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Towards understanding the global magnetism of the Sun and solar-like stars

Allan Sacha Brun
DSM/IRFU/SAp & UMR AIM, CEA-CNRS-Université of Paris 7, France

Abstract. The Sun and solar-like stars possess intense and cyclic magnetic activity. In order to understand how this comes about we have developed series of 2-D and 3-D models in order to simulate their global dynamics and magnetism. We here report on our latest findings.

Keywords. Sun: activity, interior, magnetic fields; stars: activity; convection, MHD, turbulence

1. Simulating the magnetism of the Sun and solar-like stars

The Sun possesses striking magnetic and dynamical properties, such as its turbulent convective envelope, large-scale surface differential rotation, 22-yr cycle of magnetic activity, butterfly diagram of sunspot emergence, hot corona, etc. (Stix 2002). Understanding how the physical processes operating in the solar turbulent plasma nonlinearly interact to yield this wide range of dynamical phenomena is very challenging. One successful and powerful approach is to rely on multi-dimensional magnetohydrodynamics (MHD) numerical simulations. Today, despite tremendous advances in building powerful supercomputers, it is still not possible to compute a fully integrated 3-D MHD model of the Sun starting from its core up to its corona. One is thus forced to study individually complementary pieces of the full solar MHD puzzle and to progressively incorporate them in a more nonlinearly coupled model. One important characteristic of the Sun that needs to be understood is the origin of its magnetic activity because it has direct societal impact by impairing satellites, damaging electric power grids, interfering with high frequency radio communications and radars. It is currently believed that the solar magnetism is linked to an internal "fluid" dynamo (Parker 1955). More precisely the Sun is the seat of both a small scale and irregular dynamo and a large scale and cyclic dynamo that generate and maintain its magnetic field and lead to the various magnetic phenomena observed at its surface (Parker 1993; Cattaneo & Hughes 2001; Ossendrijver 2003). Developing numerical models of the solar dynamo has thus been a very active field of research. This has mainly involved two types of numerical experiments:

- *kinematic solar dynamo models* that solve only the induction equation in its mean field approximation and assume the velocity field as given (Stix 1976; Moffat 1978; Krause & Radler 1980; see Charbonneau 2005 and Solanki *et al.* 2006 for recent reviews). These models rely on the parametrization of two important effects that are thought to be at the origin of the solar global dynamo, the α and Ω effects. They provide a useful and fast tool to model the solar 22-yr magnetic cycle and its associated butterfly diagram since no feedback from the Lorentz force on the motion is accounted for.
- or *dynamical solar dynamo models* that solve explicitly the full set of MHD equations (see for instance Gilman 1983; Glatzmaier 1985; Cattaneo 1999; Brun *et al.* 2004; Vögler & Schüssler 2007). These models self-consistently compute all the physical processes in three dimensions allowing significant progress to be made on the intricate interactions operating in a turbulent magnetized plasma. The cost of 3-D models and the large number

of degrees of freedom needed to model the whole Sun make it difficult, as of today, to provide quantitative predictions such as the cycle period.

Clearly, both methods (2-D vs 3-D) are complementary and are needed to better understand the magnetic solar activity. They have also often been extended to other stellar spectral type (see for instance Charbonneau & Saar 2001; Brun 2009). We propose in this paper to report on our latest findings using this complementary approach.

2. Solar-stellar connection

Another important information to better constraint the magnetic Sun is to compare its activity to other stars. Observations of the magnetism of 'solar type' stars, i.e. stars possessing a deep convective envelope and a radiative interior (late F, G, K and early M spectral type) are becoming more and more available (Giampapa 2005) and may provide additional constraints on our understanding of global scale stellar dynamos. One difficulty of such observational programs is that they require long term observations since stellar cycle periods are likely to be commensurate to the solar 11-yr sunspots cycle period (or 22-yr for a full cycle including two polarity reversals of the global poloidal field). Thanks to the data collected at Mount Wilson Observatory since the late 60's, such data is available (Wilson 1978; Baliunas *et al.* 1995). Among the sample of 111 stars (including the Sun as a star) originally observed between the F2 to M2 spectral type, it is found that about 50% of the stars possess a cyclic activity, with cycle (starspot) periods varying roughly between 5 to 25 yrs, i.e. between half to twice the sunspot cycle period. They further indicate that among the inactive stars of the sample some are likely to be in a quiet phase (as was the Sun during the Maunder minimum). Overall activity cycles seem to be more frequent for less massive stars K than for F stars. More recent observational programs have been pursued that now even provide information on the field topology as a function of the rotation rate, such as the one using the Espadons and Narval instruments and the Zeeman Doppler Imaging technique (Donati *et al.* 2006). Applying this observational technique over a sample of four solar analogues with rotation rate varying from one to three times solar, Petit *et al.* (2008) have shown that the field amplitude increases as a function of the star's rotation rate and, more importantly, becomes more and more dominated by its toroidal component (modulo possible bias in the observational technique used). If such a trend is confirmed, i.e. that the field topology is becoming more toroidal with increasing rotation rate, it is a very important and instructive result that puts strong constraints on the dynamo models.

The systematic analysis of stellar magnetism data revealed that for solar type stars there is a good correlation between the cycle and rotation periods of the stars. This correlation is even stronger when using the Rossby number ($Ro = P_{rot}/\tau$) that takes into account the convection turnover time τ at the base of the stellar convective envelope (Noyes *et al.* 1984; Baliunas *et al.* 1996). As the star rotates faster, its cycle period is found to be shorter. Typically Noyes *et al.* (1984) found that $P_{cyc} \propto P_{rot}^n$, with $n = 1.25 \pm 0.5$. Based on an extended stellar sample Saar & Brandenburg (1999) and Saar (2002) have argued that there is actually two branches when plotting the cycle period vs the rotation period of the stars. They make the distinction between the primary (starspot) cycle and Gleissberg or grand minima type modulation of the stellar activity. For the active branch they found an exponent $n \sim 0.8$ and for the inactive stars $n \sim 1.15$ (Charbonneau & Saar 2001). It is also found that this correlation breaks at high rotation rate with the possible appearance of a super active branch. Further at very high rotation rate saturation of the X-ray luminosity seems to limit the validity of the scaling found at more moderate rotation rates (Pizzolato *et al.* 2003). For G type stars this saturation is found for rotation rate above 35 kms^{-1}, for K type stars at about 10 kms^{-1} and for M dwarfs around 3–4 kms^{-1} (Browning 2008; Reiners, Basri & Browning 2009). How stellar magnetic flux scales with rotation rate is thus also important to understand since it is telling us how the magnetic field generated by dynamo action inside the stars emerges and imprints the stellar surface (Rempel 2008) and if it actually saturates. Both the Corot and Kepler satellites should further improve our understanding of stellar magnetism and convection (we advise the reader to read the contributions related to both satellites in these proceedings).

3. 2-D mean field models

Theoretical considerations to interpret stellar magnetism based on classical 2-D mean field α-ω dynamo models (Durney & Latour 1976, Knobloch *et al.* 1981; Baliunas *et al.* 1996) naturally find correlations between rotation rate and stellar activity. In particular it is found that both magnetic field generation and the dynamo number D (i.e. a Reynolds number characterizing the mean field α and ω dynamo effects used in the models) vary with the rotation period of the star. This is due to the fact that in these models both effects are sensitive to the rotation rate of the star. The ω-effect is a direct measure of the differential rotation $\Delta\Omega$ established in the star. It is well known both theoretically and observationally that the differential rotation in the convective envelope of solar-type stars is directly connected to the star's rotation rate Ω_0 (Donahue *et al.* 1996; Barnes *et al.* 2005; Ballot *et al.* 2007; Brown *et al.* 2008; Küker & Rüdiger 2008). However, the exact scaling n_r (i.e. $\Delta\Omega \propto \Omega_0^{n_r}$) is still a matter of debate among the observers and theoreticians, being sensitive to the observational techniques used and to the modelling approach.

The α-effect used in mean field theory is related to helical turbulence and represents a measure or parameterization of the mean electromotive force (emf) (Moffatt 1978; Pouquet *et al.* 1976). It is thus also linked to the rotation rate of the star and the amount of kinetic helicity present in its convective envelope. However in Babcock-Leighton flux transport dynamo models the standard α-effect is replaced by a surface term linked to the tilt of the active regions with respect to the east-west direction (i.e. Joy's law, Kosovichev & Stenflo 2008). This tilt is thought to be due to the action of the Coriolis force during the rise of the toroidal structures that emerge as active regions (D'Silva & Choudhuri 1993). Recent 3-D simulations in spherical shells (Fan 2008; Jouve & Brun 2007b, 2009) seem to indicate that this is not the only effect responsible for the observed tilt and that the twist and arching of the toroidal structures as well as the continuous action of the surface convection during the emergence have some influence on the resulting tilt.

Another important ingredient in flux transport models is the large scale meridional circulation (MC) used to connect the surface source term generating the poloidal field to the region of strong shear at the base of the convection zone (i.e. the tachocline) where it will be subsequently sheared by the ω-effect in order to close the global dynamo loop (i.e. $B_{pol} \rightarrow B_{tor} \rightarrow B_{pol}$) . The meridional flow (or "conveyor belt") thus plays an important role in setting the cycle period of the global dynamo in this class of flux transport model. As a direct consequence it is natural to ask how the meridional circulation amplitude and profile change with the rotation rate and how these may influence the magnetic cycle period.

Jouve & Brun (2007a) have done a systematic study of the influence on the cycle period of the various parameters of Babcock-Leighton flux transport models. For the reference model possessing only one meridional circulation cell per hemisphere they confirm the results of Dikpati & Charbonneau (1999) that the cycle period is strongly dependent on the meridional circulation amplitude. A least square fit through the ensemble of models they computed leads to the following dependency:

$$P_{cyc} = v_0^{-0.91} s_0^{-0.013} \eta_0^{-0.075} \Omega_0^{-0.014}, \tag{3.1}$$

with v_0, s_0 and η_0 being respectively, the amplitude of the meridional circulation at the surface, that of the Babcock-Leighton source term and of the magnetic diffusivity (see Jouve & Brun (2007a), Jouve, Brown & Brun (2009) for more details). We can thus expect that as we increase the rotation rate the cycle period will shorten if all the other parameters are kept constant but unfortunately not as fast as what is observed (Noyes *et al.* 1984; Saar 2002). However we may also expect the meridional circulation to depend on the rotation rate, and the sign of the variation of v_0 with Ω_0 will most certainly dominate over the weak dependence of P_{cyc} on Ω_0. It is thus crucial to assess how the meridional circulation amplitude and profile vary with the rotation rate Ω_0.

Early work by Dikpati *et al.* (2001) have assumed that the amplitude of the meridional flow is proportional to the rotation rate of the star. With such scaling they seem to be able to reproduce some of the rotation vs cycle period correlation observed in solar type stars. Charbonneau & Saar (2001) have also studied the sensitivity of various mean field dynamo models in reproducing stellar activity observations assuming for the Babcock-Leighton type that the meridional flow amplitude increases either as Ω_0 or as $\log(\Omega_0)$. Similarly Nandy & Martens (2007) have explored the solar-stellar connection with B-L mean field dynamo models and have reached the same conclusion: only a positive scaling of the amplitude of meridional flows with the rotation rate can reconcile the models with observations of magnetism of solar-like stars. However recent theoretical work by Ballot *et al.* (2007) and Brown *et al.* (2008) indicate that this is unlikely to be the right scaling. Instead 3-D simulations reveal that the amplitude of the meridional flow weakens as the rotation rate is increased.

In Jouve *et al.* (2009) we have explored how mean field Babcock-Leighton dynamo models may be used to explain the observational relation between the rotation and magnetic activity period of solar-like stars by incorporating scaling law for the amplitude of the meridional circulation coming from the 3-D simulations of Brown *et al.* (2008). In this 3-D study they found that the energy and amplitude of the meridional circulation decreases with the rotation rate.

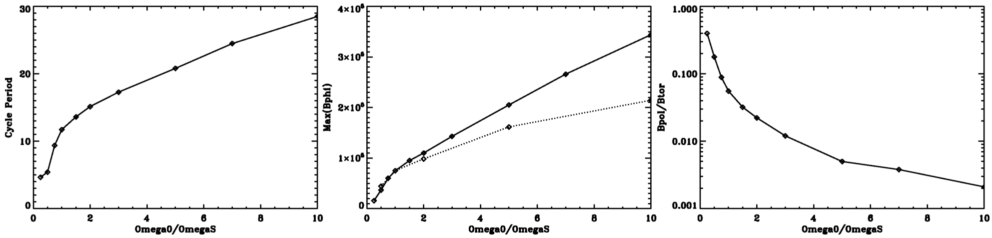

Figure 1. Left panel: Variation of the magnetic cycle period (in years) wih respect to the solar rotation rate for the cases published in Jouve *et al.* (2009). On the 2nd panel we display the variation of the maximum of toroidal field at the base of the CZ for two series of models. The solid line corresponds to the models using the scaled down meridional flow as a function of the rotation rate deduced from Brown *et al.* (2008) whereas the dotted line corresponds to another set of models for which v_0 was kept constant. On the third panel, we show the ratio Bpol/Btor wih respect to the solar rotation rate for the scaled cases.

On Figure 1, we display the results of our series of mean field stellar dynamo models at various rotation rate. First we find that indeed the magnetic field topology tends to

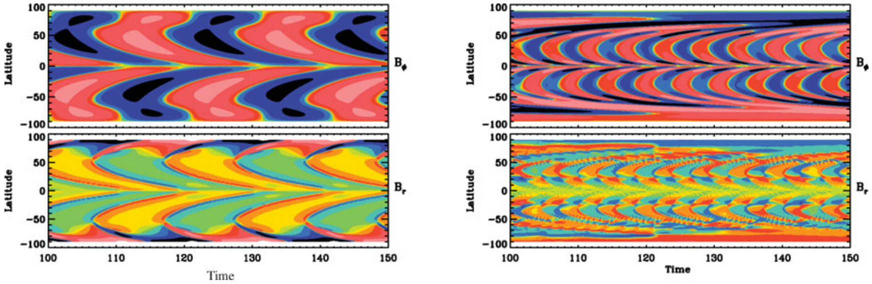

Figure 2. The top panel shows the time-latitude cut of the toroidal field at the base of the convection zone and the bottom one represents the evolution of the radial field at the surface for two cases rotating at 5 times the solar rate. Both models possess a scaled down MC flow but in the right panels the meridional flow is multicellular, with the poleward cell extending from the equator up to 20 degree. Time is in years. The scale of the color table is the same for all: between -5×10^3 G and 5×10^3 G for the radial field and between -5×10^5 G and 5×10^5 G for the toroidal field.

be more and more toroidal, with B_ϕ increasing in strength due to a larger ω-effect and a slower flow at the base of the convection zone, that allows the fields to be stretched further more. Unfortunately we find that Babcock-Leighton flux transport dynamo model based on single cell meridional circulation does not reproduce the P_{cyc} vs P_{rot} relation when using scaling for the MC deduced from 3-D global simulations.

However the strong relation between the meridional circulation speed and the cycle activity is an indication that shortening the advection path may result in shorter cycle period. Dikpati *et al.* (2004), Bonanno *et al.* (2005) and Jouve & Brun (2007a) have studied in details the influence of various meridional circulation profiles on the butterfly diagram and other large scale magnetic properties of the Sun. In their study Jouve *et al.* (2009) demonstrate that adding cells in latitude may resolve the apparent disagreement between B-L dynamo models and observations of stellar activity cycles.

On the butterfly diagrams shown on Figure 2, we clearly see that we have significantly reduced the length of the activity cycle: the period is now 5.2 yr instead of 22 yr. So shortening the advection path seems to be an interesting solution but it also modifies the butterfly diagram. The cyclic activity tends now to be more concentrated at low latitudes. The radial field appears to be busy, exhibiting a periodic but yet complex pattern at low latitudes. We moreover note that above 55 degree, the cyclic activity is lost and we are left with some regions where the magnetic field evolves much more slowly than the time scale of the main cycle.

Another way to reconcile B-L models and observations may be to replace the meridional circulation as the main process to transport the field from the surface down to the tachocline. We refer to the work of Guerrero & Gouveia Dal Pino (2008) and Yeates *et al.* (2008) that have considered respectively turbulent pumping or magnetic diffusion as the dominant transport process. The weaker dependancy of the cycle period to those alternative transport processes may result to a better agreement between B-L dynamo models and stellar cycle observations. We intend to explore further these alternatives. We now turn to discussing our recent nonlinear 3-D MHD studies of solar and stellar convection, rotation and magnetism.

4. 3-D global MHD models

The Sun and solar-like stars are complex magnetohydrodynamic objects that require state-of-the-art observations and numerical simulations in order to pin down the physical processes at the origin of such diverse activity and dynamics. We propose here to make a brief summary of the recent advances made with the Anelastic Spherical Harmonic (ASH) code (Clune *et al.*(1999); Brun *et al.* 2004) in modelling in 3-D the global non-linear solar magnetohydrodynamics. We refer to Ballot *et al.* (2007) and Brown *et al.* (2008, 2009) for the study of the dynamics of solar-like stars rotating at various rotation rate.

- Solar Global Convection

A series of papers has been published on this important topic (Miesch *et al.*(2000); Elliott *et al.*(2000); Brun & Toomre 2002), the most recent being by Miesch *et al.*(2008). In that later paper for the first time a global model of the solar convection with a density contrast of 130 from top to bottom and a resolution equivalent to 1500^3 has been obtained. This has lead to significant results regarding the turbulent convection spectra from large scale (giant cells like) down to supergranular like convection patterns and their correlation with the temperature fluctuations leading to a large (150% $L_\odot$) convective luminosity.

- Solar Differential Rotation and Meridional Circulation

A very recent review by Brun & Rempel (2008) discusses the respective role of Reynolds stresses, latitudinal heat transport and baroclinic effect in setting the peculiar conical differential rotation profile observed in the solar convection zone. Indeed basic rotating fluid dynamic considerations imply that the differential rotation should be invariant along the rotation axis thus yielding a cylindrical rotation profile. Since this is not observed it is necessary to find the source of the breaking of the so called Taylor-Proudman constraint. In particular in a recent paper by Miesch *et al.*(2006) we have been able to show that baroclinic effects are associated to temperature latitudinal variation and that convection by transporting heat poleward contributes for a significant part of that variation but not all. A temperature contrast of about 10 K is compatible with helioseismic inference of the inner solar angular velocity profile. Meridional flows in most cases are found to be multi cellular and fluctuate significantly over a solar rotation. This flow contributes little to the heat transport and to the kinetic energy budget (accounting for only 0.5% of the total kinetic energy). However it plays a pivotal role in the angular momentum redistribution by opposing and balancing the equatorward transport by Reynolds stresses (Brun & Toomre 2002, Brun & Rempel 2008).

- Solar Global Dynamo

In the continuation of the work by Gilman (1983) and Glatzmaier (1985), we have studied at much higher resolution dynamo action in turbulent convective shells (Brun 2004; Brun *et al.* 2004). We have found that dynamo action is reached above a critical magnetic Reynolds number and that the magnetic field is mostly intermittent and small scale (see Figure 3 (right panel)), the large scale axisymmetric field only contributing for about 3% of the total magnetic energy. Reversals of the field occur on a time scale of about 1.5 yr as opposed to the observed 11 yr cycle of solar activity. This is in part due to the absence of a tachocline at the base of the convective envelope. In an attempt to resolve that issue, we have in Browning *et al.* (2006), computed the first 3-D MHD model of a convection zone with an imposed stable tachocline. In that layer the field that has been transported or pumped down from the above turbulent convection zone, is organized in strong axisymmetric toroidal ribbons with a dominant antisymmetry with respect to the equator (see Figure 3 (left panel)). The poloidal field in the convection zone is stabilized by the presence of that layer with much less frequent, if any,

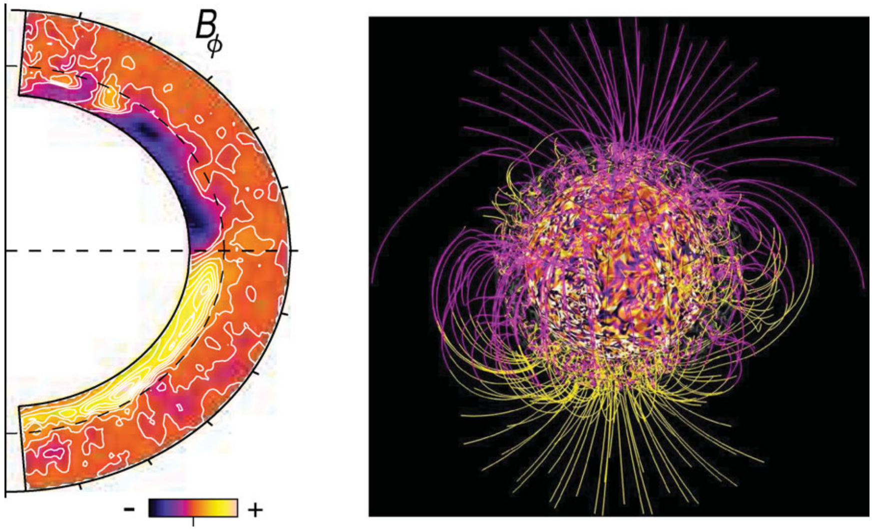

Figure 3. Left panel: Azimuthal and temporal average of the toroidal magnetic field in a 3-D MHD simulation of the solar convective envelope coupled to a stable sheared tachocline (Browning *et al.* 2006). We note the antisymmetric magnetic layers at the base of the convection zone represented by the dash line. Right panel: 3-D magnetic field lines rendering (yellow corresponds to positive Br, i.e the line is directed toward the observer) inside the convection zone (below the transparent sphere) and in a current-free corona (potential field line reconstruction) for the purely convective dynamo case M3 of Brun *et al.* (2004). We note the intricacy of the field lines in the convection zone, often located in strong downflows and the connectivity of the field over large distances at the surface.

reversal. The magnetic energy reaches in both cases about 10% of the total kinetic energy. We also find that the differential rotation is reduced in amplitude due to the nonlinear feedback of the field on the flow via the Lorentz force. In a recent study by Jouve & Brun (2007b), Jouve & Brun (2009), we have also studied flux emergence in isentropic and turbulent rotating convection zone. We confirmed that a minimum amount of field concentration and twist is required for the structure to emerge at the surface and that convection may pin down and recycle weak field. At the surface in the convective case, horizontal converging motions modify the orientation of the emerging structure. We find that Maxwell stresses associated with the emerging structure modify locally the horizontal flows, changing their amplitude and direction.

- Solar Radiation Zone and its coupling to CZ

The thinness of the solar tachocline ($h < 0.05R_\odot$) implies the presence of processes that transport angular momentum horizontally in order to prevent its viscous and thermal spread along the solar evolution. Horizontal turbulence in stratified layers, the presence of a fossil magnetic field or internal waves may provide such a transport (Spiegel & Zahn 1992, Gough & McIntyre 1998). Brun & Zahn (2006) have studied for the first time in 3-D the nonlinear evolution of an horizontal shear (similar to the solar angular profile) in a stable radiative zone and the role of a fossil field. They found that such field by connecting to the surface shear will ease rather than oppose the spread of the tachocline. It is thus difficult to invoke only the magnetic field to explain the solid body rotation of the radiative zone and the thin tachocline. Another important finding in Brun & Zahn (2006) is that simple magnetic configurations undergo non axisymmetric MHD instabilities first discovered by Talyer in the 70's. In particular a purely poloidal (dipolar) fossil

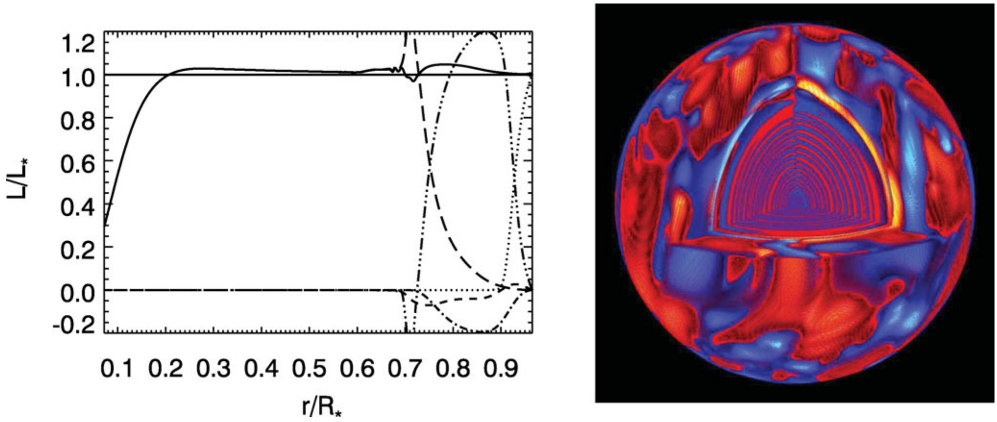

Figure 4. 3-D integrated solar model coupling nonlinearly the convective envelop to the radiative interior (Brun *et al.* 2009, in preparation). The base of the convection zone is located at 0.72 R. Left panel: Radial energy flux balance. Shown are the enthalpy (convection) (dash three dots), radiative (long dash), kinetic (dash dot), viscous (dash), unresolved (dotted) and total (solid) fluxes converted to luminosity and normalized to the solar luminosity. We note the large enthalpy flux in the convection zone and the sharp penetration at the case of the CZ. Right panel: Shown is the a 3-D rendering of the density perturbations with red corresponding to positive fluctuations. We have omitted an octant in order to be able to see the equatorial and meridional planes within the domain. We note the clear presence of internal waves in the radiative zone.

field would undergo high azimuthal wavenumber instabilities. In order to obtain a stable configuration for a magnetic field burried in the solar radiative interior it is necessary to have a coupled poloidal/toroidal topology. Such non axisymmetric instabilities may lead to dynamo action in radiative interiors has also been advocated by Spruit (2002) and discussed in details by Zahn *et al.* (2007).

- Towards a 3-D integrated model of the Sun and solar-like stars

Coupling nonlinearly the convection zone with the radiative interior is key to understand the solar global dynamo and inner dynamics. Brun *et al.* (2009, in preparation) have developped the first 3-D solar integrated model from $r = 0.07R_{\odot}$ up to $0.97R_{\odot}$. We show on Figure 4 (left panel) the energy flux balance realized in such coupled models. We note the dominant role played by the convective flux in transporting most of the energy in the convective envelope, while the radiative flux dominates in the deep interior. A thin convective penetration region exist at the base of the convection zone (as indicated by the negative convective flux around 0.7 R), overlapping with the tachocline of shear self consistently generated in the simulation. On Figure 4 (right panel), we show a 3-D rendering of the density fluctuations over the whole computational domain. The presence of internal waves is obvious in the radiative interior. The penetrative convection is at the origin of these gravito-inertial waves. We are currently studying in details the source function at every depth in the model and the resulting power spectrum at different locations in the radiative interior and find that a large spectrum near the base of the convection zone is excited. The tachocline is kept thin in this model by using a step function at the base of the convection zone for the various diffusion parameters making the thermal and viscous spread of the latitudinal shear imposed by the convective envelope slow with respect to the convective overturning time. We intend in the near future to reproduce the study of Brun & Zahn (2006) by introducing in the integrated model a fossil field taking advantage of the more realistic boundary conditions realized in this new class of models (Strugarek *et al.* 2010, in prepration). We will also adapt our latest solar models to solar-like and more massive stars in order to span the H-R diagram (see for instance Brun 2009).

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