Solution by J. W. Moon, University of Alberta, Edmonton

It is clear that $n \geq k + 2$. We may suppose that G has some vertex x of valence at most k, for otherwise the result is certainly true. Then the graph obtained from G by removing x and its incident edges has n - 1 vertices and more than $k(n-k) + \binom{k}{2} - k = k[(n-1)-k] + \binom{k}{2}$ edges. The result now follows immediately by induction, since it is trivially true when $n = k + 2$.

Also solved by W. G. Brown and the proposer.

Editor's comment. The result is vacuously true for $n = k, k + 1$ since then no graph has as many edges as the problem requires; but as stated, it is false for $n < k$, the complete graph furnishing a counter-example.

P 90. Let $\log_s x$ be the log function iterated s times, and let m be the smallest positive integer such that $\log_4 m > 1$. Then show that the sum

$$\Sigma_{k=m}^{\infty} \frac{1}{k(\log k)(\log_2 k)(\log_3 k)(\log_4 k)^2}$$

is approximately 1 - correct to more than one million decimal places!

John D. Dixon, California Institute of Technology

Solution by S. Spital, California State Poly. College.

Since the series in question

$$S = \sum_{k=m}^{\infty} u(k) = \sum_{k=m}^{\infty} \frac{1}{k \log k \log_2 k \log_3 k (\log_4 k)^2}$$

is composed of positive decreasing terms, and since

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$$\int_m^\infty u(t)dt = \int_m^\infty \frac{d(\log_4 t)}{(\log_4 t)^2} = \frac{1}{\log_4 m}$$

is convergent, the integral test demands that S be convergent and bounded by

$$\frac{1}{\log_4 m} < S < \frac{1}{\log_4 m} + u(m).$$

Defining $\log_4 t_o = 1$ and recalling that m is an integer such that $\log_4 m > 1$, we can bound u(m):

$$0 < u(m) < u(t_o) = \frac{1}{e^{e^{e^{e^{e^{e^{e}}}}}}\, e(1)^2} < 10^{-1.5\times 10^6}.$$

Noting that $u(t) = -\frac{d}{dt}\left(\frac{1}{\log_4 t}\right)$ we use the mean value theorem to begin the bounding of $1/\log_4 m$:

$$\frac{1}{\log_4 t_o} - \frac{1}{\log_4 m} = (m-t_o)u(\tau), \quad t_o < \tau < m.$$

From the monotonicity of u, $u(t) < u(t_o)$ and since m is the <u>smallest</u> integer greater than t_o, $(m-t_o) < 1$; therefore

$$0 < 1 - \frac{1}{\log_4 m} < u(t_o) < 10^{-1.5\times 10^6}$$

or

$$1 - 10^{-1.5\times 10^6} < \frac{1}{\log_4 m} < 1.$$

Thus, combining the two bounds,

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$$1 - 10^{-1.5\times 10^6} < S < 1 + 10^{-1.5\times 10^6} .$$

Also solved by the proposer.