MISSING MATTER IN THE VICINITY OF THE SUN

John N. Bahcall

The Institute for Advanced Study, Princeton, New Jersey 08540

ABSTRACT

The combined Poisson-Boltzman equation for the gravitational potential is solved numerically for a detailed Galaxy model. The main result - obtained by comparing the calculated densities with observations of F dwarfs and K giants - is that about half of the mass density in the vicinity of the Sun has not yet been observed.

I have solved the combined Poisson-Boltzman equation for the gravitational potential of Galaxy models consisting of realistically large numbers of individual isothermal disk components in the presence of a massive unseen halo. The calculations were carried out with different assumptions about the unseen matter and the predicted number densities of F dwarfs versus height above the plane were compared with the observed distribution given by Hill, Hilditch, and Barnes (1979). I have also calculated the expected distribution of K giants and have compared these results with the observations described by Oort (1960).

The basic result I obtain is that the amount of *unobserved* material in the disk is at least as large as 50% of the observed material for all of the models that are discussed. In the best-estimate model, the amount of unobserved material is approximately equal to the amount of observed matter in gas, dust, and stars. This unseen disk material may be different from the missing mass inferred to be in extended galactic halos. Also, the disk material must be dissipational.

The special features of the work described here are: (1). the solutions are self-consistent (the star densities are determined by the common potential they create); (2). the Galaxy models contain realistically large numbers of disk components (from 10 to 23); (3). a massive halo is included; and (4). quantitative estimates of the uncertainties are determined. The details are given in Bahcall (1984) [Paper I]; only the results are summarized here. Earlier work is reviewed by Oort (1965).

Table 1 describes a Standard Galaxy model for the observed mass components. The disk luminosity function and the z-velocity dispersions are taken from Wielen (1974). The decomposition of the faint disk stars is based on the grouping according to H and K intensities described by Wielen (1974). The other components are taken from the standard Bahcall and Soneira (1980, 1984) model (hereafter,

M. Capaccioli (ed.), Astronomy with Schmidt-Type Telescopes, 241–246.

referred to as the B&S Galaxy model). The mass fractions are defined in terms of the total *observed* mass density (in stars, gas, and dust), i. e.,

$$A_i = \frac{\rho_i(0)}{\rho_{obs}(0)}. \tag{1}$$

Table 1. The Galaxy Model for Observed Components

Component	B & S Mass Fraction (A_i)	$\langle v_z^2 \rangle^{1/2}$ ($km\ s^{-1}$)
Main Sequence Stars		
$M_V < 2.5^m$	0.021	4
$2.5^m \leq M_V \leq 3.2^m$	0.015	8
$3.2^m \leq M_V \leq 4.2^m$	0.031	11
$4.2^m \leq M_V \leq 5.1^m$	0.035	21
$5.1^m \leq M_V \leq 5.7^m$	0.025	20
$5.7^m \leq M_V \leq 6.8^m$	0.037	17
$M_V \geq 6.8^m$	0.0358	8
	0.0626	13
	0.0536	15
	0.0626	20
	0.0834	24
Giants	0.016	~20
White dwarfs	0.052	21
Atomic H and He and Molecular H and dust	0.469	4
Spheroid	0.001	~100 km/sec
Total	0.0958 $M_\odot pc^{-3}$	

The basic equation used is the combined Poisson-Boltzman equation for the potential. This equation describes how the gravitational potential at a given height above the plane can be calculated from the mass densities and velocity dispersions that are specified in the plane of the disk for any number of isothermal disk components - some observed as stars, dust, or gas and some unobserved - plus a halo mass density (constant, to first approximation, with height above the plane). The *dimensionless* form of the combined Poisson-Boltzmann equation is:

$$\frac{d^2\varphi}{dx^2} = 2\left[\sum_{i=1}^{N_{obs}} A_i\, e^{-\alpha_i\varphi} + \sum_{j=1}^{N_{unobs}} B_j\, e^{-\beta_j\varphi} + \varepsilon\right] \quad , \tag{2}$$

with $\varphi(0) = \left(\frac{d\,\varphi}{d\,x}\right)_0 = 0$. The gravitational potential has been divided by the square of a velocity dispersion which is taken here to be $(10\ \mathrm{km\ s^{-1}})^2$ for numerical convenience. The quantities $\alpha_i = \left[(10\ \mathrm{km\,s^{-1}})^2 / <v_z^2>_i\right]$, with a similar definition for the unobserved β_j. The height z above the plane is taken to be $z = z_0 x$, where the unit of length is $z_0 = \left[(10\ \mathrm{km\,s^{-1}})^2 / 2\pi G \rho_{obs}(0)\right]^{1/2}$. The quantity N_{obs} is the total number of observed mass components (15 for the standard case considered here, see Table 1) and N_{unobs} is the number of unobserved mass components. For the B&S Galaxy model, $z_0 = 196.6$ pc. The unobserved mass fractions B_j are defined, by analogy with equation (1), as the ratio of the mass density in component j to the total observed mass density. Finally ε is defined as the ratio of $\rho_{halo}^{eff}(0)$ to $\rho_{obs}(0)$. The effective halo mass density is equal to the total halo mass density for a constant rotation curve but is slightly different if the rotation curve is not exactly flat.

The isothermal approximation adopted here requires that the absolute value of the logarithmic derivative of the velocity dispersion be much less than the absolute value of the logarithmic derivative of the density, i.e., $\left|\frac{\Delta\left(<v_z^2>\right)}{<v_z^2>}\right| << \left|\frac{\Delta\rho(z)}{\rho}\right|$. The fractional change in the velocity dispersion of the F stars is less than or of order 0.1 over the first 200 pc in z (cf. the first four rows in Table 6 of Hill *et al.* 1979), while the density changes by a factor of 3. Thus the isothermal approximation appears to be well satisfied for the Hill *et al.* (1979) sample of F stars. Radford (1976) found that the velocity dispersion of G and K giants is constant to an accuracy of about 10% for z less than 400 pc. Hartkopf and Yoss (1982) obtained a similar result for the separate velocity dispersions of metal-poor and normal composition giants (cf. their Figure 8). Eggen (1969) found an approximate constancy of the velocity dispersion of A stars to about 300 pc. It is well known that at moderate and large values of z each separate disk component can be described by an exponential density profile (see the many references to the original data that are given in the caption of Figure 2 of B&S), which suggests that the isothermal approximation is also reasonable for the other disk stars.

I have neglected also the cross terms involving $<v_z\, v_R>$. Assuming Oort's (1965) hypothesis of a tilted velocity ellipsoid pointing in the direction of the Galactic center, the ration of omitted to include terms is of order $(z\, z_s / h\, R)$, where z_s is the scale height of the stars and h is the scale length of the disk. At the solar position, the correction due to the cross terms is less than 0.01 for all $z \lesssim 1$ kpc.

The results obtained with the Galaxy model described in Table 1 are $\rho_{Total}(0) = 0.188 \pm 0.02\, M_\odot pc^{-2}$, σ_{Disk} (to infinity) $=$ 64 $\pm 5\, M_\odot pc^{-2}$, and $(M/L)_{Disk} = 2.7 \pm 0.4$ solar units. For this calculation, the unobserved matter density was assumed proportional to the observed matter density everywhere. The best-estimate volume density given above is 4% larger than the best-estimate when all of the disk stars fainter than $M_V \geq 6.8$ mag are combined (the best-estimate column density is 2% smaller in the present case).

I have solved (cf. Paper I) numerically the differential equation (2) for a number of assumed distributions of the unseen matter and, in each case, for many values of the parameter characterizing the amount of unobserved material. The results of the theoretical models were fit to the observed spatial densities of F stars tabulated by Hill et al. (1979). The total number of mass components that were included varied between 11 and 31, depending on the assumption made about the unobserved matter. In Paper I, all of the faint disk stars ($M_V \geq 6.8$ mag) were combined.

Table 2 summarizes the requirements on the missing mass in the disk assuming that it is distributed like the observed interstellar matter, faint M-dwarfs, white dwarfs, or young massive stars.

Table 2. Some Candidates for the Missing Mass in the Disk

Candidate for the Missing Mass	Observed Mass Density ($M_{\odot} pc^{-3}$)	*Unobserved* Mass Density ($M_{\odot} pc^{-3}$)	$\frac{\rho_{unobs.}}{\rho_{obs.}}$
Interstellar Matter	0.045	0.144	3.2
Faint M - dwarfs ($M_V \geq 12.5^m$)	0.0093[a]	0.07	7.4[a]
White dwarfs	0.005	0.07	14
Young massive stars ($M > 1.6 M_{\odot}$)	0.002	0.144	35

[a] Assumes that the number of stars per absolute magnitude is constant for $16.5^m \geq M_V \geq 13$ and is equal to the value given by Wielen (1974) at $M_V = 12.5^m$.

The total volume density is better determined than the total column density. The extreme range of volume densities that were found in Paper I corresponds to a ratio of (maximum allowed/minimum allowed) = 1.5, versus 2.9 for the corresponding ratio of extreme column densities. Moreover, the theoretical uncertainties (isothermal approximation, separable potential) affect the calculated column density more strongly than the volume density.

Oort (1960) gives an often reproduced curve (smoothed) distribution of K-giants as a function of height above the plane for z=0 to 3 kpc. I have begun a reinvestigation of the K-giant distribution using this data set.

The amount of disk matter can be inferred in a simple way only if the tracer stars are essentially uncontaminated by spheroid stars.

https://doi.org/10.1017/S0252921100106669 Published online by Cambridge University Press

In the B&S Galaxy model, the ratio (spheroid K-giants)/(disk K-giants) is: 0.00 at z=0, 0.02 at z=0.6, 0.09 at z=0.9 kpc and reaches unity at 1.2 kpc. For larger values of z, the spheroid K-giants are much more numerous than the disk K-giants, the ratio reaching 72 at 3 kpc. It is not known what fraction of the spheroid K-giants were included in Oort's data set, which was based on the low dispersion HD and BSD catalogs. In fact, the fraction of spheroid K giants that are listed as K giants in these catalogs must depend upon the - as yet unknown - metallicity gradient of the spheroid. I have, therefore, come to the conclusion, reluctantly, that Oort's data cannot be used beyond a maximum distance which is somewhere between 0.6 to 0.9 kpc.

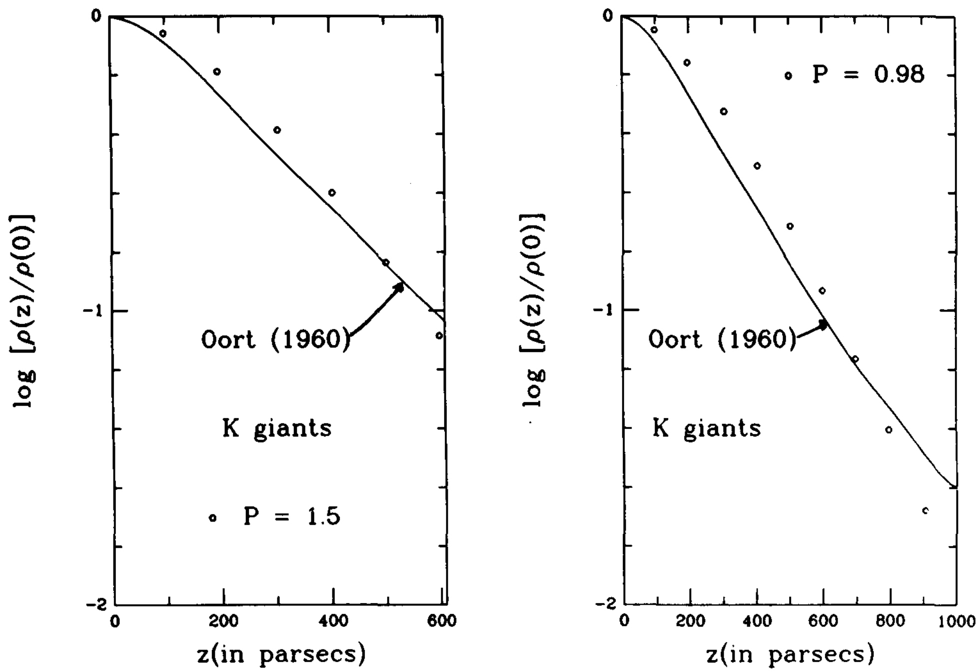

Figure 1 shows the comparison between a best-fitting theoretical model and Oort's data for two-values of the maximum distance. For the models shown in Figure 1. I have assumed a velocity dispersion for the K-giants of 20 km/s [Radford 1976, Hartkopf and Yoss 1982] and a halo mass density corresponding to ε=0.1. The unseen disk material was assumed proportional to the observed disk material.

The results for the K-giant sample are consistent with the analysis, described above, of the F dwarfs. If Oort's curve is used to 0.6 kpc, the best-fitting total mass density is $\rho_{Total}(0) = 0.24\, M_\odot\, pc^{-3}$ and the column density is $\sigma(\infty) = 65\, M_\odot\, pc^{-2}$. If the analysis is extended to 0.9 kpc, the best-fitting values are $\rho_{Total}(0) = 0.20\, M_\odot\, pc^{-3}$ and $\sigma(\infty) = 69\, M_\odot\, pc^{-2}$. Unfortunately, the results are rather sensitive to the maximum distance above the plane that is accepted in the calculation and to the not very well known velocity dispersion of the K-giants ($\rho_{Total} \sim \langle v_z^2 \rangle^{3/2}_{K-giants}$).

The missing matter at the solar position must be in a disk since if it were in a spheroidal component the calculated rotation velocity at the solar position would be

https://doi.org/10.1017/S0252921100106669 Published online by Cambridge University Press

much larger than the observed value (see Section 5b of Bahcall and Soneira 1980). The reason that so much spheroidal material would be required is that a given amount of matter is much less efficient at producing the needed z-acceleration if it is placed in a spheroidal rather than a disk configuration, approximately in the ratio of the scale heights (3 kpc to 0.3 kpc).

The slope of the faint end of the disk luminosity function that is required in order to hide all of the missing matter in stars fainter than 0.1 $M_\odot$ can be calculated easily. Suppose that the disk luminosity function has the form: $\Phi(M_V) \propto 10^{\gamma M_V}$ stars per absolute visual magnitude, for faint ($M_V \geq 13$ mag) stars. Then the minimum γ that is required varies from about 0.01 to about 0.05, depending mostly on the assumed mass - visual luminosity relation for faint dwarfs.

The observationally important implication is that all of the missing mass in the disk could be in faint stars *if* the slope of the disk luminosity function has a small *positive* value for large absolute visual magnitudes ($M_V > 13$ mag). This work was supported by the National Science Foundation grant no. PHY-8217352 and NASA grant no. NAS8-32902.

REFERENCES

Bahcall, J. N. 1984, *Ap. J.* **276** (Paper I).
Bahcall, J. N. and Soneira, R. M. 1980, *Ap. J. Suppl.* **44**, 73-110.
Bahcall, J. N. and Soneira, R. M. 1984, *Comparisons of a Standard Galaxy Model With Stellar Observations In Five Fields*, (submitted to Ap. J.)
Eggen, O. J. 1969, *P. A. S. P.* **81**, 741.
Hartkopf, W. I. and Yoss, K. M. 1982, *Astron. J.* **87**, 1679.
Hill, G. Hilditch, R. W., and Barnes, J. V. 1979, *M.N.R.A.S.* **186**, 813.
Oort, J. 1960, *Bull. Astr. Inst. Netherlands* **15**, 45.
Oort, J. 1965, in *Galactic Structure*, ed. A. Blaauw and M. Schmidt (Chicago: University of Chicago Press), p.455.
Radford, G. A. 1976, PhD Dissertation (esp. Table 4, page 26), Cambridge University.
Wielen, R. 1974, *Highlights of Astronomy*, **Vol. 3**, 395, ed. Contopoulos, G., (Dordrecht, Holland: D. Reidel).