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BOOK REVIEWS

DAVIES, K. M. and CHANG, Y.-C. *Lectures on Bochner–Riesz means* (London Mathematical Society Lecture Note Series 114, Cambridge University Press, Cambridge, 1987), pp. x + 150, 0 521 31277 9, paper, £15.

The Bochner–Riesz means, defined in the series case by $S_R^\alpha f(\theta)=\sum(1-|k|^2/R^2)_+^\alpha \hat{f}(k)e^{2\pi ik\theta}$ and originating of course in classical summability theory, are of considerable interest in harmonic analysis at the present time. For example, the problem of whether or not $(1-|x|^2)_+^\alpha$ is an $L^p(R^n)$ multiplier for $2n/(n+1+2\alpha)<p<2n/(n-1-2\alpha)$, which would be the optimal range, is still open and attracting considerable attention although the case $n=2$ was settled affitmatively by L. Carleson and P. Sjölin in 1972. The book under review aims to make available to postgratuate students some of the results in the area which have appeared in journals since about that time, and in this succeeds well.

After an opening chapter on the elementary properties of multipliers of Fourier series and integrals, which includes proofs of the Riesz–Thorin and Marcinkiewicz interpolation theorems, the authors give a modern and relevant treatment of the Hilbert transform, including maximal functions and Calderon–Zygmund decomposition. This is followed by a short chapter on good lambda and weighted norm inequalities, including the A_p condition. The treatment of multipliers with singularities begins with proofs of the Hörmander–Mihlin and Marcinkiewicw multiplier theorems and the Littlewood–Paley decomposition theorem, and then has a discussion of singularities on curves, in particular on $|x|=1$. The final chapters are on restriction theorems, C. Fefferman's celebrated resolution of the multiplier problem for the characteristic function of the unit ball, and Cordoba's 1979 proof of (essentially) Carleson and Sjölin's result.

The typescript is pleasing and it is reproduced well, though there are many slips and misprints. These are mainly of a trivial nature, but it is annonying to find the optimal range for L^p boundedness given differently in the introduction to Chapter 6 and in the statement of the actual result, Theorem 6.9. Again, in the case $n=2$, Corollary 8.10 simply says "Bochner–Riesz means are bounded on the optimal range", without making it clear that this refers not to $4/(3+2\alpha)<p<4/(1-2\alpha)$ but to the universal range $4/3\leq p\leq 4$ valid for all α. It is also disconcerting to meet numbers such as 52^j and 2002^j (the reader has to realise that these denote multipliers of 2^j by 5 and 200) and to have n used in Theorem 1.25 for the dimension and also as the index of the Fourier coefficient. The authors' style is informal in the extreme. This is no bad thing, but occasionally, where they lead the reader up a blind alley for didactic reasons, it meant that the reviewer was puzzled about the exaxt status of an argument; there is also a sentence on page 43, 'ine 9, which is offensive and should not have been allowed by the editors to appear. However, a good graduate student, or indeed a more advanced researcher, will learn a great deal in finding his way through the book, and its apprarance is to be welcomed, particularly at such a reasonable price.

PHILIP HEYWOOD

OLIVER, R. *Whitehead groups of finite groups* (London Mathematical Society Lecture Note Series 132, Cambridge University Press, Cambridge 1988) x + 349 pp, paper: 0 521 33646 5, £19.50.

The first generation of invariants of algebraic topology were homotopy invariants involving free modules, such as the homology groups of chain complexes of such modules. The study of homotopy-equivalent manifolds which were not homeomorphic (such as lens spaces) led to a second generation of invariants, involving based free modules, such as the torsion invariant $\tau(f)\in \mathrm{Wh}(\pi)$ introduced by J. H. C. Whitehead some 50 years ago for a homotopy equivalence $f:X\to Y$ of finite simplicial (or CW) complexes, with $\pi=\pi_1(X)$ the fundamental group. The

Whitehead group $Wh(\pi)$ is an abelian group which measures the extent to which an invertible matrix with entries in the group ring $\mathbb{Z}[\pi]$ can be diagonalized by generalized Gaussian elimination. Whitehead torsion has found wide application in the study of high-dimensional manifolds, such as in the s-cobordism theorem.

The computation of the Whitehead group $Wh(\pi)$ was initiated by G. Higman in 1940. He showed that $Wh(\pi)$ is trivial for some groups π, such as the trivial group $\{1\}$, and the infinite cyclic group $\mathbb{Z}$, and non-trivial for others, such as the finite cyclic group $\mathbb{Z}_5$. This set the pattern for later developments, due to Bass, Milnor, Swan and others. By 1970 it had become clear that the computation of $Wh(\pi)$ for finite groups π could be attempted using the methods of algebraic number theory, such as localization and completion, and representation theory. These methods tend to fail for infinite groups π: instead the methods of combinatorial group theory and topology are used to prove that $Wh(\pi)$ is trivial for suitable classes of torsion-free groups.

In the last ten years Oliver has written a series of research papers on the Whitehead groups of finite groups, providing the most powerful computational technology available, at such a rate that it has been difficult for even experts in algebraic K-theory to keep up with the wealth of material. This book is a welcome opportunity to take stock, and survey the landscape. It is basically a pure algebra book, aimed at the reader who wants to learn the modern techniques, especially the p-adic logarithm introduced by Oliver. The author is quite high-minded, eschewing the simple device of a page or two tabulating the Whitehead groups for some particular finite groups, developing instead all the results needed to work out such a table. The only application considered here is to the congruence subgroup problem. However, the main applications have been to the computations of the Wall surgery obstruction groups of finite groups, over which the book draws a veil. Perhaps it is unreasonable to require a detailed account, but it is the important work of Wall and Bak in the early 70s which made the need for the new computational techniques apparent, and it is the L-theorists who are the largest category of users.

Overall, this is an excellent book for any K- or L-theorist who has to work with computations of the Whitehead groups of finite groups, and is already familiar with the classic text of Bass.

A. RANICKI

Evans, D. E. and Takesaki, M. *Operator algebras and applications*, Volume 1: Structure theory; K-theory, geometry and topology; Volume 2: Mathematical physics and subfactors (London Mathematical Society Lecture Note Series 135, 136, Cambridge University Press, Cambridge 1988) Vol. 1, viii+244 pp, paper: 0 521 36843 X, £17.50; Vol. 2, viii+240 pp, paper: 0 521 36844 8, £17.50.

There is a tradition from the sixties that there are occasional, but irregular, conferences on operator algebras and some aspects of their applications. A number of these conferences have produced excellent proceedings afterwards: the most important was the Kingston Conference in 1980, and more recently there was the US–Japan Seminar, Kyoto, July 1983. This tradition continues. The two volumes under review contain research and expository articles from the participants in a UK–USA Joint Seminar on Operator Algebras held in July 1987 in Warwick and a few papers from those that had given talks at the symposium earlier in the year at the University of Warwick. The papers in these volumes provide good coverage of the material in the titles in a random way within each broad subject. There are expository articles alongside research calculations on technical problems; the standard of the papers is good. Unfortunately the volumes are a typographical muddle as they were produced from camera-ready copy supplied by the authors (the title on the cover of Volume 1 has an error in it); the advantage of the production is the informal style of some of the papers.

This review is not the place to provide a detailed assessment of the individual articles in these two volumes, a service which is fulfilled by the specialist reviewing journals. However the scope of these notes may be illustrated by mentioning a few papers and a brief view of what is in them. P. Baum and A. Connes in 'K-theory for discrete groups' discuss and motivate their conjecture that a certain natural homomorphism, defined by them, from a suitable geometrical K-theory group associated with a discrete group G into the corresponding C*-algebra K-theory group of