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SHIN'S FORMULAS FOR EIGENPAIRS OF SYMMETRIC TRIDIAGONAL 2-TOEPLITZ MATRICES

KHAKIM D. IKRAMOV

A relationship is pointed out between the results in a recent paper of Shin's and those in a previously published paper by M.J.C. Gover.

A recent paper [**5**] by Shin deals with a special class of real symmetric tridiagonal $n \times n$ matrices, namely, with matrices $A = (a_{ij})$ such that, for all i,

$$(1) \qquad a_{ii} = b, \quad a_{2i-1,2i} = a_{2i,2i-1} = c, \quad a_{2i,2i+1} = a_{2i+1,2i} = d$$

with $cd \neq 0$, other entries a_{ij} being zero. Shin gives explicit formulas for eigenpairs of matrix (1) when the order n is odd, and implicit formulas, for n even.

The purpose of this note is to draw attention to a paper [**3**], with which Shin was obviously unfamiliar. In [**3**], the class of so-called tridiagonal 2-Toeplitz matrices is studied. These are tridiagonal matrices that satisfy the relation

$$(2) \qquad a_{i+2,j+2} = a_{ij}, \quad i, j = 1, 2, \dots, n-2.$$

[**3**, Theorem 2.3] gives explicit formulas for eigenvalues of tridiagonal 2-Toeplitz matrices of odd order. When applied to class (1), these formulas virtually coincide with formulas [**5**, (2.8a),(2.8b)]. For even n, implicit formulas for eigenvalues in [**3**, Theorem 2.4] are closely related to formulas [**5**, (2.28),(2.30)]. Finally, both papers give formulas for the entries of eigenvectors, although formulas in [**3**] and [**5**] are expressed in different terms. The techniques used by both authors in the corresponding proofs are different as well, the approach in [**3**] relying on polynomials of Chebyshev's type.

It is observed in [**3**] that applications involving 2-Toeplitz (and, more generally, r-Toeplitz matrices) are to be found in the field of sound propagation. Shin mentions another application of class (1), namely, a cubic collocation method designed for the numerical solution of a partial differential equation in [**4**].

Two concluding remarks are relevant here. First, formulas in [**3, 5**] reveal that the spectrum of Shin's matrix (1) is symmetric about b. This is caused by the fact that the main diagonal is constant in (1). Indeed, it has long been known that the spectrum

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of a symmetric tridiagonal matrix with zero main diagonal consists of symmetric pairs $(\lambda_i, -\lambda_i)$ (see, for example, [**6**, Chapter 5, Theorem 2.2]). Second, Shin's matrix of an even order is centrosymmetric. By a well-known orthogonal similarity (see [**1**] or [**2**, Section 2]), this matrix can be transformed into the direct sum of two matrices of half the order. Each of these matrices is "nearly" a Shin's matrix differing from a genuine Shin's matrix by only the entry (n, n). This makes the symmetry of the spectrum about the point b even more obvious.

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Faculty of Numerical Mathematics and Cybernetics
Moscow State University
Moscow 119899
Russia
e-mail: ikramov@cmc.msk.su