The preceding remarks give some indication of the traps that beset the reader. If he perserveres and is prepared to treat every statement with due caution he will find much to reward his efforts. To the reviewer the section (§9.8) on algebraic independence of arithmetic functions is one of the most interesting and owes much to the author's own work. Moreover, there is a wealth of bibliographical information at the end of each chapter to encourage the reader to further study.

R. A. RANKIN

MINC, H., *Permanents* (Encyclopedia of Mathematics and its Applications, vol. 6, Addison-Wesley Advanced Book Programme, 1978), xviii+205 pp., $21.50.

In his preface, the author states: "Permanents made their first appearance in 1812 in the famous memoirs of Binet and Cauchy. Since then 155 other mathematicians contributed 301 publications to the subject, more than three quarters of which appeared in the last 19 years. The present monograph is the outcome of this remarkable re-awakening of interest in the permanent function." (In fact, 303 publications are quoted in the Bibliography.)

In an attempt to give a complete account of the theory of permanents, the author has traced their development from their inception until the present day (1978). Chapter 1 contains a historical survey in which only the classical results are discussed in detail, while Chapter 2 covers the basic elementary properties of permanents and Chapter 3 is devoted to the permanent of (0, 1) matrices, including the classical theorem of Frobenius and König (Let A be an $n\times n$ matrix with non-negative entries. Then $\text{per}(A)=0$ if and only if A contains an $s\times t$ zero submatrix such that $s+t=n+1$.)

The next three chapters are given over entirely to inequalities involving permanents, either upper and lower bounds for permanents of (mainly) non-negative matrices, or, in the case of Chapter 5, the Van der Waerden conjecture. This states that if S is a doubly stochastic $n\times n$ matrix then $\text{per}(S)\geqslant n!/n^n$. The whole chapter is devoted to a discussion of this, giving the then most recent developments in the pursuit of a solution to this conjecture. In 1980 it was proved to be true by a Russian author, Egoritsjev, in Russian. However, J. H. Van Lint has produced an account of Egoritsjev's proof in Linear Algebra and its Applications, **39** (1981), pp. 1–8, entitled "Note on Egoritsjev's Proof of the Van der Waerden Conjecture."

Chapter 7 discusses methods of evaluating permanents and compares their efficiency, while the final chapter deals with a variety of applications of permanents, e.g. to the estimation of the number of latin squares of a given order, the number of non-isomorphic Steiner triple systems of a given order and to the n-dimensional dimer problem. A list of conjectures and unsolved problems completes this final chapter.

It has to be said that this book is well-written and beautifully produced, with few misprints. To anyone working with, or requiring knowledge of, permanents, it should be regarded as essential. It is likely to become the standard reference on permanents.

E. SPENCE

ASIMOW, L. and ELLIS, A. J., *Convexity Theory and its Applications in Functional Analysis* (London Mathematical Society Monograph 16, Academic Press, 1980), x+226 pp. £23.20.

Convexity theory is a beautiful subject, combining geometry with algebra and analysis, and providing a unified approach to classical and modern results in areas such as potential theory, ergodic theory and operator algebras. It tells us that it is always possible to decompose points in a compact convex set into suitable combinations of extreme points (vertices), and when it is possible to do so uniquely. In finite dimensions this is a classical theorem of Carathéodory; an infinite-dimensional example is Bochner's Theorem on positive-definite functions.

After the discovery of Choquet's Theorem in 1956 and the Bishop-de Leeuw Theorem in 1960, there was much activity in general convexity theory for about 10 years, before a change of emphasis occurred in the 1970s. Alfsen's book "Compact convex sets and boundary integrals" (Springer, 1971) therefore appeared at a propitious time, and it has become a standard reference in view of the scarcity of Phelps' excellent introductory "Lectures on Choquet's Theorem" (van Nostrand, 1966).

The switch in direction did not cause neglect. The last decade has produced the extension of the theory to complex function spaces, the proof of the uniqueness and universality of the Poulsen simplex, the geometric characterisation of state spaces of operator algebras, and connections between the Radon-Nikodym and Krein-Milman properties in Banach spaces. Of these four topics only the last had been expounded in a monograph until the appearance of the book under review. After two chapters of general theory, the authors have devoted one chapter to each of the other three topics as outstanding examples of the subject ("Applications" in the title does not ring true).

So it would be expected that this book would be welcomed, but it comes as a great disappointment. The beauty of the subject has been obliterated by a canopy of unintuitive abstraction. Definitions and results are introduced in the greatest possible generality, and remarkable geometrical properties are stated starkly without attention being drawn to their true nature. The same faults affect the mathematical details; many simple facts are deduced from the "Gauge Lemma", an elaborate version of the Uniform Boundedness Principle; the authors' fascination for duality leads to the exclusion of the extension version of the Hahn-Banach Theorem, even though this is more natural than the Bipolar Theorem as a tool for Choquet's Theorem. The reader is also frustrated that the book falls narrowly short of being self-contained. It is understandable that the lengthy proofs of Alfsen-Shultz theory should be omitted, but the absence of Lazar's Selection Theorem seriously devalues the proof of uniqueness of the Poulsen simplex. Neither is the book suitable for reference, since the results as stated often depend on assumptions and notational conventions which are buried in the text. There are also signs of carelessness in checking the completeness of the arguments, and eliminating misprints and ambiguities. The symbol "=" is used in juxtaposition with different logical meanings.

In his enlightening survey article "The Choquet representation in the complex case", Bull. Amer. Math. Soc. **83** (1977), Phelps referred to a new expanded edition of his book, but this has not yet materialised. One's eagerness for such a republication is not removed by the work under review. In the meantime, a novice can still be advised to introduce himself to the subject via the original edition if available, and then Choquet's "Lectures on Analysis II" (Benjamin, 1969) and Alfsen's book. Asimow and Ellis have an advantage only in being able to include more recent work on simplex spaces and complex function spaces.

C. J. K. BATTY

COLLINS, MICHAEL J. (ed) *Finite Simple Groups II* (Academic Press, London, 1980), xvi+346 pp. £25.

This book is officially the proceedings of London Mathematical Society's 1978 Durham Symposium on Finite Simple Groups but its contents have been revised up to the date of publication in order to include some recently-proved results.

The first part of the book is devoted to an exposition by various authors of the problem of classifying the finite simple groups, the methods used and the results obtained. The quality of the exposition is very high in general. The best contributions are those of Gluberman and Goldschmidt, which have a notable clarity. On the other hand, I feel that Gorenstein, Lyons and Griess have not motivated their work enough with the result that their contributions tend to deteriorate into a catalogue of an increasing complexity of definitions and notation. An introductory chapter to the book was desirable in order to link the various contributions together into a cohesive and comprehensive account of the classification project. The editor took upon himself the task of providing this and has greatly expanded an introductory lecture given at the symposium by Gorstein into a very satisfactory survey article.

The second part of the book is concerned with the representation theory of the groups of Lie type. There are good "surveys for group theorists" of the current knowledge of both ordinary and modular representations together with a somewhat more specialised article on the structure of Weyl modules.