PIONS IN NEUTRON STAR MATTER

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Abstract. Arguments are given for pions becoming an important constituent of neutron star matter at densities greater than a critical density lying between 5×10^{14} and 10^{15} g cm^{-3}. It is anticipated that this will lead to a substantial softening of the equation of state.

Most of this work was done in collaboration with D. Scalapino. The final conclusions are not yet firm. What we think we have established is that there will be some very interesting phenomena involving real pions in the ground state of neutron star matter. However we have looked carefully at only a few of the possible ways pions can enter, and there are many other possibilities to be considered. The game is to look for the way in which the energy is minimized and it is likely that someone will soon find a way of using pions to give even a lower energy than that of our 'ground state.'

The possibility of real pions in the ground state of superdense matter is of interest both for the possible effects on the zero temperature equation of state of the matter, and for the effects on the transport properties of the matter. If pions are present they will surely occur in a condensed mode with many pions in the same state. The strong interactions of pions will determine whether such a condensation occurs; free π^-'s would not begin replacing e^-'s in matter of density less than 10^{16} g cm^{-3} under any set of assumptions. However a π^- meson still is, *a priori*, the most likely pion, since it needs to acquire negative potential energy equal only to the pion rest energy minus the electron chemical potential.

Therefore we begin by asking what is known about π^- interactions in nuclear matter. A π^- at rest in a sea of protons and neutrons acquires a potential energy from *s*-wave pion-nucleon interactions equal to (Bethe, 1971),

$$\delta E = 219\,(\varrho_n - \varrho_p)\ \text{MeV}, \tag{1}$$

where ϱ is in fm^{-3}.

Thus a single π^- in dense neutron rich matter has an enormous positive potential energy. This has been quoted as an argument against the development of a π^- phase in neutron star matter. However if we have, for example, an equal number of protons and neutrons, the proton charge neutralized by negative pions, the potential energy from *s*-wave π^- nucleon interactions vanishes and the residual interactions will determine whether the state is energetically preferred over a pure neutron state.

A moving pion will undergo strong *p*-wave interactions in nuclear matter. The optical potential of Auerbach *et al.* (1967), the real part of which is based on low energy *p*-wave phase shifts, gives a good fit to the scattering of a π^- particle from $A = 2Z$ nuclei. We can express the effect of the real part of this potential as a modification of the dispersion relation for the pion moving in a uniform medium,

$$\omega^2 = k^2(1 - 6\varrho) + m_\pi^2. \tag{2}$$

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At densities slightly greater than nuclear, $\varrho > 0.16\,\mathrm{fm}^{-3}$, this dispersion relation exhibits a striking property: as k is increased from zero, ω decreases. Of course the scattering length approximation underlying (2) is only valid for small k and should be cut off for high k, so when ϱ is only slightly above $0.16\,\mathrm{fm}^{-3}$ we would expect a shallow dip in energy followed by a rise as k is increased. But for larger ϱ the pion energy will dip far enough for some moderate value of k to make it economical to trade an electron for a π^-; or even dip to $\omega = 0$. In the latter case the optical potential approach is inapplicable for a number of reasons, but the result is strongly suggestive of an instability toward formation of a large number of π^-'s.

Instead of pursuing the optical potential approach further we turn to an approach based directly on the Hamiltonian for emission and absorption of single pions, which is thought to underlie the p-wave πN scattering amplitudes. Solving exactly for the behavior of a single pion moving in a medium of protons and neutrons is out of the question in this approach, but the problem of many pions condensed in the same mode is much simpler and admits a solution.

For non-relativistic nucleons the Hamiltonian for the interaction of one plane wave π^- mode with momentum $k\hat{z}$ with nucleons is

$$H = H_0 + H_\pi = H_0 + \frac{ifk\sqrt{2}}{m_\pi\sqrt{2\omega_k V}} \times \\ \times \int \mathrm{d}^3x\,[\bar{\mathrm{n}}(x)\,\sigma_3 \mathrm{p}(x)]\,a(k\hat{z})\,e^{ikz} + \mathrm{H.C.}, \tag{3}$$

where $f = 1.1$; $\bar{\mathrm{n}}$ and p are the creation and annihilation operators for neutron and proton; $a(k\hat{z})$ the annihilation operator for a π^- of momentum $k\hat{z}$. H_0 is the Hamiltonian for free pions and nucleons.

If we have a pion condensation in the mode $k\hat{z}$ we can make the c-number replacement for the operators a and a^+,

$$a,\, a^+ \to \sqrt{N_\pi}\,.$$

We begin by comparing the lowest energies of the following two systems, with a fixed number of baryons in a volume V, (a) pure neutrons, and (b) neutrons, protons and a condensed pion mode, where the number of π^-'s is constrained to be equal to the number of protons.

From (3) we can see that if we can find a state for the nucleons in which

$$\langle \bar{\mathrm{n}}(x)\,\sigma_3 \mathrm{p}(x)\rangle = \mathrm{const.}\; i\varrho_{\mathrm{nucl}}\,e^{-ikz}$$

without undue sacrifice in kinetic energy, then for sufficiently large density the interaction terms in (3) will overcome the free meson energy terms and give a lower energy to the state with condensed pions. In what follows the kinetic energy sacrifice per proton in the condensed state will be seen to be of order $k^2/2\,M_{\mathrm{nucleon}}$ and to be negligible in the domains of interest.

To solve the problem we minimize $\langle H\rangle$ subject to constraint of neutrality,

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$N_p = N_\pi \equiv XN$, where N is the total number of baryons. Writing $\mathscr{H} = H_0 + H_\pi + \mu N_p$, we see that $\mathscr{H}$ can be almost diagonalized by a canonical transformation on the nucleon fields,

$$\begin{aligned} u(x) &= \sqrt{1-\theta^2}\,\mathrm{n}(x) + i\theta\sigma_3\mathrm{p}(x)\,e^{ikz}, \\ v(x) &= i\theta\mathrm{n}(x) + \sqrt{1-\theta^2}\,\sigma_3\mathrm{p}(x)\,e^{ikz}, \end{aligned} \tag{4}$$

where θ is a known function of the proton chemical potential, μ. Discarding a term (which is small and which we pick up later in perturbation theory) coming from the non-diagonalization of H_0 under the above transformation we obtain

$$\begin{aligned} \mathscr{H} \cong H_0 &+ \lambda_-(\mu) \int \bar{u}(x)\,u(\mathrm{x})\,\mathrm{d}^3x, \\ &+ \lambda_+(\mu) \int \bar{v}(x)\,v(x)\,\mathrm{d}^3x, \end{aligned} \tag{5}$$

where $\lambda_- < \lambda_+$.

The ground state is thus a sea of 'u particles'

$$|\psi_0\rangle = \prod_k^{K_F} \bar{u}_k\,|\mathrm{vac}\rangle \tag{6}$$

instead of a sea of neutrons. The parameter μ (and therefore θ) is determined by the requirement that the average charge of the baryons be given by the parameter, X, which fixes the value of the pion field,

$$\begin{aligned} \langle\psi_0|\,N_p\,|\psi_0\rangle &= NX = \theta^2 X \\ N_\pi &= NX. \end{aligned} \tag{7}$$

Although the state $|\Psi_0>$ does not have a definite electric charge, the charge fluctuations around the value given by (7) die off in the $V\to\infty$ limit sufficiently fast to make the Coulomb energy per nucleon vanish.

The energy per nucleon can now be calculated from (5) and turns out to be

$$\frac{\text{Energy}}{\text{Nucleon}} = \frac{3}{5}\frac{k_F^2}{2M_N} + X\omega_k + \frac{k^2}{2M_N} - 2\frac{\sqrt{\varrho}f}{\sqrt{\omega_k m_\pi}}\,X\sqrt{1-X}. \tag{8}$$

The first term on the right-hand side is the Fermi energy of a non-interacting neutron gas. We shall refer to the sum of the next three terms as the condensation energy. When it is negative the ground state will contain condensed pions. As the density, ϱ, is increased from nuclear densities the condensation energy becomes negative first for a particular value of k, $k = 1.2\ m_\pi$. In this case the onset of the condensation would be at $\varrho = 0.25$ baryons fm^{-3}. The onset in this model is gradual (second class phase transition) beginning at $\varrho = 0.25$ from $X = 0$ and increasing at $\varrho = 1\ \mathrm{fm}^{-3}$ to about $X = 1/2$ (one half protons).

As an example of the potential mischief which could be caused by the pion condensation we can calculate the pressure loss (relative to the pressure of the free Fermi

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gas) implied by (8). We find a 30% pressure loss at $\varrho=0.3\ \mathrm{fm}^{-3}$ and a 73% loss at $\varrho=0.5\ \mathrm{fm}^{-3}$. However, we think that it is premature to try to estimate effects on neutron star masses and radii, for reasons which should become clear in what follows.

Having given a detailed treatment of the simplest model which shows the condensation, we now give a qualitative discussion of several ways in which the model can be made more realistic, and of several outstanding problems which must be solved before a believable equation of state can be found.

The most essential extension of the model is the inclusion of a realistic nuclear force. It is not immediately obvious, for example, that the demands made on the nucleonic wave function in order to sustain the pion condensation are not completely in conflict with the demands made by the short range repulsions. However, we are spared the indignity of having to do a new nuclear matter calculation in the presence of a condensed pion field by the following extraordinarily lucky circumstance; if the two-body nucleon-nucleon force is spin and isospin independent, as the hard core repulsion and a good deal of the medium range attraction are supposed to be, then the *difference* in energies between the ground state of a pure neutron gas and that of our condensed pion state is exactly the same as we calculated for the case of no nuclear force. This is because the two-body force is invariant under the canonical transformation, (4), in the sense that it has the same form in the quasi-particle fields u and v as in the nucleon fields n and p. The additional terms in the Hamiltonian for the condensed pion case measure only the total number of u and v particles (as in [5]). So the ground state in the condensed state consists of exactly the same configuration of 'u particles' as the pure neutron case does of neutrons, the energies differing by exactly the same amount as in the case of no two-body interactions.

Thus in calculating the condensation energy we need to concern ourselves with the spin and isospin dependent part of the two-nucleon force only. Single pion exchange is the most obvious source of spin and isospin dependent terms. Accordingly we calculate the second order effects of pion emission and absorption (non-condensed pions; that is, all modes).

The wave function of the nucleons does matter for this calculation. We have considered only the case of the free Fermi sea of 'u particles.' The effect of correlations, if the short range repulsions had been taken into account, would probably have been to reduce the second order pion effects. In any event our calculation gives a positive term in the condensation energy which, if added to our earlier result, would delay onset of the condensation until a density between 0.3 baryons fm^{-3} and 0.5 baryons fm^{-3}, depending on a cutoff.

It is of interest to note that only about 60% of this effect is attributable to the one pion exchange 'potential.' There is an additional term of second order in the coupling constant of the non-condensed pions which can be described as a medium dependent self-energy effect. This kind of effect is significant in our calculation and we suspect that similar terms should be included in any high density nuclear matter calculation. They are clearly omitted in variational approaches based on phenomenological two-body forces.

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Next we briefly consider the effects of pion-nucleon and pion-pion s-wave interactions. According to (1), s-wave interactions in a pure neutron medium are very unfriendly to π^-'s, so that one would expect the small X onset of condensation to be considerably delayed or perhaps eliminated altogether in favor of a first class phase transition to an $X \approx 1/2$ state at higher density. However, with this concentration of π's we can expect $\pi^-\pi^-$ s-wave forces to be significant also. Information on this subject is purely theoretical. We have taken the formula analogous to (1) for the energy due to I$=2$ s-wave $\pi^-\pi^-$ interactions, using the scattering length of Weinberg (1966) and combined it with the πN term and the free condensation energy formula. This combination of terms turns out to lead to a second order phase transition with onset at a density of $\varrho \approx 0.36\ \mathrm{fm}^{-3}$.

Thus the pion condensation survives the inclusion of all two-body interactions which are thought to act between the different constituents of the matter. Unfortunately there are many corrections which could become important at high densities and which might favor either a condensed pion ground state or a pure neutron ground state. Three-body forces are one possible source of such effects, but a more important source of differences between the condensed state and the normal state is probably a different kind of term: the s-wave π^--nucleon interaction which underlies (1) is equivalent to a direct interaction of fields $\Psi\ \psi\ \phi^*\ \phi$. The energy of interaction was calculated by putting in the condensed pion field for ϕ and ϕ^*. If only one pion field is replaced by the condensed field the direct interaction term becomes an effective vertex for the emission or absorption of a non-condensed pion, and there will be a new set of terms in the energy coming from the emission and absorption of pions by this mechanism. The second order terms in this case clearly favor the condensation, but we have made no numerical evaluation as yet. Furthermore, if we started from a typical chiral dynamic non-linear Lagrangian for multi-pion emission from nucleons, a nightmarish collection of new terms would result.

Another complication, at densities greater than $1\ \mathrm{fm}^{-3}$, or so (depending on the equation of state), is the intrusion of species of baryon other than neutrons and protons. The Λ, Σ^- hyperons are the first to show up, and since they can communicate *via* π^- emission and absorption $\Sigma^- \to \Lambda + \pi^- \to \Sigma^-$, they should be as effective as neutrons and protons in sustaining the condensation.

Another class of questions which has not yet been answered completely is whether a single plane wave mode of π^- gives the lowest energy under all circumstances. Perhaps a standing wave mode will be preferred, or perhaps a condensate consisting of a mixture of π^+, π^- and π^0 will take over in some region in density. Let us leave out magnetic energies for the moment (later we discuss how a filamentary structure can solve the magnetic energy problems for the plane wave case) and ask which sort of solution minimizes the strong interaction energy. The following conclusions are only preliminary:

(a) A standing wave mode would have a potential onset at exactly the same baryon density as a plane wave mode, but would be slightly less favored energetically for densities somewhat above the onset density. At sufficiently high densities a standing

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wave solution apparently has a lower energy than the plane wave solution. The problem in this case cannot be solved analytically and a numerical calculation is in progress.

(b) Even near onset a state with a certain small fraction of π^+ mesons may be preferred, with the fractions of π^+'s growing as the density is increased.

(c) π^0's may develop at higher densities.

(d) There never arises a catastrophic situation in which it would lower the energy to introduce indefinitely many pions, even in the absence of any repulsive pion-pion interactions.

Pending resolution of these questions we shall say only a little about including electromagnetic interactions and building a macrostructure out of condensed pion matter. If it turns out that a plane wave structure is greatly preferred by the strong interactions, then according to our estimates filaments could be formed of diameter from 10 to 20 fm, with currents moving in opposite directions on neighboring filaments. In this configuration the lowering of energy in the condensed state due to strong interactions wins out over the positive magnetic energy and filament surface energy terms. It is not clear what the length of the filaments should be, nor how the problem of connecting the ends should be resolved. If the matter breaks up into fairly small domains, randomly oriented in space, then we could probably use our equation of state for homogeneous matter in the absence of electromagnetic interactions in order to discuss the properties of a neutron star core in the large. If on the other hand the directionality of the matter (determined by the direction of the wave vector for the condensed mode) were preserved over the entire core one would anticipate very exotic mechanical properties, as well as transport properties, for the core of the star.

If the standing wave solutions turn out to be dynamically preferred then we presumably can have a homogeneous phase, instead of the filamentary structure. The matter would still have the directionality in this case.

To summarize, we believe that we have very strong arguments for pions becoming an important constituent of superdense matter at some baryon density between 0.4 fm^{-3} and 1 fm^{-3}. We do not yet know, however, what the best configuration will be to minimize the energy, or what the exact effects will be on the equation of state. However we can anticipate a substantial softening of the equation of state, which should lead to a lower value of the maximum mass of a neutron star.

References

Auerbach, E. H., Fleming, D. M., and Sternheim, M. M.: 1967, *Phys. Rev.* **162**, 1683.
Bethe, H. A.: 1971, *Ann. Rev. Nuclear Sci.* **21**, 93.
Weinberg, S.: 1966, *Phys. Rev. Letters* **17**, 168.

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