ABSTRACTS

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Über die Beziehung zwischen strikter und strenger Implikation, W. Ackermann. In this paper the author enters into the particulars of the relations of the concept of "strict" implication introduced by C. I. Lewis with the concept of "strenge" implication introduced by himself. He points out that within the system of "strenge" implication a further concept of implication may be defined which has all the qualities of strict implication. The definition is the following: A implies B in this sense if the conjunction of A and non-B is impossible, which is not the same as the "strenge" implication between A and B. The system of strict implication is taken in the form as formerly given by Arnold Schmidt.

"Cogito ergo sum"—raisonnement ou intuition?, E. W. Beth. According to Descartes, the demonstrative power of an argumentation may result either from the application of the universal rules of logic or from a particular intuition. This doctrine permits us to avoid the reproach of circularity which is often raised as an objection to certain argumentations in epistemology. On the other hand, it implies the rejection of the method of the counter-example. The acceptance of certain argumentations based on a particular intuition never creates a permanent situation; it rather constitutes a stage in the genesis of what Bernays has characterized as an *évidence acquise*.

Beobachtungssprache und theoretische Sprache, R. Carnap. Among the non-logical constants of the language of science two kinds are distinguished, the observation terms (e.g., "blue") and the theoretical terms (e.g., "electric field"). The latter terms are introduced, not by definitions, but by postulates of two kinds, theoretical postulates, e.g., basic laws of physics, and correspondence postulates which connect the theoretical terms with observation terms. As Hilbert has explained, both mathematics and theoretical physics can in this way be constructed in the form of uninterpreted calculi. It is here briefly indicated that by this method of construction also the mathematical terms have meanings (in a wider sense) assigned to them. The theoretical terms obtain at least an incomplete interpretation by means of the correspondence postulates. It is shown how the distinction between analytic and synthetic sentences can be defined also for the theoretical language.

Calculuses and formal systems, H. B. Curry. Lorenzen, in his book *Einführung in die operative Logik und Mathematik* has given a relatively precise form of syntactical system which he calls a calculus. The present paper deals with the relationship of Lorenzen's notion of calculus with the notion of formal system (as explained, for example, in *Outlines of a formalist philosophy of Mathematics*, Amsterdam, 1951). It is shown that the obs of a formal system can be represented as the theses of a calculus of a certain type just when the calculus has a property called the tectonic property, and conditions are given under which one form of system can be transformed into the other.

Über eine bisher noch nicht benützte Erweiterung des finiten Standpunktes, K. Gödel. P. Bernays has pointed out that, in order to prove the consistency of classical number theory, it is necessary to extend Hilbert's finitary standpoint by admitting certain abstract concepts in addition to the combinatorial concepts referring to symbols. The abstract concepts that so far have been used for this purpose are those of the constructive theory of ordinals and those of intuitionistic logic. It is shown that the concept of a computable function of finite simple type over the integers can be used instead, where no other procedures of constructing such functions are necessary except simple recursion by an integral variable and substitution of functions in each other (starting with trivial functions).

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On the nature of mathematical systems, R. L. GOODSTEIN. The crux of the dispute between formalism and intuitionism, it is held, is not whether certain entities exist or not, but how the term function shall be used in mathematics. The identification of effective definition with general recursion fails because an undefined function lies concealed beneath the requirement of a finite number of substitutions, and a fresh characterization of effective definition is sought in terms of a hierarchy of ordinal recursions.

A correspondence exists between primitive recursive properties and direct proofs, of irrationality and transcendence for instance, and between general recursive properties and indirect proofs.

Mathematics is a concept creating activity and the distinction between a formal mathematics devoid of meaning, at one level, and a meaningful metamathematics at the next is considered to be untenable.

Zum Einfachheitsprinzip in der Wahrscheinlichkeitsrechnung, H. HERMES. Shimony, Lehman and Kemeny recently developed a foundation of the theory of confirmation by reduction to the concept of rational betting. This procedure yields essentially only the axioms first stated by Kolmogoroff. It is well known that these are not sufficient for application. Thus it is necessary to search for a new principle if one wants to motivate new axioms. This can be done by a principle of simplicity, which expresses that the probability of a hypothesis increases with its degree of simplicity. A critical survey is given about several attempts which have been tried in Münster especially by Kiesow and W. Oberschelp with the aim to make the notion of simplicity precise. The simplicity is reduced to syntactical properties of propositions.

Blick von der intuitionistischen Warte, A. HEYTING. The paper contains remarks on intuitionism and its relations with other domains of foundational research. Inside the intuitionistic mathematics, in connection with Griss' criticism against the use of negation, different degrees of evidence are distinguished, depending upon the way in which conditioned constructions are admitted. Some difficulties in the theory of finite species are discussed. Concerning the foundational research in general it is observed that it has separated intuitive, formal and platonistic constituents in classical mathematics. Some remarks are made on Church's thesis in the theory of recursive functions.

Hilbert's programme, G. KREISEL. Hilbert's plan for understanding the concept of infinity required the elimination of non-finitist machinery from proofs of finitist assertions. The failure of the original plan leads to a hierarchy of progressively less elementary, but still constructive methods instead of finitist ones (modified Hilbert programme). A mathematical proof of this failure requires a definition of "finitist".—The paper sketches the three principal methods for the syntactic analysis of non-constructive mathematics, the resulting consistency proofs and constructive interpretations, modelled on Herbrand's theorem, and their mathematical and logical consequences. A characterization of finitist proofs is sketched. A remark on the completeness of the predicate calculus concludes the paper. Throughout open problems and alternative approaches are emphasized.

Graphschemata und rekursive Funktionen, RÓZSA PÉTER.

Relative model-completeness and the elimination of quantifiers, A. ROBINSON. Most of the early proofs of the decidability or completeness of certain mathematical theories were based on the method of eliminations of quantifiers. Various more recent results on completeness were obtained independently of such procedures. However, it is shown in the present paper that, conversely, the completeness of a mathematical theory will in certain circumstances entail the existence of an elimination method. The proof involves the application of the extended first ε-theorem of Hilbert-Bernays.

Über einige neuere Untersuchungen zur Modalitätenlogik, H. A. SCHMIDT. In the present report on some papers of the author and on a sequel to them by G. Emde, Marburg, a series of results concerning the combination of the basic modalities "possibility" and "necessity" are treated.

Starting from some very general framing-codifications, the list of the finitely many implicative modal logics with idempotent "possibility" which is obtainable through basis reduction is discussed. Besides basis reduction, in the case of some important subclasses of the not-necessarily-idempotent implicative modal logics, attention is given to the special decision problems.

Aussagenlogische Grundeigenschaften formaler Systeme, K. Schütte. In a formal system without types which contains no contradictions and is able to represent all notions of classical mathematics, all the laws of classical propositional calculus cannot be valid. From this fact ensues the problem generally to explore the properties of formal systems from the point of view of the propositional calculus. Under certain assumptions concerning the propositional expressions the "propositional completeness" and the "propositional consistency" of a formal system is characterized through the *tertium non datur*, respectively through the inference rule of the *ex falso quodlibet*. There are decision procedures for the syntactical inference rules which in every formal system are valid under the given assumptions and for the syntactical inference rules of every formal system which is complete or consistent with respect to the propositional calculus.

Dualität, E. Specker. The axiom system of plane projective geometry is dual in the sense that it is transformed into itself by exchange of the notions "point" and "line." It follows that for every theorem the dual sentence is also a theorem. However, from the duality of the axiom system one cannot conclude that in a model the truth of a sentence implies that of the dual sentence; even less can one conclude that each model admits a 1-1-transformation interchanging points and lines and preserving the incidence relation. For projective geometry, models of this kind are well known. For the simple theory of types (where duality is replaced by ambiguity of types) it is shown that the existence of such models is equivalent to the consistency of "New Foundations."

Additional remark. The following theorem answers both of the questions proposed in the paper: If it is complete, then a theory with an automorphism has a model with a corresponding automorphism. NF is therefore consistent if simple theory of type with the additional axioms $S \equiv S^*$ (in the notation of the paper) is consistent.

Eighty years of foundational studies, H. Wang. A survey is made of work since 1879 on foundational problems viewed as an analysis, by reduction and formalization, of the concepts proof, feasible, number, set, and constructivity. It is suggested that there are five domains of concepts and methods, viz., anthropologism, finitism, intuitionism, predicativism, and platonism. It is also suggested that the central problem is to characterize these domains by formalization and to determine their interrelations by different forms of reduction. Finally, the range of logic in the narrower sense is discussed, and applications of mathematical logic are briefly outlined.