naturally occurring variations in drilling speed. At such places, diameter measurements at intervals of about 10 mm reveal a pattern that is preserved for at least a year, in firn as well as ice.

Diameter was measured by a Pollak and Skan caliper as described by Hansen and Landauer (1958). Three spring-loaded arms, in contact with the walls at points 120° apart, vary a linear potentiometer whose resistance is a function of the diameter. The instrument was mounted on the cable used for the drill. The drum on which this cable is wound is normally driven by a motor, but there is also a hand wheel in case the power fails. The gearing

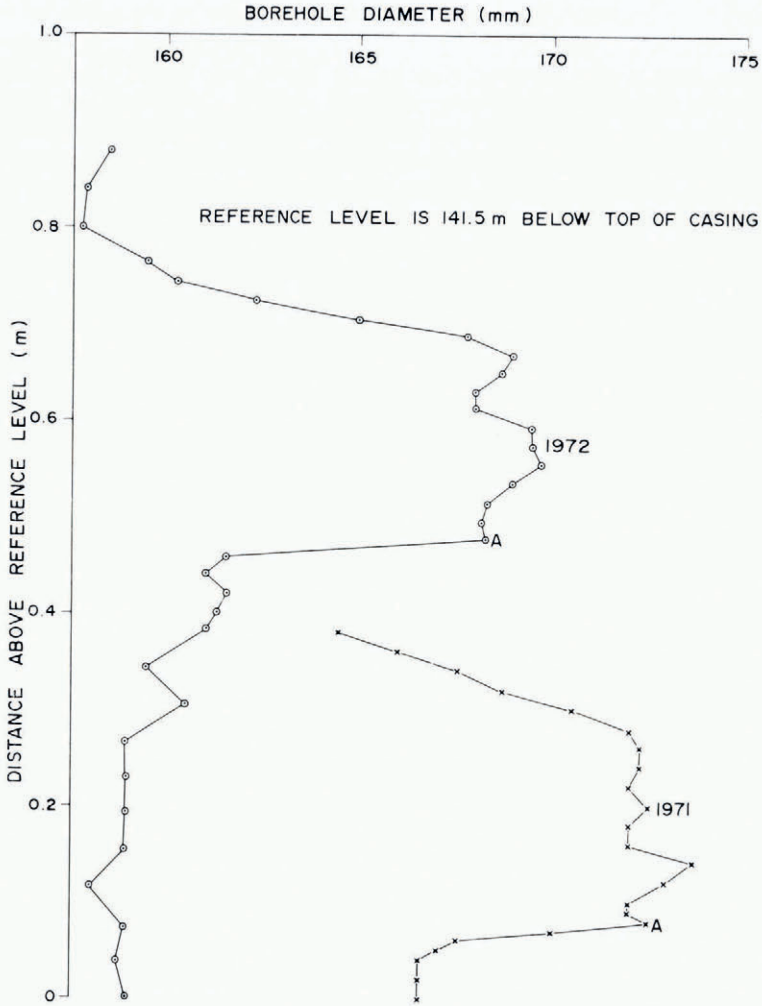

Fig. 1. Diameters measured in successive years over a short section of bore hole at the same distance below the top of the casing. Each point represents one measurement. The displacement of recognizable features such as point A measures the contraction of the bore hole between that point and the casing.

is such that one turn of the hand wheel raises the cable by approximately 20 mm. The distance corresponding to one turn varies slightly according to the amount of cable on the drum, which depends on the depth; the figure has to be determined accurately for each measurement.

At each point where a measurement is to be made, the cable is marked at the point level with the top of the bore-hole casing. The hole diameter is then measured every 5 or 10 mm over a distance long enough to show a recognizable pattern. The measurements are repeated a year later starting at the same cable marks. Figure 1 shows a record typical of depths down to about 200 m. Below 200 m the patterns were less easily recognized because the sides of the hole appeared to have become rougher during the year. This probably resulted from re-crystallization of the ice; this should occur more rapidly as depth increases because both the temperature, and the stress tending to close the hole, increase with depth. Two sets of measurements were made at each depth in each year. Measurements were made at four points in hole 71 in June 1971 and May 1972, and at depth intervals of about 25 m in hole 72 in June 1972 and April 1973. By 1973, the lowest 4.9 m had become too narrow for the caliper to penetrate. The total length of the hole was therefore measured with a weight on a wire line. This was compared with its length when drilled.

Although the method described proved satisfactory, and useful data were obtained, improvements seem possible. A system might be devised to record diameter continuously on a chart recorder, driven by the motor on the hoist, as the caliper is brought up the hole. Strain-rates in the Greenland and Antarctic ice sheets are at least an order of magnitude less than those on the Devon Island ice cap. Comparable precision could be obtained, however, by spacing the measurements at 250 m rather than 25 m and by increasing the time interval. The problem of the blurring of the pattern by recrystallization of the ice at depth could perhaps be overcome by making, during drilling, sudden changes in diameter more marked than those that normally occur. However, the method of Rogers and LaChapelle (1974) is probably preferable, provided that their suggestion of using inductive heating to implant the markers in cold ice can be made to work. Wire strain meters, of the type recently used on the surface of a glacier (Goodman and others, 1975), offer another possibility if a means of installing them in bore holes can be devised.

Data reduction and results

Velocity

Figure 2 shows the contraction, in one year, measured between the top of the casing and points in the two bore holes. The estimated standard error of each measurement is 10 mm a^{-1}, except for the lowest point; the error there may be about 30 mm a^{-1} because the total length of the hole was not measured by the same method each year. However, the fact that the lowest point lies on the smooth curve increases confidence in the value. In the upper 100 m, contraction rates in hole 71 appear slightly greater than in hole 72. The point at 170 m in hole 72 does not lie on the smooth curve. The reason for this discrepancy is not clear; the most likely explanation is that the number of turns on the hand wheel on the hoist was miscounted during one set of measurements.

Two factors may have perturbed the measurements: the casing in the uppermost 3.2 m of each hole and the additional load of the camp on the snow surface. The casing was placed in a wide hole which was then filled in and water poured down the outside of the casing to refreeze and anchor it securely in the snow. Drilling was then carried out with the thermal drill inside the casing. The additional load on the surface consisted of a building of floor area 5 m × 5 m erected round the drill site to house the rig, plug several other buildings in the vicinity and snow drifts that accumulated around them. At the end of the field season, which lasted about two months, the building around the bore hole was left in position but the other buildings were removed.

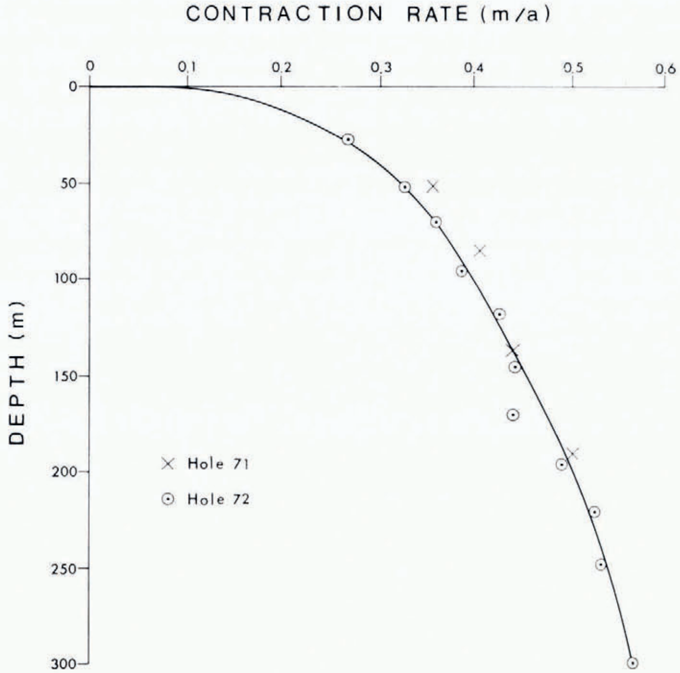

Fig. 2. Measured contraction in one year as a function of depth in two bore holes. The estimated standard error of each point is 0.01 m a⁻¹, except for the lowest point where it is 0.03 m a⁻¹. The smooth curve is drawn through the points from hole 72.

Table I provides evidence on the effects of these factors. The "calculated" values are the firn compaction calculated from the variation of density with depth as described below (Equation (2)). The contraction of 270 mm was measured between the *top* of the casing and the first marked position in the bore hole. It has been tabulated for the depth range 3.2 to 27.5 m because it should not include snow compaction along the length of the rigid casing. This should be measured by the displacement of the floor of the building relative to the top of the casing, the quantity tabulated for the range 0 to 3.2 m. This quantity is very much less than the calculated value; on the other hand, totals for the range 0 to 27.5 m are in approximate agreement. We conclude that (1) the casing was securely anchored in the near-surface snow layers and moved downwards with them and (2) the additional surface load had a negligible effect on the compaction rate. The measured displacement of 20 mm a^{-1} has been taken to represent compaction along the length of the casing and added to the measured compaction (relative to the casing) at each depth.

The contribution of shear to the measured contraction of the bore hole was calculated by the formula of Harrison (1975, equation (9)). The derivation of this formula rests on the assumption that the bore hole is initially vertical. This holds in the present case: inclinometer

TABLE I. COMPACTION RATES IN UPPER PART OF HOLE 72

Depth interval m	Measured compaction rate mm a^{-1}	Calculated compaction rate mm a^{-1}
0.0– 3.2	20	140
3.2–27.5	270	160

measurements made shortly after drilling showed that the maximum inclination in hole 72 was 0.67° and that the horizontal displacement between top and bottom of the hole was only 0.7 m. Another set of measurements a year later determined the shear strain-rate as a function of depth. Its maximum value was only 3.4×10^{-3} a^{-1}, except in the lowest 5 m where it reached 10^{-2} a^{-1}. The contribution of shear to the total shortening of the hole was only 15 mm.

The measured contractions in one year, corrected for shear, give $v-v_s$ where v is the velocity component perpendicular to the surface, measured positive downwards, and v_s is its surface value. (Harrison's formula includes a rotation of axis from vertical to normal to the surface.) Because the ice sheet is frozen to its bed, the velocity there is zero. Thus the change in length of the complete bore hole gives v_s and the values of v are then determined. Figure 3 shows v as a function of depth. The estimated standard error of v, obtained by combining those of v_s and $(v-v_s)$ is 32 mm a^{-1}.

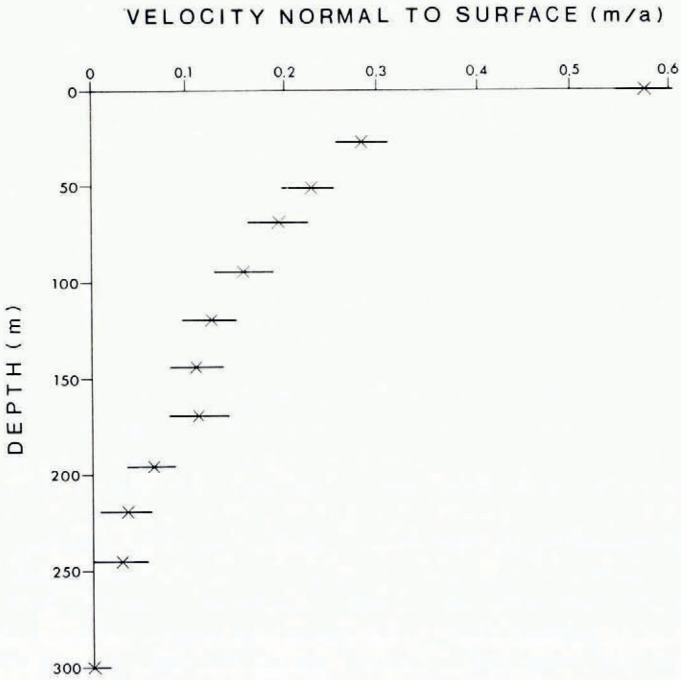

Fig. 3. Downward velocity perpendicular to ice-cap surface v as a function of depth in hole 72. The bars represent one standard error on each side of the point.

Strain-rate

The strain-rate perpendicular to the surface averaged over intervals of roughly 25 m (the distance between measurement points) was calculated by numerical differentiation of v. The values of v were first smoothed by fitting a cubic regression equation to them; higher-order regressions made no significant improvement to the fit. This strain-rate includes that due to firn compaction and to compression of air bubbles in the ice, terms that can be computed separately as follows:

Take the x-axis in the surface, pointing down the direction of maximum slope, the y-axis normal to the surface, positive downwards, and the z-axis so as to make the system righthanded. Let u, v, w, be the x, y, z velocity components. Let $\dot{\epsilon}$ denote strain-rate, t time and ρ density. The equation of mass conservation is

$$\dot{\epsilon}_{xx}+\dot{\epsilon}_{yy}+\dot{\epsilon}_{zz} = -\frac{1}{\rho}\frac{D\rho}{Dt}$$

where D/Dt is the material time derivative. If it is assumed that $\partial\rho/\partial t = 0$, that is that the density at a given point does not change with time (Sorge's Law), the equation reduces to

$$\dot{\epsilon}_{xx}+\dot{\epsilon}_{yy}+\dot{\epsilon}_{zz} = -\rho^{-1}(u\,\partial\rho/\partial x+v\,\partial\rho/\partial y+w\,\partial\rho/\partial z). \qquad (1)$$

It seems plausible to assume that density variations in the horizontal direction are negligible compared with the vertical variation. Moreover, w is negligible and, since the bore hole is near the ice divide, u and v are comparable. Equation (1) thus becomes

$$\dot{\epsilon}_{yy}' = -\dot{\epsilon}_{xx}-\dot{\epsilon}_{zz} = \dot{\epsilon}_{yy}+(v/\rho)\,\partial\rho/\partial y \qquad (2)$$

where $\dot{\epsilon}_{yy}'$ is the strain-rate corrected for firn compaction. The correction term was evaluated from the measured values of v and the depth–density curve which was obtained from measurements on cores from the bore holes. Firn compaction ceases at a depth of 60 m where the density reaches 820 kg m^{-3}. Between 60 m and 90 m, density increases as a result of compaction of air bubbles in the ice. No measurable density change occurred below 90 m. Because the measured values of $\dot{\epsilon}_{yy}$ are averages over depth intervals of roughly 25 m, the correction term was integrated numerically over the same intervals.

Figure 4 shows $\dot{\epsilon}_{yy}'$ as a function of depth. The surface point is the measured surface value of $-\dot{\epsilon}_{xx}-\dot{\epsilon}_{yy}$. No point has been plotted for the interval 0 to 25 m because the calculated firn compaction was slightly greater than the measured strain (see Table I). This discrepancy

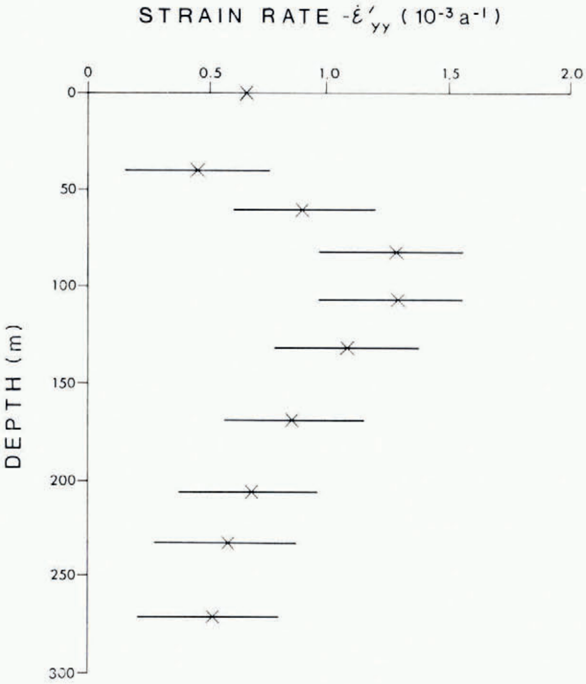

Fig. 4. Component of strain-rate measured perpendicular to ice-cap surface, corrected for compaction of firn and air bubbles, as a function of depth in hole 72. All strains are compressive. The bars represent one standard error on each side of the point. The standard error of the surface point is 0.02×10^{-3} a^{-1}.

probably arises because, near the surface, local variations make it difficult to obtain representative measurements of density and a small density change makes a considerable difference to the calculated firn compaction. The curve has not been extrapolated to the bed because the strain-rate there is unknown. Because the ice is frozen to its bed the velocity components will be zero everywhere. However the strain-rate components will only be zero if the slope of the bed, averaged over a certain distance, is the same as that of the surface. The appropriate distance is uncertain but it should probably be comparable with the ice thickness. Radio-echo sounding (Paterson and Koerner, 1974; personal communication from C. S. M. Doake) shows that the bed is rough and, although the mean slope is approximately the same as that of the surface over a distance of about 150 m up-stream from hole 72, this is not the case over longer or shorter distances.

The standard errors of $\dot{\epsilon}_{yy}'$ were calculated as follows. The root-mean-square deviation of the measured values of v from the regression curve fitted to them is 5 mm a^{-1}. Thus the standard error of the difference between two values is 7 mm a^{-1} and thus that of a strain-rate over a depth interval of 25 m is 0.3×10^{-3} a^{-1}. The standard errors of the values at 40 and 60 m may be slightly greater than this as a result of inaccuracies in the correction for firn compaction.

Discussion

Change in ice thickness with time

The relevant equation is (e.g. Nye, 1975, equation (2))

$$\partial h/\partial t = c - v_s. \tag{3}$$

Here h is ice thickness, t time, c accumulation rate measured as snow thickness per unit time, and v_s the downward component of velocity at the surface. All quantities are measured perpendicular to the surface. At hole 72, the annual accumulation is 220 kg m^{-2} and the mean surface density is 330 kg m^{-3} (personal communications from R. M. Koerner). Thus $c = 0.67$ m a^{-1}. Also $v_s = 0.57$ m a^{-1}. Thus $\partial h/\partial t = +0.10$ m a^{-1} of snow. The estimated standard error of v_s is 0.03 m a^{-1}. The precision of the value of c is difficult to estimate as a result of the problem of sampling both in space and time. The value is obtained from pit studies covering 11 years. However, the thickness of an annual layer varies significantly from year to year and from point to point within a short distance of the bore hole. The surface density also varies, and it is difficult to allow for the effect of ice layers in the firn. In spite of these uncertainties, the measurements suggest that the ice cap in the vicinity of the bore hole is thickening at present. An attempt will be made to check this by repeated gravity measurements as Bentley (1971) did at the South Pole; the first measurement was made in 1971. The radio-echo method (Nye and others, 1972; Nye, 1975) would also be worth trying.

The fact that the ice cap is thickening under the present accumulation rate suggests that this rate may be abnormally high. If it had persisted for hundreds of years, ice flow should have adjusted to it. Further information on this point can be obtained by calculating the age t of the ice at depth y from the formula $t = \int_0^y \mathrm{d}y/v$ using numerical integration of the measured values of v (Fig. 3). The calculated ages agree, within two standard errors, with three out of four radio-carbon dates spanning the last 6 000 years, obtained by down-bore-hole extraction of CO_2 from air bubbles in the ice (personal communication from P. Bucher). The discrepancy occurs at the youngest date, and the radio-carbon value may be wrong. This suggests that the value of v has not changed appreciably over the past 6 000 years. Thus an annual accumulation of 190 kg m^{-2}, corresponding to the present value of v_s, should be more typical of this period than the present value of 220 kg m^{-2}.

The present thickening at the bore hole does not necessarily conflict with the fact that the mass balance of the north-west part of the ice cap, in which the drill site lies, was negative for the period 1961–71 (Koerner, 1970 and personal communication). Mass balance depends not only on accumulation rate, which controls the thickness at the drill site, but also on the ablation rate at lower elevations, particularly on the large outlet glaciers.

Variation of strain-rate with depth

The general trend of the points in Figure 4 suggests that, even when allowance is made for the relatively large standard errors, the strain-rate $\dot{\epsilon}_{yy}'$, corrected for firn compaction, varies significantly with depth. In particular the strain in the firn appears to be less than that in the ice immediately below it. This cannot result merely from inaccuracies in calculating firn compaction, because compaction is zero below 90 m. Moreover, the surface value was obtained from surface measurements of $-\dot{\epsilon}_{xx}-\dot{\epsilon}_{zz}$ and is independent of any measurements in the bore hole. This result suggests that a form of extrusion flow can occur, at least locally, near an ice divide. Inclinometer measurements, made over a one-year interval, of the variation of horizontal velocity with depth support this conclusion; velocity increases from a minimum of 0.57 m a^{-1} at 60 m to a maximum of 0.64 m a^{-1} at 110 m. However, as the standard error of these measurements is 0.03 m a^{-1}, we prefer to postpone further discussion of these observations until results of another set of inclinometer measurements, planned for 1976, are available. Moreover, because the amount of the extrusion flow is small, the possibility that it has been induced in some way by the presence of the bore hole cannot be completely excluded.

A theoretical analysis of glacier flow by Nye (1957), based on the assumptions that flow is confined to the x–y plane and that the stresses are independent of x, leads to velocity solutions such that the strain-rates $\dot{\epsilon}_{xx}$ and $\dot{\epsilon}_{yy}$ are constant with depth. Subsequently this assumption has been widely adopted and a value of $\dot{\epsilon}_{xx}$ measured at the surface has been taken as the value at depth also. Because the present measurements were made near an ice divide, where the longitudinal stress is unlikely to be independent of x, it is perhaps not surprising that the theoretical prediction fails. However, it has also been found to fail in the only other places where it has been tested, namely Athabasca Glacier (Savage and Paterson, 1963; Raymond, 1971), Blue Glacier (Shreve and Sharp, 1970; Harrison, 1975) and near the edge of Barnes Ice Cap (Hooke, 1973). Thus the strain-rate averaged over the depth cannot be determined accurately by measuring the strain-rate at the surface.

One case in which these two strain-rates have been assumed equal is a method for determining whether an ice sheet is thickening or thinning proposed by Shumskiy (1965) and also used by Mellor (1968), in both cases with unconvincing results. For an ice sheet deforming in plane strain, so that $\dot{\epsilon}_{xx} = -\dot{\epsilon}_{yy}$, the average value of $\dot{\epsilon}_{xx}$ over the ice thickness, denoted by $\langle\dot{\epsilon}_{xx}\rangle$, is given by

$$h\langle\dot{\epsilon}_{xx}\rangle = \int_0^h \dot{\epsilon}_{xx}\,\mathrm{d}y = -\int_0^h \dot{\epsilon}_{yy}\,\mathrm{d}y.$$

But $\dot{\epsilon}_{yy} = \partial v/\partial y$, and if the ice is frozen to its bed $v = 0$ there. Thus the integral on the right-hand side is equal to v_s. The method consists in assuming that $\langle\dot{\epsilon}_{xx}\rangle$, the strain-rate averaged over the depth, is equal to the strain-rate measured at the surface. Thus v_s is determined and then $\partial h/\partial t$ from Equation (3). However, $\partial h/\partial t$ is usually the small difference between two approximately equal terms and a surface value of $\dot{\epsilon}_{xx}$ will probably be inadequate.

To calculate the age of the ice at different depths in a core, the form of the variation of vertical strain-rate with depth must be assumed. The simplest assumption is that strain-rate is constant. As an alternative, Dansgaard and Johnsen (1969) used a model in which strain-rate is constant down to a certain depth and then decreases linearly to zero at the bottom.

A time scale based on measured vertical strain-rates should be an improvement over theoretical models. However, one still has to make the steady-state assumption namely that strain-rates have remained constant throughout the time period represented by the core. This is of course unlikely, at least for periods of a few thousand years or more, and the major errors in the time scale will probably arise from the failure of this assumption rather than from differences between the various possible relations between strain-rate and depth.

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