PROJECTING DEVELOPMENT OF LOSSES DURING AN ACCIDENT YEAR

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Abstract

A method is presented for projecting the development of reported losses, or of paid losses, or the number of reported cases, for each of the first 12 months of an accident year. The method is based on the relationship between development patterns after the first 12 months of an accident year and development patterns during the first 12 months. A way of deriving the development pattern of an individual accident quarter or accident month is also described.

It is pointed out that the method can be useful for underwriting decisions for which the most recent experience is relevant, or for estimating loss reserves at the end of a quarter or when earned premium differs greatly by month or quarter.

Keywords

Quarterly loss development; accident quarter.

Sometimes data can be collected to indicate what average percentage of an accident year's ultimate losses is reported or paid, or what average percentage of claims is reported, by the end of each quarter or month of the accident year, but this information is not always available.

The percentages of ultimate losses expected at the end of each year after the start of the accident year are more likely to be available both for individual company and industry-wide experience. These yearly percentages generally give a better picture of what the expected percentages of ultimate losses are at the end of each month after the end of the accident year than of what they are during the accident year. This is because these expected percentages generally increase at a decreasing rate after the end of the accident year, but at an increasing rate during the accident year, for the following reason. During the year the increase each month equals the expected losses for the latest accident month as of one month, plus the development from an increasing number of previous months.

A method will be presented to derive development patterns for incomplete accident years from later development patterns, as well as to derive individual development factors for an accident quarter or month. Deriving development patterns for accident year quarters or months can be useful if earned premiums, loss ratios, or development patterns are expected to differ greatly from quarter

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to quarter or month to month, owing for example to a change in the mix of business. The theorem on which the method of this discussion is based will be presented, and then some points about its application will be discussed.

The word losses in the following can be interpreted to mean amount of reported losses, amount of paid losses, or number of reported losses.

THEOREM. *If, for an accident year, the expected ultimate losses and the expected loss development pattern are the same for each accident quarter, then if the accident year's losses develop to ultimate in p years, the fraction M_n of the accident year's ultimate losses which is expected to have developed after n quarters satisfies the following formula for $n \leqslant 4$:*

$$M_n = \frac{n}{4} - \sum_{k=1}^{p-1} (M_{4k+n} - M_{4k}) \tag{1}$$

PROOF. We will refer to the following diagram:

Accident quarter	Development Quarter									
	1	2	3	4	...	...	$4p-3$	$4p-2$	$4p-1$	$4p$
1	$e_{1,1}$	$e_{1,2}$	·	·	·	·	$e_{1,p-3}$	0	0	0
2	0	$e_{2,2}$	$e_{2,3}$	·	·	·	·	$e_{2,p-2}$	0	0
3	0	0	$e_{3,3}$	$e_{3,4}$	·	·	·	·	$e_{3,p-1}$	0
4	0	0	0	$e_{4,4}$	$e_{4,5}$	·	·	·	·	$e_{4,p}$

Let $e_{i,j}$, for positive integers $i \leqslant 4$, $j \leqslant 4p$, be the expected fraction of the accident year's ultimate losses which is expected to develop for the ith accident quarter during the jth quarter after the start of the year. For any $1 \leqslant j \leqslant 4p-3$, let S_j be the set of all elements $e_{1+k,j+k}$, for $0 \leqslant k \leqslant 3$. This set is on a diagonal line across the $4 \times 4p$ matrix of elements $e_{i,j}$. All elements of S_j are equal since each element depends on the expected development of losses of an identical accident quarter in the jth quarter after its start. We are only concerned with $1 \leqslant j \leqslant 4p-3$, because later diagonals extend beyond p years. For integers k such that $1 \leqslant k \leqslant 4p$, let C_k be the set of elements $e_{i,k}$. This is a column of the matrix of elements $e_{i,j}$. For $1 \leqslant n \leqslant 4$, the sum of the elements which are in the union of all sets C_{4k+j} for $0 \leqslant k \leqslant p-1$, $1 \leqslant j \leqslant n$ equals

$$M_n + \sum_{k=1}^{p-1} (M_{4k+n} - M_{4k}).$$

Since each S_j intersects four consecutive columns, exactly n of its four equal elements are in the above union of sets. But the sum of the elements in all diagonals S_j is 1, so

$$M_n + \sum_{k=1}^{p-1} (M_{4k+n} - M_{4k}) = \frac{n}{4}. \tag{2}$$

The theorem follows immediately. Q.E.D.

Let M'_n be the fraction of an accident quarter's ultimate losses which are expected to have developed n quarters after it begins. Since $M_n - M_{n-1}$ equals the sum, as a percentage of ultimate accident year losses, of the nth, $(n-1)$th, ..., 1st quarters of development, respectively, of the 1st through nth equal accident quarters, therefore

$$M'_n = 4(M_n - M_{n-1}) \qquad \text{for } n \leqslant 4. \tag{3}$$

For $n > 4$, the losses for an accident quarter, as of n quarters, equal the losses for an accident year as of n quarters minus the losses for the second, third, and fourth quarters as of $n-1$, $n-2$ and $n-3$ quarters, respectively. So we have, for $n > 4$,

$$M'_n = 4M_n - M'_{n-1} - M'_{n-2} - M'_{n-3}. \tag{4}$$

In this way, M'_n is defined inductively.

If Q_n represents the fraction of the accident year's ultimate losses which is expected to have developed after n months, and Q'_n represents the fraction of an accident month which is expected to have developed n months after it begins, the formulae analogous to (1), (3) and (4) are

$$Q_n = \frac{n}{12} - \sum_{k=1}^{p-1} (Q_{12k+n} - Q_{12K}), \qquad \text{for } n \leqslant 12 \tag{5}$$

$$Q'_n = 12(Q_n - Q_{n-1}), \qquad \text{for } n \leqslant 12 \tag{6}$$

$$Q'_n = 12Q_n - Q'_{n-1} - Q'_{n-2} - \cdots - Q'_{n-11}, \qquad \text{for } n > 12 \tag{7}$$

These could be proved in a manner analogous to the proofs of (1), (3) and (4).

In practice, if the M_n for $n \geqslant 4$ are chosen in such a way that they would not actually result from any smoothly developing sequence of quarterly factors $M'_1, M'_2, \ldots$, then the M'_n derived from the M_n will not be reasonable. Using (1), it is then necessary to estimate adjustments for three numbers, i.e. $M_{4k+n} - M_{4k}$, for $n = 1, 2$, and 3, in order to get more reasonable estimates for M_1, M_2, and M_3, and thereby for M'_1, M'_2, $M'_3, \ldots$

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