KIRKWOOD GAPS AND RESONANT GROUPS

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Abstract. This paper is a short review of the dynamics of the asteroidal resonances as currently determined from maps and simulations over $10^6 - 10^7$ years. The main recent results concern the extensive exploration of the phase space to determine domains of initial conditions leading to close approaches to the inner planets, the topological dynamics of the planar Sun-Jupiter-asteroid problem at very high eccentricities and the differences amongst 2/1 and 3/2 resonances able to explain the existence of a gap in the asteroidal belt at the 2/1 resonance and of a group of asteroids in the 3/2 resonance. Current results point to a confirmation of Wisdom's theory for the formation of the gaps by gravitational evolution and scattering by the inner planets.

1. Introduction

Several features in the distribution of the asteroids are associated with resonances. They are groups and gaps appearing in plots where the horizontal axis displays the semi-major axis or the mean motion. The gaps are located at positions corresponding to the 4/1, 3/1, 5/2, 7/3 and 2/1 commensurability between the orbital period of Jupiter and that of the asteroid, while the groups are located in the neighbourhood of the 3/2, 4/3 and 1/1 (fig.1).

The gaps were discovered by Kirkwood in 1867 and are known as Kirkwood gaps. The groups in the neighbourhood of the 3/2 and 4/3 resonances have 57 and 2 members currently known (EMP 1994), respectively. We propose to call them Palisa groups after the name of the astronomer who discovered their two paradigms: (153)Hilda and (279)Thule. The asteroids moving in the neighbourhood of these commensurabilities have similar dynamics and are considered in this review. The dynamics of asteroids in the 1/1 resonance, the Trojans, is very particular and is treated in a separated review (Milani, this volume). The techniques used to study mean-motion resonances have been recently reviewed by Henrard (1988) and Froeschlé and Greenberg (1989). Therefore, the present review is limited to the results directly concerning the occurrence of gaps and groups.

Throughout the 20th century, several hypotheses have been formulated to explain the Kirkwood gaps. One of them, the statistical hypothesis, claims that the gaps are only apparent: asteroids near resonances oscillate about the exact resonance value and expend most of the time far from the commensurabilities. This motion, known as libration or σ-libration, is characterized by the oscillation of the angle

$$\sigma = (r+1)\lambda_{Jup} - r\lambda - \varpi$$

(λ and ϖ denote, respectively, the mean longitude and the longitude of the perihelion and r is a rational number; $(r+1)/r$ is the commensurability ratio) around a fixed value. It was recognized since the early work of Poincaré, and does indeed

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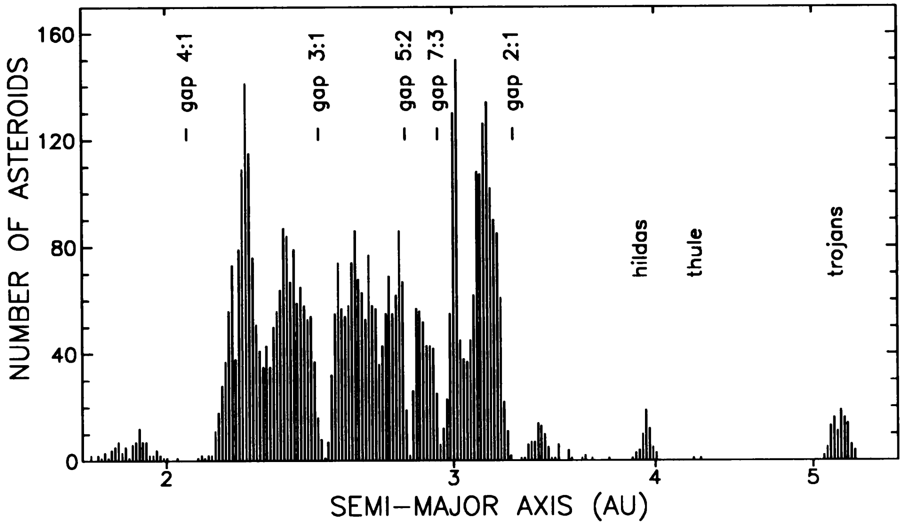

Fig. 1. Histogram of the number of asteroids *vs* semi-major axis showing the location of the main gaps and groups.

exist: it has, for some asteroids, a very large amplitude (up to 0.15 AU, see Ferraz-Mello, 1988). However, it is not large enough to provoke a gap. A complete analysis done by Schweizer (1969) shows that the distribution of the osculating elements in the neighbourhood of the gaps does not significantly differ from the distribution of the mean elements. This is at variance with what should be expected, if the origin of the gaps were of statistical nature. The final argument comes from current numerical simulations over 10^7 years. No regular oscillation has been found with an amplitude large enough to originate a statistical gap as wide as those observed.

The other hypotheses assume that the gaps are real and are either primordial or the result of the orbital evolution of the asteroids. They are usually classed in three groups: cosmogonic, collisional and gravitational. The gravitational theory says that pure gravitational evolution is sufficient to explain the gaps. This theory is difficult to be directly confirmed because of the large time interval elapsed since the origin of the asteroidal belt. However, it stands high in favor since Wisdom's work on the formation of the gap in the 3/1 resonance. Wisdom shows that this gap may be explained by a chaotic diffusion of resonant orbits due to separatrix crossing. This diffusion can drive an asteroid to orbits approaching Mars closely in a very short time-scale ($10^5 - 10^6$ years).

The collisional hypothesis assumes that asteroid collisions are more frequent at resonances. A direct proof or disproof of this hypothesis is difficult. A satisfactory theoretical approach does not exist and numerical simulations are still beyond the existing computational capabilities. Collisions may have played a role in the initial stages of the solar system, but are certainly negligible now, given the small asteroidal masses and space densities. However, if we consider not only collisions amongst asteroids but also with inner planets as well, the collisional hypothesis is part of Wisdom's theory.

At last, there are the cosmogonic hypotheses. They are generally made plausible by the assembling of assumptions in each theory. They generally consist in finding some scenario able to produce gaps in the asteroidal distribution. To the extent that Wisdom's theory of gravitational evolution and scattering of asteroids in resonances is accepted as a general explanation, cosmogonic hypotheses become less important. Indeed, if the efficiency of gravitational evolution mechanisms were as large as pointed out by some recent investigations, the present state of the asteroidal belt at resonances would weakly depend on its primordial state and could provide only a little cosmogonic information. They are not considered in this review.

The crucial point concerning Kirkwood gaps and Palisa groups is that, in spite of their opposite distribution characteristics, they have very similar dynamics. For instance, the theoretical models of the 3/2 and 2/1 resonances are identical and differ only in the numerical value of one parameter (see, for instance, Henrard and Lemaître, 1987; Lemaître and Henrard, 1988). Therefore, in order to be fully accepted, a theory must explain both, the existence of gaps at the 4/1, 3/1, 5/2, 7/3 and 2/1 resonances and the existence of groups at the 3/2 and 4/3 resonances. It must also explain the broadness of the 2/1 and 4/1 gaps, the non-existence of apocentric librators at the 3/2 resonance (while they are abundant at the 2/1 resonance), the existence of some long-lasting librators at the 2/1 gap (while they are absent in other gaps) and the bounded eccentricity range of the 3/2 librators.

2. The 3/1 gap

The first gap to receive an acceptable explanation was the 3/1 gap. Its depletion mechanism was completely unraveled by Wisdom (1982, 1983, 1985). He shows the existence of intermittencies associated with chaos. They appear in this problem as sudden increases – jumps – of the eccentricity (fig.2). Wisdom's simulations show that an asteroid may have a seemingly regular motion for long times and suddenly have a large eccentricity increase due to a separatrix crossing. During these jumps the eccentricity reaches values larger than 0.35 and the asteroid orbit crosses the orbit of Mars. Therefore, the asteroid can have a close approach to the planet. This behaviour is intermittent and the asteroid may escape from close approach many times. However, the close approach eventually occurs and the asteroid exchanges a great amount of energy with Mars, leaving the resonance. The time scale associated with these intermittencies is short ($\sim 10^5$ years) as compared to the age of the solar system. Therefore, the theory is consistent with the absence of permanent asteroids in the 3/1 resonance. The few asteroids presently observed in this resonance are believed to have come from recent captures and are bound to close approaches with the inner planets (Milani *et al.*, 1989).

The work of Wisdom on the 3/1 resonance was extended by Ferraz-Mello and Klafke (1991) and Hadjidemetriou (1992) to very high eccentricities. Using averaged potentials given by semi-numerical expansions, Ferraz-Mello and Klafke mapped the structure of the phase space of the restricted elliptic model in the neighbourhood of this resonance. They show that, besides the region of homoclinic chaos discovered

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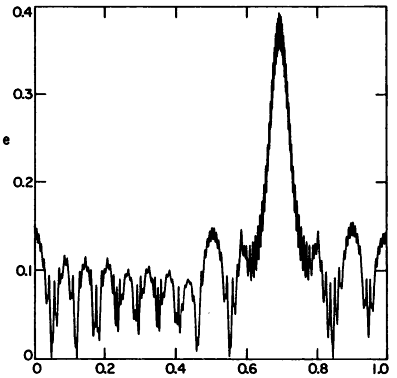

Fig. 2. Intermittencies (jumps) of the eccentricity of a fictitious asteroid in the 3/1 resonance. Total integration time: 200,000 years (Wisdom, 1983).

by Wisdom, there is another at high eccentricities. They also show that, for some values of the energy, a heteroclinic bridge allows the solutions to go from one region to another (fig.3). The intermittencies associated with orbits going through this heteroclinic bridge can drive the asteroidal eccentricity to values as high as 0.9 and then back to a lower eccentricity region. Orbits of fictitious asteroids going through the heteroclinic bridge to high eccentricities were actually computed by Saha (1992). In fact, they are very common: the orbit of (4179) Toutatis lies in this region.

The 3/1 resonance has become a paradigm in the study of asteroidal gaps and has been studied by many authors. In particular we quote Henrard and Caranicolas (1990) who studied the role played by secondary resonances in the complex separatrix-crossing mechanism causing the chaotic behaviour at low eccentricities.

3. The 5/2 and 7/3 gaps

The second-order techniques devised by Wisdom to study the 3/1 resonance use an abridged Hamiltonian that contains only the leading terms of the classical Laplacian expansion of the disturbing potential of Jupiter. The convergence radius of this expansion in the outermost 5/2, 7/3, 2/1 and 3/2 resonances is $e \sim 0.28$, $e \sim 0.26$, $e \sim 0.20$ and $e \sim 0.09$, respectively (it is $e \sim 0.35$ in the 3/1 resonance; see Ferraz-Mello, 1994). When these values are compared to the eccentricities which may lead to collisions with Mars – 0.41, 0.44, 0.49 and 0.58, respectively – we see that theories based on the classical Laplacian expansion can no longer give a verdict on the occurrence, or not, of gravitational mechanisms with amplitude large enough to allow the depletion of a neighbourhood of these resonances.

Wisdom's mapping and perturbative models allowed several authors to show that the resonances 3/1 and 5/2 are quite similar. Šidlichovský and Melendo (1986) show that a low-eccentricity chaotic region exists in the 5/2 resonance and must account for the existence of the gap. Yokoyama and Balthazar (1992) studied the dynamics

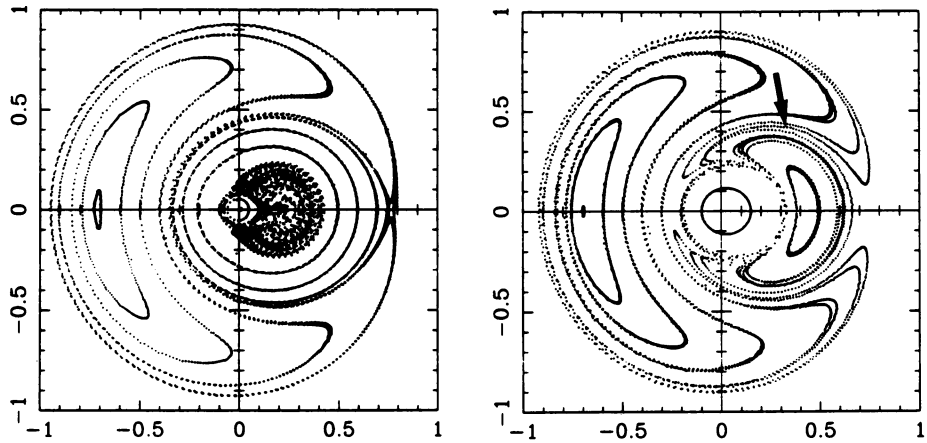

Fig. 3. Poincaré maps ($\sigma = \pi/2$, $\dot{\sigma} < 0$) of the resonance 3/1 in the frame of the planar averaged Sun-Jupiter-asteroid problem at two different energy levels. Coordinates are $x = e.\cos(\varpi - \varpi_{Jup})$, $y = e.\sin(\varpi - \varpi_{Jup})$. At left, the chaotic domain found by Wisdom is seen in the inner part of the figure. At right, the tori confining this region no more exists and a heteroclinic bridge (arrow) is seen (Ferraz-Mello and Klafke, 1991).

of this resonance. They show the separatrix crossing mechanism leading to chaotic behaviour, which is able to transfer one orbit from the low-eccentricity regime (up to 0.1 – 0.2) to an outermost one where it may reach 0.4. The comparison of the results with numerical integrations (Šidlichovský, 1993) shows that Wisdom's techniques, at their limit for this resonance, give a qualitatively correct picture of the solution, in spite of some important quantitative differences (fig 4a). We should also mention some early results of Scholl and Froeschlé (1975) who showed increases of eccentricity up to 0.3 in relatively short time-scales ($\sim$ 20,000 years), in agreement with the results reported above.

The chaotic origin of the gap was later confirmed by Ipatov (1990, 1992) by means of an extensive simulation. The simulation shows that the domain of initial conditions corresponding to chaotic orbits coincides with the region depleted in the distribution of the asteroids in the plane eccentricity *vs.* semi-major axis (fig. 4b). Furthermore, using averaged potentials given by semi-numerical expansions, Klafke *et al.* (1992) mapped the structure of the phase space of the restricted elliptic model in the neighbourhood of this resonance. They show that, similar to the 3/1-resonance, there is a chaotic region at high-eccentricities, which is linked to the low-eccentricity region by a heteroclinic bridge for a wide range of energies.

In the case of the 7/3 resonance, Yokoyama and Balthazar (1992) have shown that, in the Sun-Jupiter-asteroid model, there are gravitationally confined low-eccentricity orbits. However, Yoshikawa (1991) studied the distribution of the observed asteroids in the neighbourhood of this commensurability and found that the boundary of the depleted region in the plane eccentricity *vs.* semi-major axis almost

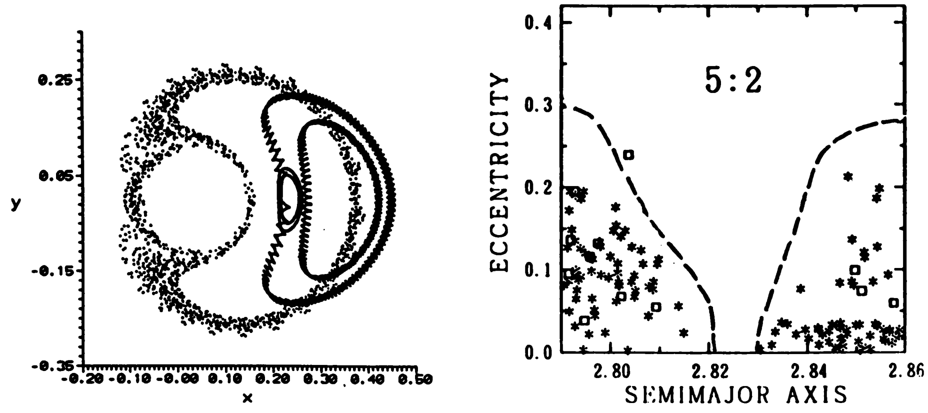

Fig. 4. (a) Comparison of surfaces of section obtained with maps in the 5/2 resonance and real trajectories for high eccentricities. Coordinates as in Fig. 3 (Šidlichovský, 1993). (b) Distribution of the numbered asteroids around the 5/2 resonance when $\sigma = 0$ and $\varpi - \varpi_{Jup} = \pi$ (after Yoshikawa, 1991); the curves show the boundaries of the region corresponding to Mars crossing asteroids with $i = 10°$ (after Ipatov, 1991).

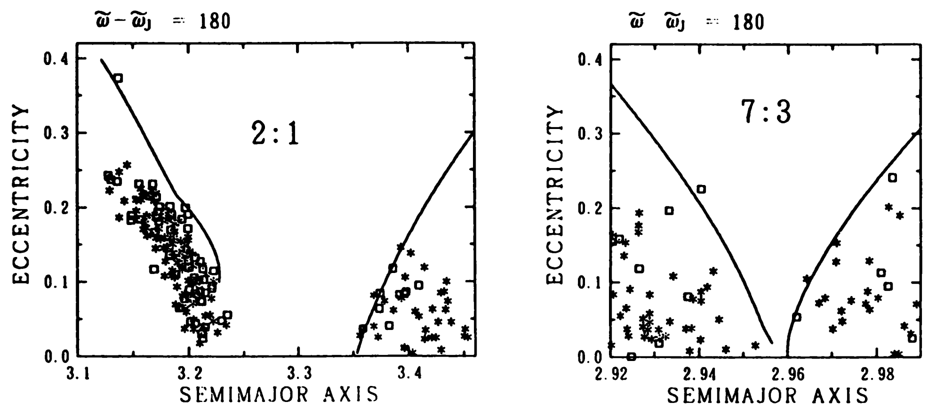

Fig. 5. Distribution of the numbered asteroids around the 7/3 and 2/1 resonances when $\sigma = 0$ and $\varpi - \varpi_{Jup} = \pi$. Two curves drawn in each diagram indicate the boundaries of the resonance region in the planar case. In the diagram of the 2/1 resonance, the asteroids in the lower left region were omitted (asterisk: $i < 15°$, square: $i > 15°$) (Yoshikawa, 1991).

coincides with the boundary between librating and circulating solutions (fig.5). The situation is quite similar to that described below for the 2/1 gap. Even if it does not privilege any particular theory, we may conjecture, by analogy with the 2/1 resonance, that the inclusion of the outer planets in the model may introduce the possibility of intermittencies allowing the low-eccentricity asteroids to migrate to regions of large orbital eccentricity.

4. The 2/1 gap

The existence of low-eccentricity chaotic solutions in the neighbourhood of the 2/1 resonance was first found by Giffen (1973). It was confirmed by Froeschlé and Scholl (1976, 1981), who also showed that those solutions are confined to low-eccentricities by regular motions starting near $e = 0.2$. They were also found by Murray (1986) and Wisdom (1987). Lemaître and Henrard (1990) identified the origin of the chaotic behaviour both in the transition between large-amplitude σ-librations and circulations, at the borders of the resonance, and in the secondary resonances between the libration frequency and the perihelion motion. The topological dynamics of this resonance was recently studied by Moons and Morbidelli (1993) with Poincaré maps up to $e = 0.3$. Their results show the low-eccentricity chaotic region and the quasi-integrable tori confining this region. These results constitute one important part of the current knowledge of the dynamics of the 2/1 resonance, but they are limited to eccentricities smaller than the necessary 0.49 to allow the asteroid orbit to intersect the orbit of Mars.

The difficulties concerning the non-existence of a globally valid expansion of the disturbing function were overcome in different ways.

Morbidelli and Moons (1993) use an approach based on local non-central evaluations of the disturbing function (see Ferraz-Mello and Sato, 1989). They include the two main harmonics of Jupiter's secular resonances, usually denoted as ν_5 and ν_6, and consider inclined orbits. The resulting system has a high number of degrees of freedom and they successively eliminate them by semi-numerical averaging. The final step is a juxtaposition of averages obtained with the separate elimination of the angles associate with ν_5 and ν_6. The solutions are shown on surfaces of section which, as pointed out by the authors, are not Poincaré sections. The results are consistent with the existence of a family of asteroids similar in distribution to that of the Hildas in the resonance 3/2. They do not explain why these asteroids are absent and rather support the need of a cosmogonic hypotheses to explain the 2/1 gap.

We present in figure 6 some Poincaré maps of the planar Sun-Jupiter-asteroid problem obtained with numerical integrations done over 1 Myr and smoothed by filtering out the high frequencies (see Michtchenko and Ferraz-Mello, 1993). These smoothed integrations were interpreted as solutions of an averaged dynamical system with 2 degrees of freedom. The sections show a large region of seemingly ordered motions, even for large libration amplitudes ($\Delta a \sim 0.15$ AU) and eccentricities as large as 0.5.

The chaotic regions identified by Lemaître and Henrard (1990) are seen in (B) and (D). In (B), chaotic motions due to the overlapping of secondary resonances

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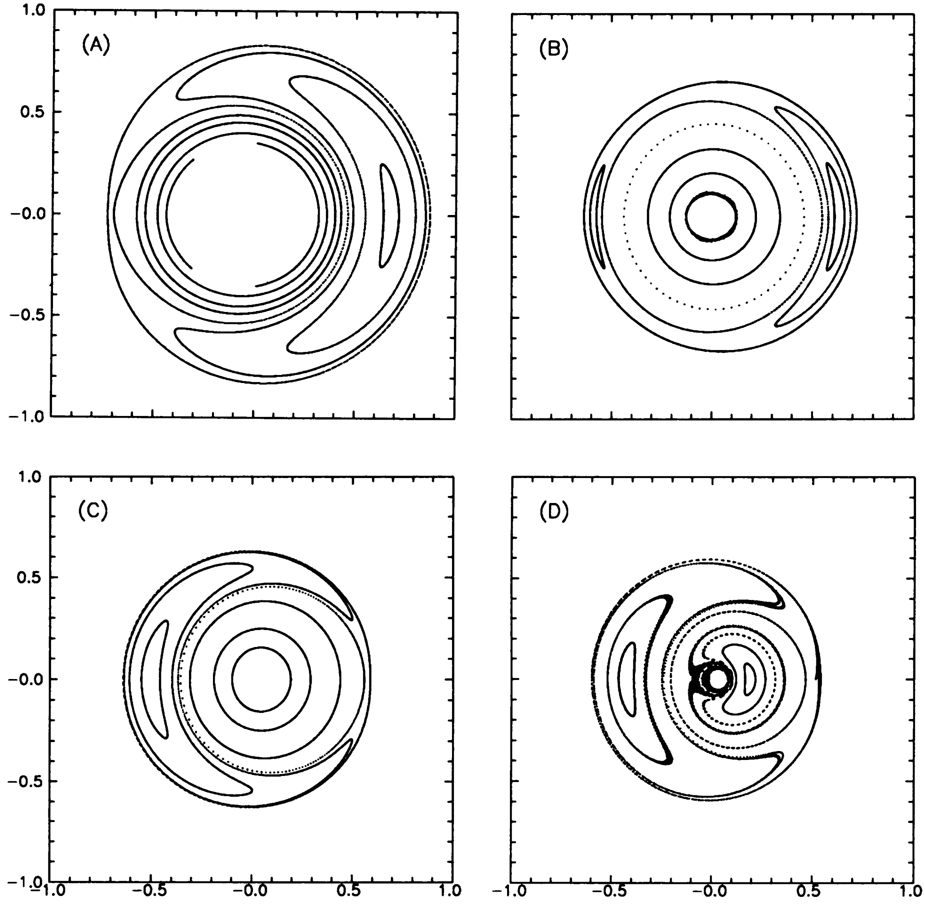

Fig. 6. Poincaré maps ($\sigma = 0$, $\dot{\sigma} > 0$) of the resonance 2/1 in the frame of the planar averaged Sun-Jupiter-asteroid problem. Coordinates as in Fig. 3.

are seen in the middle of the section; in (D), they mark the transition between σ-librations and circulations. These sections extend to high eccentricities the well-known facts that made many authors to find impossible to explain the Kirkwood 2/1 gap by the same mechanisms explaining the 3/1 gap. For instance, in the section (B) one may see that the low-eccentricity chaos is confined by seemingly regular tori, as already mentioned, and is not able to produce intermittencies reaching high-eccentricities. No new chaotic region is visible inside the resonant region.

The results on the Sun-Jupiter-asteroid model do not explain the 2/1 gap. However, when the perturbations due to Saturn are included and the initial conditions assume an inclined orbit ($i = 5°$), all solutions become clearly chaotic. The corresponding maximum Lyapunov exponents were estimated from some fifty 5–7 Myr integrations for initial eccentricities in the interval $0.1 < e < 0.4$ and semi-major axes ranging from the middle to the border of the resonance region. The results are generally in the range $10^{-5} - 10^{-3.5}\mathrm{yr}^{-1}$, except for a few orbits starting from the very middle of the resonance. These results show that Saturn triggers the destruction of the regular structures seen in fig. 6. Even if KAM tori may exist, given the high number of degrees of freedom, they no more divide energy manifolds. Some barriers may still exist (see Wiggins, 1990) but, in general, they are not expected to avoid the occurrence of intermittencies allowing orbits to reach high eccentricities and encounter the inner planets.

The actual occurrence of such diffusion in this system, when all the outer planets are considered, is confirmed by numerical simulations. Wisdom (1987) reports the integration of 5 test particles with initial eccentricities 0.05 or 0.1, for 5 Myr. Three of them reached $e \sim 0.4 - 0.5$. Current investigations of Scholl *(personal communication)* show that the range of initial conditions leading to high eccentricities in 12 Myr or less is broad and excludes only orbits close to the libration centers (Fig. 7)

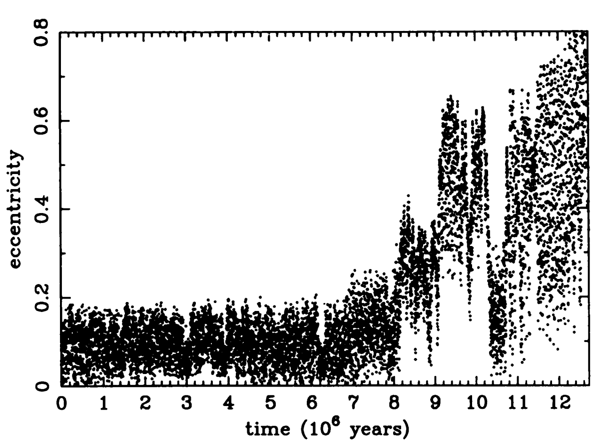

Fig. 7. Evolution of the eccentricity of a fictitious asteroid in the resonance 2/1. (Scholl, *personal communication*)

These results are completed by other evidences of the association of the depleted region with the region where large Lyapunov exponents were found. For instance, Yoshikawa (1991) studied the distribution of the observed asteroids in the neighbourhood of this resonance and found that the boundary of the depleted region in the plane eccentricity *vs.* semi-major axis almost coincides with the boundary of the libration zone (see fig.5).

The above results may be interpreted as responsible for orbital changes able to deplete the resonance at a time rate consistent with the fact that some asteroids are librating inside the 2/1 resonance. Many of them may be considered as temporary but at least one, (3789) Zhonguo, has a smaller eccentricity and seems to be able to remain inside the resonance for a long time. In fact, the long term evolution of this asteroid is not yet known.

However, one should keep in mind that we are dealing with very slow processes, at the limit of the available numerical and analytical techniques. A great deal of investigation is still necessary to confirm the results and to identify all dynamical mechanisms at work in this resonance.

5. The 3/2 and 4/3 groups

The results found in the 2/1 resonance cannot repeat themselves in the 3/2 resonance. Otherwise this resonance should be as depleted as the 2/1 resonance, at variance with the observation of some 60 asteroids moving near the exact commensurability. On the basis of the seemingly regular short-term evolution of some computed orbits in the Sun-Jupiter-asteroid model, Giffen (1973) conjectured that this regularity should be responsible for the existence of this group. A longer integration performed by Wisdom (1987) also shows regular motions in the region where the Hildas are found.

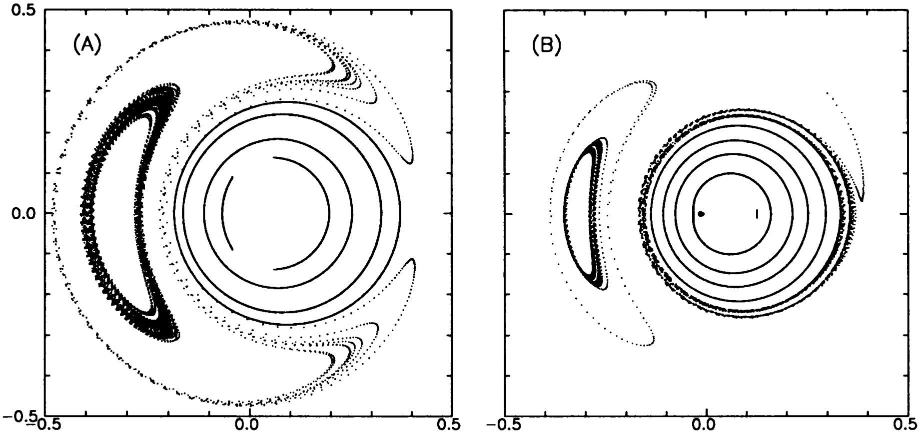

Fig. 8. Poincaré maps ($\sigma = 0,\ \dot{\sigma} > 0$) of the resonance 3/2 in the frame of the planar averaged Sun-Jupiter-asteroid problem. Coordinates as in Fig. 3. Orbits in the perihelion libration lobe are highly chaotic and are bound to close approaches to Jupiter in short times. Orbits in the innermost part remains regular for long terms even when inclined orbits and the perturbations of Saturn are taken into account. The actual Hildas are in the inner region.

Figure 8 shows Poincaré maps of the planar Sun-Jupiter-asteroid problem obtained numerically. They are different from those obtained for the 2/1 resonance in several aspects: (a) the region of perihelion circulation is devoid of chaotic activity

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(confirmed by Lyapunov exponents tending to zero in numerical integrations over 10 Myr); (b) there is a persistent saddle point at the right side and one lobe of libration of the perihelion at the left side (in the 2/1 resonance there is a reversion of this structure and, then, transition portraits showing two lobes of libration, one at the right and the other at the left side; see section (B) in fig. 6); (c) the bifurcation between the two modes of oscillation of the perihelion is the source of appreciable chaoticity spreading itself over a large part of the perihelion libration lobe and also over the outer circulations.

Figure 8 also shows that other planets are not needed to explain the depletion of the orbits with mean eccentricities larger than ~ 0.3. All sections show a fast diffusion in the libration lobe whose outer orbits are scattered by approaches to Jupiter itself. The inclusion of Saturn only accelerates this phenomenon allowing the orbits to be scattered in less than 1 Myr. The results of Morbidelli and Moons (1993) for this resonance also show an extended chaotic region surrounding seemingly regular motions with $e < 0.25$.

When the perturbations due to Saturn are taken into account and the asteroid is left to move in an inclined orbit, the maximum Lyapunov exponents of the inner regular orbits still tend to very small values. They are in the range $10^{-5.5} - 10^{-7}\mathrm{yr}^{-1}$. Franklin *et al.* (1993) extended to this system some results on the time necessary for the occurrence of sudden orbital transitions, in the N-body asteroidal problem, when the diffusion process in the asteroid eccentricity is assumed as unbounded. They concluded that, in this case, this time is generally much larger than the age of the Solar System and that there is no contradiction in the fact that most of the real Hildas are formally chaotic.

The reported results show many similarities in the dynamics of the 2/1 and 3/2 resonances. In many respects, the 2/1-gap orbits appear as more regular than those of the 3/2 resonance. The only instance in which this fact is reversed is when Lyapunov exponents are computed in the Sun-Jupiter-Saturn-asteroid problem. The fact that the maximum Lyapunov exponents of the orbits in the 2/1 gap are at least 2 orders of magnitude larger than those in the 3/2 group is, likely, one reason for the observed distribution differences.

One open question concerns the existence of chaotic low-eccentricity orbits close to the exact commensurability. For low eccentricities, it is possible to calculate the locus of secondary resonances similar to those responsible for the low-eccentricity chaos of the 2/1 resonance (Lemaître and Henrard, 1988; Michtchenko, 1993). Schubart (1990) computed 3 orbits in this region and found no special features other than secondary resonances. Michtchenko (1993) computed a large number of orbits in this region and analyzed them by means of Fourier Transforms. Her results point to the existence of thin layers of chaotic orbits with very small eccentricities. Some of these layers are clearly associated with secondary resonances.

The Palisa group at the 4/3 resonance is recognized as a group, in spite of its only 2 members. They are the only known asteroids with a mean period between 8 years (period of the Hildas) and 11.8 years (period of the Trojans and Jupiter). The structure of the 4/3 resonance was recently studied by Ries (1993) in the frame of the planar restricted three-body problem. Her results point toward the survival of low-eccentricity objects ($e < 0.075$).

6. The 4/1 gap

In this review, we have not considered the particular problems related to the 4/1 gap. The dynamics in the neighbourhood of this commensurability is complex even when Jupiter is kept moving in a fixed ellipse (see Klafke *et al.*, 1992). Moreover, this resonance is situated at 2.06 AU, just where the secular resonances ν_6 and ν_{16}, as derived from general secular theories, occurs for low-eccentricity asteroids (fig. 9). Scholl and Froeschlé (1991) have shown that the overlapping of these resonances is responsible for a complex dynamic behaviour in the whole 2.0 – 2.1 AU region. In this region, the necessary eccentricity for close approach to Mars is only 0.2. Thus, even without a complete dynamical study, one can say that this is certainly a reason for the huge observed depletion.

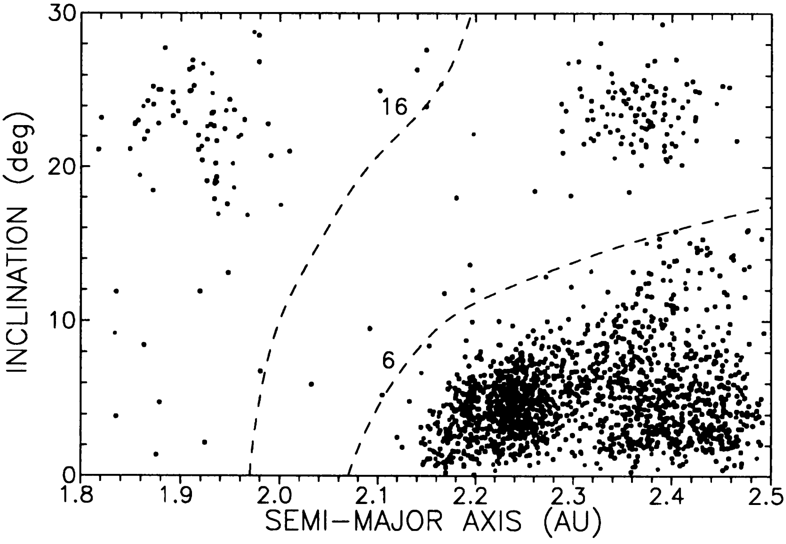

Fig. 9. Distribution of the asteroids in the neighbourhood of the 4/1 resonance showing the wide gap at 2.0 – 2.1 AU. The locus of the secular resonances ν_6 and ν_{16}, as derived from general secular theories, is also shown.

7. Conclusion

We reviewed the main period resonances. Whenever possible, we emphasized the results concerning the topological dynamics of these resonances rather than those that are only generic simulations. The large quantity of simulations conducted in the past 20 years has already shown that asteroids can be found in these places only if some protective mechanisms related to σ-libration or secular resonance are acting.

The collected results refer to current maps and simulations extending over $10^6 - 10^7$ years. They show that several conclusions, obtained in the past on the

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basis of simulations over $10^4 - 10^5$ years, are not correct. This fact may serve to prevent us from assuming that current conclusions are final. They are certainly an improvement on the previous scenario but we do not know what will be unraveled when our theories become able to show evolutions over a time span as long as $10^8 - 10^9$ years.

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