

THE BULLETIN OF SYMBOLIC LOGIC

Edited by

James Cummings, *Managing Editor*

Matthias Baaz

Ali Enayat

Guram Bezhanishvili

Leonid Libkin

Katalin Bimbó

Øystein Linnebo

Paola D'Aquino

Arno Pauly

Peter Dybjer

Reviews Editors

Graham Leach-Krouse, *Managing Editor for Reviews*

Albert Atserias

Dugald Macpherson

Mark van Atten

Antonio Montalbán

Benno van den Berg

Christian Retoré

Clinton Conley

Marion Scheepers

Bradd Hart

Nam Trang

Thesis Abstracts Editor

Christian Rosendal

VOLUME 28 • NUMBER 1 • MARCH 2022 • ISSN 1079-8986

Copyright © 2022 by the Association for Symbolic Logic. All rights reserved.
Reproduction by photostat, photo-print, microfilm, or like process by permission only.



PUBLISHED BY CAMBRIDGE UNIVERSITY PRESS FOR
AND ON BEHALF OF THE ASSOCIATION OF SYMBOLIC LOGIC.

The BULLETIN, the JOURNAL and the REVIEW OF SYMBOLIC LOGIC are the official organs of the Association for Symbolic Logic, an international organization for furthering research in logic and the exchange of ideas among mathematicians, computer scientists, linguists, and others interested in this field. The BULLETIN encourages submissions of *Articles* and *Communications* in all areas of logic, including mathematical or philosophical logic, logic in computer science or linguistics, the history or philosophy of logic, and applications of logic to other fields. It also encourages the submissions of abstracts for doctoral level theses in logic. The BULLETIN also publishes reviews of publications in logic.

Editors

James Cummings (Managing Editor), Department of Mathematical Sciences, Carnegie Mellon University, Wean Hall 6113, Pittsburgh, PA 15213, USA

jcumming@andrew.cmu.edu

Matthias Baaz, Institute for Discrete Mathematics and Geometry, Vienna University of Technology, Weidner hauptstrasse 8-10, 1040 Vienna, Austria

baaz@logic.at

Guram Bezhanishvili, New Mexico State University, New Mexico, USA

Katalin Bimbó, University of Alberta, Alberta, Canada

bimbo@ualberta.ca

Paola D'Aquino, Department of Mathematics and Physics, Università della Campania "L. Vanvitelli", viale Lincoln 5, Caserta 81100, Italy

paola.daquino@unia2.it

Peter Dybjer, Department of Computer Science and Engineering, Chalmers University of Technology, SE 412 96 Göteborg, Sweden

peterd@chalmers.se

Ali Enayat, Department of Philosophy, Linguistics, and Theory of Science, University of Gothenburg, Box 200, SE 405 30, Gothenburg, Sweden

ali.enayat@gu.se

Leonid Libkin, Laboratory for Foundations of Computer Science, University of Edinburgh, Informatics Forum, IF 5.33, Crichton Street, Edinburgh, EH8 9AB, UK

libkin@inf.ed.ac.uk

Øystein Linnebo, Department of Philosophy, IFIKK, 0315 Oslo, Norway

oystein.linnebo@ifikk.uio.no

Arno Pauly, Swansea University, Wales

arno.m.pauly@gmail.com

Reviews editors

Graham Leach-Krouse (Managing Editor), Kansas State University, Kansas, USA, gkrouse@ksu.edu

Albert Atserias atserias@cs.upc.edu,

Mark van Atten mark.vanatten@univ-paris1.fr,

Benno van den Berg bennovdberg@gmail.com,

Clinton Conley clintonc@andrew.cmu.edu

Bradd Hart hartb@mcmaster.ca,

Dugald Macpherson pmthdm@maths.leeds.ac.uk,

Antonio Montalbán antonio@math.berkeley.edu,

Christian Retoré christian.retore@limm.fr

Marion Scheepers mscheepe@boisestate.edu, and

Nam Trang namtrang35@gmail.com.

Thesis abstracts editor

Christian Rosendal rosendal@math.uic.edu

Articles present topics of broad interest in a way that is accessible to a large audience. They can be purely expository, survey, or historical articles or they may contain, in addition, new ideas or results or new approaches to old ones.

Communications should be announcements of important new results and ideas in any aspect of logic; they may be short papers in their final form or preliminary announcements (extended abstracts, position papers) of longer, full papers that will be published elsewhere. In any case, they should include, in addition to a description of the new results or ideas, enough history, background, and explanation to make the significance of the work apparent to a wide audience. *Communications* will be quickly refereed and published within six months of the submission of final version.

To submit *Articles* or *Communications* please follow the procedure described at <https://asonline.org/journals/the-bulletin-of-symbolic-logic/the-bulletin-of-symbolic-logic-submission-guidelines>.

To submit *Thesis Abstracts* please follow the procedure described at <https://asonline.org/journals/the-bulletin-of-symbolic-logic/logic-thesis-abstracts-in-the-bulletin-of-symbolic-logic>.

Books for review in the BULLETIN should be sent to **ASL, Department of Mathematics, University of Connecticut, 341 Mansfield Road, U-1009, Storrs, CT 06269-1009, USA.**

TABLE OF CONTENTS

ARTICLES

Level theory, part 3: A Boolean algebra of sets arranged in well-ordered levels by TIM BUTTON	1
Degrees of randomized computability by RUPERT HÖLZL AND CHRISTOPHER P. PORTER	27
An introduction to the Scott complexity of countable Structures and a survey of recent results by MATTHEW HARRISON-TRAINOR	71
Invariance criteria as meta-constraints by GIL SAGI	104

COMMUNICATION

Theorems of hyperarithmetic analysis and almost theorems of hyperarithmetic analysis by JAMES S. BARNES, JUN LE GOH, AND RICHARD A. SHORE.....	133
---	-----

IN MEMORIAM

In Memoriam: Gerald E. Sacks, 1933–2019.....	150
Notices.....	156