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ARTICLE

Conspicuous leisure, time allocation, and obesity Kuznets curves†

Nathalie Mathieu-Bolh[1] and Ronald Wendner[2]

[1]Department of Economics, University of Vermont, Burlington, VT, USA
[2]Department of Economics, University of Graz, Graz, Austria
Corresponding author: Ronald Wendner; Email: ronald.wendner@uni-graz.at

Abstract
Our growth model explores the complex relationship between income, obesity, and changes in exercise-related behavior. Combining Becker's theory of time allocation (*The Economic Journal* 75(299), 493–517, 1965) with Veblen's theory of conspicuous leisure (*The Theory of the Leisure Class*, 1st ed. New York: Macmillan, 1899), we determine conditions for dynamic and static obesity Kuznets curves. Considering food consumption and exercise choices, we show that dynamic and static Kuznets curves result from the rising opportunity cost of exercise and peer influence, both increasing with income. Focusing on calorie expenditure, we investigate the rise and slowdown in obesity prevalence in the USA and the correlation between obesity and income per worker. Our numerical simulations indicate that, as the economy grows, exercise choices slow down the rise in obesity prevalence but do not generate a dynamic Kuznets curve in the USA. By contrast, they generate a static Kuznets curve for a population cross section. We discuss policy implications of our findings.

Keywords: Obesity; status; conspicuous leisure; inequality; Kuznets curve; economic development

1. Introduction

Obesity is a multifaceted problem to address as it is affected by biological, psychological, environmental, cultural, social, and economic factors. In this paper, we concentrate on the socioeconomic factors of obesity. Even under this focus, obesity remains a complex issue. Indeed, the literature review by Mathieu-Bolh (2022) suggests that the link between income and obesity is non-monotone and follows a Kuznets curve pattern. To explain this pattern, the theoretical literature has essentially focused on food consumption choices influencing calorie intake, assuming exogenous preferences. By contrast, while modeling food consumption choices simply, we focus on the specificities of physical activity choices influencing calorie expenditure, assuming changing preferences over time [Bowles (1998)]. Indeed, physical activity is an important factor influencing body weight, and the empirical literature indicates that an important aspect of physical activity is tied to choices individuals make with their leisure time, such as time spent watching TV or exercising [Harvard School of Public Health (2022)].[1] Our theory rationalizes physical activity choices in relation to income levels and explains their contribution to the Kuznets curve pattern of obesity. Our model is the first theoretical growth model that combines Becker's (1965) theory of the allocation of time and Veblen's (1899) theory of conspicuous leisure. We numerically simulate our

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model to describe obesity patterns in the USA. We discuss the fact that our model also provides a tool for policy analysis.

So far, most theoretical explanations of the link between income and obesity concentrate on the role of food consumption (calorie intake). Some contributions explain the historical rise in obesity [Dragone (2009); Dragone and Ziebarth (2017); Burke and Heiland (2007); Levy (2002, 2009); Dragone and Savorelli (2012); Strulik (2014); Mathieu-Bolh (2020, 2021)]. Other contributions explain the inverse relation between income and obesity for population cross sections in rich countries [Cutler et al. (2015); Fuchs (1982); Grossman (1972); Cawley and Ruhm (2012); Mathieu-Bolh (2021)]. Few articles provide insights on the changing link between income and obesity focusing on food consumption. For Philipson and Posner (1999), as economies develop, the change in the relation between income and obesity, from positive to negative, relates to the assumption that rich individuals exogenously care more about their health and weight than poor individuals and to complementarity between consumption and weight. Mathieu-Bolh and Wendner (2020) is the only article that includes endogenous preferences with respect to food consumption. As economies develop, the dominating income effect makes individuals increase calorie consumption. After a certain income threshold, an endogenous status effect makes individuals prefer low-calorie food over high-calorie food and decrease their calorie intake. In addition, only a few theoretical contributions explore the role of calorie expenditure in models with exogenous preferences. The static model by Yaniv et al. (2009) analyzes optimal choices made by weight conscious and weight unconscious consumers accounting for interactions between food consumption choices and physical activity. Dynamic models by Lakdawalla et al. (2005) and Lakdawalla and Philipson (2009) argue that in rich countries, people are heavier than in poor countries because technological progress leads to more sedentary work and raises the cost of physical activity. In rich countries, where work-place technology is more uniform, rich people are thinner than poor people because the authors assume that the demand for thinness is exogenously higher among rich individuals. Therefore, there is to this date no dynamic model that includes exercise choices (calorie expenditure) with endogenous preferences and food consumption choices (calorie intake) to explain the changing link between income and obesity.

We emphasize that our purpose is not to incorporate the various causes of obesity and quantify their respective effects on obesity as an empirical contribution would do. Instead, we build a theoretical model that sheds light on the specific mechanisms through which the interaction between exercise and consumption choices influences the income obesity links and can generate Kuznets curves.

Our model directly extends the theoretical literature by combining Becker's (1965) theory of the allocation of time and Veblen's (1899) theory of conspicuous leisure in a growth model. We extend Becker's (1965) theory [followed by Gossen (1983); Le Van et al. (2018); Ha-Huy et al. (2019)] in two ways. In the first place, we acknowledge that there are two types of consumption goods, those that do not require time and those that do. For tractability reasons and consistent with previous theoretical contributions [e.g. Levy (2002, 2009)], we associate consumption expenditures that do not require time to food consumption. By contrast, we associate time-consuming expenditure to exercise and distinguish between sedentary leisure and non-sedentary leisure (exercise) for proper counting of calorie expenditure. For example, the purchase of a gym or ski pass is associated to spending time exercising. Exercise choices therefore involve a double cost, the exogenous price paid for exercising (such as the cost of a pass), and the endogenous opportunity cost of exercise. The opportunity cost of exercise is the forgone wage. With capital accumulation, the equilibrium wage rises and the rising opportunity cost of exercise pushes individuals to exercise less. Assuming that exercise is a time-consuming expenditure therefore reinforces the overall cost of exercise. This mechanism is important for two reasons. First, it can help explain the empirical decrease in calorie expenditure and the rise in obesity. Second, it has implications for policy analysis as exercise choices can be influenced through two channels, consumer prices and wages.

However, accounting for the rising opportunity cost of exercise alone would yield counterfactual results with respect to two empirical facts: in population cross sections, high-income earners tend to exercise more and weight less than low-income earners; obesity prevalence declines beyond a certain income threshold as captured by the empirical obesity Kuznets curve [e.g. Clément (2017)]. Thus, in the second place, we introduce comparison utility in our model, which reflects the fact that individuals' exercise choices are influenced by their peers. The genealogy of this idea is Veblen's (1899) theory of conspicuous leisure, which we incorporate in our growth model. According to Veblen, conspicuous leisure is visible leisure in which people engage with the objective of displaying and reaching a certain status. We apply the idea of visible leisure to time-consuming exercise expenditure. Consistent with the literature on peer effects, in order to capture such comparisons, we introduce a reference level for exercise. The taste for exercise relates to the extent to which people compare themselves to the reference, described as the degree (strength) of peer influence. The implication of such a reference level is that the own marginal utility of exercising increases with the reference level, which is endogenously determined in our model. In other contexts, this type of mathematical formulation is referred to as positionality. Specifically, we extend the literature on endogenous positionality [e.g. Dioikitopoulos et al. (2019, 2020); Akay and Martinsson (2019); Mathieu-Bolh and Wendner (2020)], which establishes that the degree of peer influence changes over time due to changes in economic development or inequality. In accordance with this literature, in our model, the degree of peer influence has two components. An exogenous component reflects individual rank in society and an endogenous component is tied to capital accumulation. While exogenous differences in taste exist at all times, as the stock of capital accumulates, everyone's taste for exercise increases. As a result, ceteris paribus, individuals exercise more as the stock of capital increases. We refer to this change in the degree of peer influence as the dynamic peer effect. Additionally, consistent with the past literature on endogenous preferences [Mathieu-Bolh and Wendner (2020)], in our core framework, individuals experience disutility from weight gain but are not calorie conscious. This implies that individuals do not count calorie when they make consumption or exercise decision, which is key to maintain mathematical tractability. However, we also provide a brief extension and the main intuitions for the model with calorie conscious individuals.

The rising opportunity cost and the dynamic peer effect create a wedge between optimal exercise and food consumption choices of individuals with different socioeconomic status as economies develop. As a result, our model explains the changing link between income and obesity as a reflection of dynamic peer effects competing with the rising opportunity cost of exercise.

Empirical studies support that individuals have become increasingly influenced by peers with respect to exercise choices. While the empirical literature on leisure initially did not find evidence of positionality with respect to overall leisure time [Carlsson et al. (2007)], recent findings suggest that people have become less positional with respect to the goods they purchase, but more positional with respect to how they spend their time [Holthoff and Scheiben (2018)]. Furthermore, individuals are highly positional with respect to physical attractiveness [Solnick and Hemenway (1998)]. Cawley's (2015) literature review finds evidence of peer effects on women's weight and men's fitness. Individuals work-out to be thin or fit, which in empirical studies is linked to attractiveness [Fletcher et al. (2014)]. Additionally, the psychology literature shows that peers influence exercise behavior in adolescents [Chung et al. (2017)] and adults [Ingledew et al. (1998)]. Thus, it makes sense to assume that exercise choices are influenced by peer behavior in the same way as some healthy food choices are. The assumption that peer influence has increased is further supported by the increase in the portion of the population using social media, as well as the rise in the number of followers of sport and fitness influencers. For example, the number of Instagram users has increased from 100 million in 2010 to 800 million in 2017. The number of followers of the top 20 fitness influencers on Instagram reached almost 100 million individuals in 2017 [Influencer Marketing Hub (2021)]. Additionally, social fitness applications (such as Fitbit, Garmin Connect, Nike Run Club, PumpUp, and Stava) that track individuals exercise activities and compare them

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to their family, friends, or larger group of users have increasingly become popular in the USA. The number of health and fitness application users has increased from 62.7 million users in 2018 to 87.4 million in 2020 [Statista (2021)].

Furthermore, exercise choices differ according to income. First, individual's calorie expenditure through labor is tied to economic development. Based on the U.S. National Health and Nutrition Examination Surveys (NHANES), Church et al. (2011) show that in the early 1960s, almost half the jobs in the private industry in the USA used to require at least moderate-intensity physical activity, whereas nowadays, less than 20% of those jobs demand this level of energy expenditure. Since 1960, the estimated mean daily energy expenditure due to work-related physical activity has dropped by more than 100 calories for both women and men. While Lakdawalla et al. (2005) and Lakdawalla and Philipson (2009) have focused on calorie expenditure related to technological progress and work, we are focusing on calorie expenditure related to leisure time. Our model reflects that when individuals do not spend calories at work, they may chose to exercise during leisure time. Second, there are also important differences in time spent exercising based on income cross sections. Shuval et al. (2017) find that compared to those making less than \$20,000 per year, those with an annual income of \$75,000 or more engage in 4.6 more daily minutes of moderate to vigorous-intensity physical activity. Higher income earners also exhibit more intense, less frequent weekly patterns of physical activity. The 2008 Bureau of Labor Statistics Spotlight on Statistics covering time spent on sports and exercise provides insights on differences in the practice of exercise between different income groups. The study indicates the percentage of adults 25 and older who engage in those activities between 2003 and 2006, according to their educational attainment (see Figure A.13.1 in the appendix). It shows that 10% of people with less than a high-school diploma engage in those activities, while 23% of individuals with a bachelor's degree or higher engage in those activities.

If exercise choices were solely the mirror of calories spent at work, there would be no difference between overall calorie expenditure of individuals with low-income strenuous jobs and high-income sedentary jobs. Thus, exercise choices must also be the result of other factors, such as peer effects. Indeed, peer effects seem to be concentrated on high-income earners as highlighted by Western et al. (2021), who find that digital technologies targeting physical activities are not effective on individuals with low socioeconomic status but make individuals with high economic status more active.

Our main theoretical results are as follows. We determine the conditions for the existence of both a dynamic and a static Kuznets curve for obesity. A dynamic Kuznets curve describes the evolution of body weight over time as the economy grows. A static Kuznets curve describes body weight as a function of income for population cross sections. Recall that in the empirical literature, the Kuznets curve is initially presented as a dynamic relation between economic development and obesity. However, empirical studies solely demonstrate the existence of a static Kuznets curve for population cross sections [Clément (2017); Grecu and Rotthoff (2015)] or cross-country analysis [Windarti et al. (2019); Deuchert et al.'s (2014)]. To show the change in the relation between income and obesity over time, Clément (2017) resorts to two separate cross-sectional analyses covering two different time periods in China. Table 1 [Mathieu-Bolh (2022)] summarizes the results of the empirical literature. It shows the regression coefficients for the BMI, overweight, or obesity on income and income squared. A positive coefficient on income, together with a negative coefficient on income squared, implies an inverted U-shaped relationship between income and the outcome variable (obesity).

By contrast, our theoretical model generates both a dynamic and a static obesity Kuznets curve, which complements the empirical literature on this topic, and is a new concept in the theoretical literature. Our results differ from Mathieu-Bolh and Wendner (2020) who do not describe the two Kuznets curves, ignore the role of exercise choices in describing obesity patterns that arise from a different mechanism, and do not provide numerical results.

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Table 1. Obesity Kuznets curve main estimates

	Income	Income^2	Income	Income^2	Income	Income^2
	BMI		Overweight		Obesity	
Deuchert et al. (2014)	3.904*	−0.693*	0.328*	−0.058*	0.138*	−0.025*
	(0.552)	(0.134)	(0.042)	(0.010)	(0.027)	(0.006)
Grecu and Rotthoff (2015)	0.158	−0.052**			2.088*	−0.351*
	(0.152)	(0.024)			(0.741)	(0.114)
Clément (2017)	1.479*	−0.082*				
	(−0.397)	(−0.024)				
Windarti et al. (2019)			3.567*	−0.221*	2.840*	−0.151*
			(0.938)	(0.053)	(0.605)	(0.035)

*$p < 0.01$, **$p < 0.05$, standard deviation in parenthesis.

We provide a novel explanation for the dynamic and static Kuznets curves. First, the difference between the growth rates of consumption and exercise reflects two competing effects with economic development: both the opportunity cost of exercise and peer influence for exercise increases. The former (latter) renders the difference between the growth rate of food consumption and exercise larger (lower) over time.[2] Second, we formally demonstrate that there is a level of capital per worker for which the growth rate of exercise starts exceeding the growth rate of food consumption. Therefore, in a steady state, both the opportunity cost of exercise and peer influence increase with the steady-state stock of capital up to a certain threshold, beyond which the correlation between steady-state body weight and the stock of capital per worker is negative. This is one factor explaining the negative correlation between body weight and income for high income levels per worker in cross-sectional analyses of individuals or countries with different incomes per worker.

Furthermore, we show that in the presence of dynamic peer effects, for high levels of economic development, body weight gain becomes negative as the economy develops over time. In the same way, in the presence of dynamic peer effects, our steady-state analysis shows that there is a level of per capita income beyond which the link between body weight and steady-state capital stock is negative. Last, we show that accounting for calorie consciousness reinforces the choice of exercise over consumption and is likely to result in lower equilibrium body weight. However, accounting for calorie consciousness does not alter the fundamental driving forces of obesity tied to peer effects and opportunity costs.

We supplement the qualitative analysis with a quantitative analysis relying on numerical simulations. We use a standard calibration procedure, which consists in matching the model's steady-state equilibrium characteristics with the long-term characteristics of the actual US economy. We derive the optimal simulated paths toward the steady state. Our simulations confirm the existence of two different relations between body weight and the stock of capital per worker. First, our results are consistent with data on body weight evolution in the USA as the simulated economy shows that the dynamic evolution of average body weight has been monotonous. In other words, given the current degree of peer influence, choices regarding calorie expenditure are not sufficient to generate a *dynamic* Kuznets curve pattern for obesity in the USA. By contrast, they generate a *static* Kuznets curve for the USA: the steady-state level of average body weight increases with the average stock of capital up to a level of 187 pounds, corresponding to a stock of capital per worker 25% higher than its baseline, and decreases thereafter. Second, we conduct a sensitivity analysis. It highlights that the dynamic relation between weight and the per worker capital stock does not exhibit a dynamic Kuznets curve pattern for a wide range of elasticities of substitution between food consumption and effective exercise. For very high values of the degree

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of peer influence, we obtain a simulated dynamic obesity Kuznets curve, which confirms the role of the dynamic peer effect in limiting and potentially inverting body weight growth.

The rest of the paper is structured as follows. In Section 2, we present the model and provide analytical results. In Section 3, we provide numerical results. In Section 4, we conclude and discuss policy implications of our work.

2. Model

2.1. The economy

We build a continuous time dynamic general equilibrium model for a closed economy in which the capital accumulation technology exhibits decreasing returns. There is a large number of firms and households, the respective number of which is normalized to unity. Households derive utility from two types of consumption: food consumption that does not require time, and exercise-related consumption that requires time and is affected by peer effects. In our base model, households maximize utility subject to an intertemporal budget constraint, and body weight is the result of optimal food and exercise choices. We discuss the case when the effect of net calorie intake on body weight gain is endogenized in Section 3.3. In what follows, the time index t is suppressed, unless needed for clarity.

2.1.1. Time-consuming consumption

First, the representative individual distinguishes between two types of goods, those that do not require time, C (food consumption) and those that require time, X (exercise). Second, we introduce endogenous labor in the model. Individuals are endowed with one unit of time, used for endogenous labor N, endogenous exercise X, and exogenous sedentary leisure $\bar{S}$ (such as sleeping or watching television). As a consequence, we consider the time constraint:

$$1 = N + X + \bar{S}\,. \tag{1}$$

To simplify the notation, we write that the amount of time that is not spent on sedentary leisure $\bar{L} = 1 - \bar{S}$, such that:

$$\bar{L} = N + X \tag{2}$$

As a consequence, an individual's flow budget constraint accounts for both work time and consumption expenditure related to exercise:

$$\dot{K} = rK + w\left(\bar{L} - X\right) - p_C C - p_X X\,, \quad p_X, p_C > 0\,, \tag{3}$$

where w is the wage rate, r denotes the interest rate, and p_C and p_X are the respective *exogenous* prices of C and X.

2.1.2. Peer influence

Individuals are influenced by peers when choosing to exercise. We introduce a reference level of exercise, $\bar{X}$, to take into account such peer influence. In our model, the impact of the reference level of exercise is captured by *effective* exercise $\hat{X}$, which differs from absolute exercise X according to the standard subtractive specification [Ljungqvist and Uhlig (2000)]:[3]

$$\hat{X} = X - \varepsilon\left(k\right)\bar{X}\,, \quad 0 \leq \varepsilon\left(k\right) \leq 1, \tag{4}$$

where $k \equiv K/N$ denotes aggregate wealth per unit of labor (roughly, capital per worker), which is exogenous for an individual. The reference level $\bar{X} \equiv X/1$ is given by average exercise expenses

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(aggregate exercise expenses, with the population size equaling unity). It is endogenously determined in our model, but it is exogenous from an individual's point of view (as indicated by the upper bar). The degree (strength) of peer influence is captured by function $\varepsilon\left(k\right)$. When $\varepsilon\left(k\right)=0$, individuals are not influenced by peers, and effective exercise equals absolute exercise. When $\varepsilon\left(k\right)$ is high, effective exercise is low, and the marginal utility of X is high. The formulation of peer effects is similar to Mathieu-Bolh and Wendner (2020) in the sense that it includes an exogenous and an endogenous element. We use the following standard functional form to describe the degree of peer influence:

$$\varepsilon(k) = 1 - e^{-\kappa k}, \quad \kappa > 0. \tag{5}$$

The static element κ yields the property called the static peer effect. In our model, it means that, given the stock of capital k, the higher the parameter κ, the more an individual cares about exercise due to the higher reference level of exercise. The dynamic element yields the property that we call the dynamic peer effect. It captures that a higher stock of capital—either built over time due to economic growth or observed at a given point in time among different countries or population subgroups—endogenously increases peer effects :

$$\frac{\partial \varepsilon(k)}{\partial k} > 0\,. \tag{6}$$

Tying peer effects to economic development means that over time, as a country develops, individuals become on average more influenced by peers' exercise behavior, ceteris paribus.

2.1.3. Body weight

The choices of C and X have an impact on body weight change $\dot{W}$. We connect weight gain to net energy intake, the difference between energy intake and expenditure [described in a general manner by Schofield (1985)]. Energy intake is a function of food consumption. We introduce a modification in the Schofield equation to take into consideration the role of exercise, work, and sedentary leisure. Recall that the usual Schofield equation is $\dot{W} = \lambda_C C - \lambda_W W$, where the parameter $\lambda_C > 0$ represents the energy density of food (measured in joules per unit of food consumed), and $\lambda_W > 0$ reflects a metabolic rate (measured in joules per unit of weight). In this expression, calorie expenditure is a fixed proportion λ_W of body weight W. Implicitly, calorie expenditure is measured for a unit of time of one, which can be one year or one day, and each type of activity during this unit of time exerts the same amount of calories. By contrast, in our model, we take into consideration that individuals allocate one unit of time to activities that exert different amounts of calories. This unit of time is spent in exogenous sedentary leisure $\bar{S}$, endogenous exercise X, and labor N. Therefore, we rewrite the Schofield equation as

$$\dot{W} = \lambda_C C - \frac{\left(\lambda_S \bar{S} + \lambda_X X + \lambda_N N\right)}{N} W. \tag{7}$$

In this expression, $\lambda_S \bar{S}$ represents the basal metabolic rate (BMR), which is basic energy expenditure to maintain the functioning of a body at rest. The terms $\lambda_X X$ and $\lambda_N N$, with $\lambda_X > 0$ and $\lambda_N > 0$, denote the extent to which time spent on exercise and labor reduces net energy. Note that λ_C is a rate in front of the variable C. In the same way, the term in front of W is expressed as a rate. For that reason, $\lambda_S \bar{S} + \lambda_X X + \lambda_N N$ is divided by N (since W is expressed in total terms). While sedentary leisure $\bar{S}$ is fixed and proportional to weight, calorie expenditure associated to non-sedentary leisure and work $\lambda_X X + \lambda_N N$ vary with individuals' choices. Since $N = 1 - \bar{S} - X$, it is straightforward that when individuals choose to exercise more, they spend relatively fewer calories at work.

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2.1.4. Preferences and optimal choices

Individuals face a constrained intertemporal optimization problem that is described as follows. Instantaneous utility is given by

$$U(C, \hat{X}, W) = u(C, \hat{X}) - v\left[\left(W - W^I\right)^2\right]. \tag{8}$$

Utility positively depends on food consumption and exercise. An exercise reference level $\bar{X}$ increases marginal utility of exercise. Weight is not a choice variable, and conspicuous behavior is solely reflected in individuals' exercise choices. We account for the fact that disutility is caused by body weight in excess or below a reference weight, $W^I \in \mathbb{R}_+$, which could, for example, be an ideal weight from a health perspective. There are two ways to interpret this formulation. Considering that individual and average weight are equal *in equilibrium*, this formulation can denote obesity-related externalities and justifies considering policy interventions discussed in the conclusion. Alternatively, this formulation can denote that individuals do not internalize internalities related to being overweight. Individuals are in that case boundedly rational or "weight unconscious" [e.g. Mathieu-Bolh and Wendner (2020); Mathieu-Bolh (2021); Yaniv et al. (2009)]. Boundedly rational choices connect to the vast literature on biased health choices that have roots in psychological biases or informational problems [lack of information or cost of acquiring or processing information, e.g. Chetty et al. (2009)].[4]

Assumption 1. *The sub-utility function $u : \mathbb{R}^2_+ \to \mathbb{R}$ is twice continuously differentiable, increasing in both arguments and strictly concave. Moreover,* $\lim_{C\downarrow 0} u_C = \lim_{\hat{X}\downarrow 0} u_{\hat{X}} = \infty$, *and* $u(C, \hat{X}) \leq q\, \rho\, e^{\beta\rho t}$, *where $q \in \mathbb{R}_+$ and $\beta < 1$. The sub-utility function $v : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable, increasing and strictly convex in its argument $(W - W^I)^2$. Moreover,* $v\left[\left(W - W^I\right)^2\right] \leq q_w\, \rho\, e^{\beta_w \rho t}$, *where $q_w \in \mathbb{R}_+$ and $\beta_w < 1$.*

Notice that the limit conditions prevent the individual from not consuming or exercising at all, while the latter conditions imply boundedness of intertemporal utility. The boundedness condition on function v is satisfied if v is an increasing function, and there exists a maximum weight.

The intertemporal utility function is

$$\int_{t=0}^{\infty} U(C, \hat{X}, W) e^{-\rho\tau} d\tau\,, \tag{9}$$

where $\rho > 0$ is the constant rate of time preference. Assumption 1 implies boundedness of the utility integral.[5]

Given the DOP, $\varepsilon(\bar{k})$, individuals choose C and $\hat{X}$, to maximize (9), subject to their initial endowment of wealth $K_0 > 0$, their flow budget constraint [combining (4) and (3)]:

$$\dot{K} = rK + w\bar{L} - \hat{p}_X \varepsilon\left(\bar{k}\right)\bar{X} - p_C C - \hat{p}_X \hat{X}\,, \tag{10}$$

and a No-Ponzi-Game (NPG) constraint:

$$\lim_{\tau\to\infty} e^{-R(t,\tau)} K \geqslant 0\,, \tag{11}$$

where $R(t, \tau) = \int_t^\tau r(v)dv$ represents the interest factor, and:

$$\hat{p}_X \equiv w + p_X. \tag{12}$$

Price $\hat{p}_X$ represents the total cost of effective exercise that includes the cost of exercise expenditure and the opportunity cost of exercise.

We solve the model (see Section A.1) applying Pontryagin's maximum principle (Pontryagin *et al.*, 1962). We deduce the following expressions for the growth rates of food consumption and

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exercise. Noticing that in equilibrium, average exercise expenditure equals individual's exercise expenditure, $\bar{X} = X$, and combining the optimality conditions and the relation between effective and actual exercise (4), we obtain the growth rates of consumption and exercise as (see Section A.2):

$$\frac{\dot{C}}{C} = \Omega^C\left(C, \hat{X}\right)(r - \rho) + \Phi^C\left(C, \hat{X}\right)\left(\Delta\frac{\dot{w}}{w}\right), \tag{13}$$

$$\frac{\dot{X}}{X} = \Omega^X\left(C, \hat{X}\right)(r - \rho) + \frac{\dot{\varepsilon}\left(k\right)}{1 - \varepsilon\left(k\right)} - \Phi^X\left(C, \hat{X}\right)\left(\Delta\frac{\dot{w}}{w}\right), \tag{14}$$

where $0 < \Delta \equiv \frac{w}{w+p_X} < 1$, and Ω^C, Ω^X, Φ^C, Φ^X are elasticities defined in Section A.2.

The difference between the growth rate of food consumption and the growth rate of exercise expenditure is therefore expressed as

$$\frac{\dot{C}}{C} - \frac{\dot{X}}{X} = \underbrace{\left[\Omega^C\left(C, \hat{X}\right) - \Omega^X\left(C, \hat{X}\right)\right](r - \rho)}_{\text{ECE}} + \underbrace{\left[\Phi^X\left(C, \hat{X}\right) + \Phi^C\left(C, \hat{X}\right)\right]\left(\Delta\frac{\dot{w}}{w}\right)}_{\text{ROC}} - \underbrace{\frac{\dot{\varepsilon}\left(k\right)}{1 - \varepsilon\left(k\right)}}_{\text{DPE}}. \tag{15}$$

The growth rate of food consumption differs from the growth rate of exercise expenditure due to three terms. The first term represents elasticities for consumption and exercise (ECE). It includes two elasticities, respectively, $\Omega^C\left(C, \hat{X}\right)$ and $\Omega^X\left(C, \hat{X}\right)$. The second term represents the rising opportunity cost of exercise (ROC), and the third term represents the dynamic peer effect (DPE). Note that accounting for the fact that exercise is a consumption expenditure influences the ROC through two channels. One channel operates through $\Phi^C\left(C, \hat{X}\right) < 0$, which denotes that exercise is an expenditure using up resources that cannot be allocated to food consumption. Other things equal, it decreases the growth rate of food consumption and limits the ROC. The other channel operates through p_X and lowers $\Delta \equiv \frac{w}{w+p_X}$. Other things equal, it decreases the importance of ROC related to wage growth. The ultimate impact involves general equilibrium effects that will be accounted for in the next sections.

Proposition 1. *For general forms of utility functions, under Assumption 1, the difference between the growth rate of consumption and the growth rate of exercise can be positive or negative as the DPE is an offsetting force to the ROC.*

Proof. See Section A.3. □

The term ECE is neither specific to our assumption that exercise is both a consumption and a time expenditure nor to our assumption that exercise is affected by peer behavior. By contrast, the terms DPE and ROC are specific to our model and are present for all utility's functional forms. For that reason, in what follows, we will focus on the roles of DPE and ROC. On their own, those terms cause a wedge between the growth rates of consumption and exercise and generate the pattern for body weight gain. This mechanism is at the core of our results and holds for all forms of utility functions.

It is worth noticing a fundamental difference between the mechanisms driving the income obesity relation in our model compared to Mathieu-Bolh and Wendner (2020). In their contribution, the driver of obesity is an income effect, which would be captured by the term $\Omega^C\left(C, \hat{X}\right)(r - \rho)$

in (15), competing with a DPE with respect to low calorie food consumption instead of exercise. By contrast, in our model, the income effect is present only for some specific forms of utility functions. For example, the term ECE disappears when utility is a CES function.

2.2. Firms

A unit mass of competitive firms produces a homogeneous output Y. The production process is described by a Cobb–Douglas production function: $Y = F(K, N) = K^{\eta} N^{1-\eta}$, where $0 < \eta < 1$ denotes the capital elasticity of production. Factors are paid their respective marginal products:

$$\frac{\partial F(K,N)}{\partial K} = r + \delta, \quad \frac{\partial F(K,N)}{\partial N} = w\,, \tag{16}$$

where $\delta \geq 0$ denotes the rate of depreciation. We introduce a normalization (per unit of labor): $y \equiv Y/N$. Noting that $k = K/N$, by homogeneity of degree 1, we can write:

$$y = F(k, 1) \equiv f(k) = k^{\eta}\,, \quad 0 < \eta < 1. \tag{17}$$

The first-order conditions (16) then become:

$$r(k) = f'(k) - \delta\,, \quad w(k) = f(k) - kf'(k)\,. \tag{18}$$

A linear production process transforms total consumption into food consumption and exercise consumption. Output can be used for either investment I or consumption such that $Y = p_C C + p_X X + I$.

2.3. Equilibrium

Definition 1 (*Equilibrium*). A competitive equilibrium is a price vector (r, w, p_C, p_X) and an attainable allocation for all $t \geq 0$, such that:

1. Individuals choose feasible streams of C, X, K, to maximize intertemporal utility, given the stream of price vectors, initial wealth endowments, the DOP, and average capital.
2. Firms choose K and N in order to maximize profits, given the price vector.
3. All markets clear. Specifically: $\dot{K} = Y - p_C C - p_X X - \delta K$, the goods market clears; the capital market clears, and $N = \bar{L} - X$, the labor market clears.
4. Reference levels are $\bar{X} = X$.

To study stability and ultimately enable comparisons between economies or population cross sections, we express the dynamic system in per unit of labor terms, such that $c \equiv C/N$, $x \equiv X/N$, and $\mathrm{w} \equiv W/N$. Furthermore, we express the dynamic system as functions of x and k (see Section A.4 for details):

$$\frac{\dot{k}}{k} = \left[1 - x\left[\kappa k - \Phi^X\left(x, k\right)\eta\Delta(k)\right]\right]^{-1} \left[x\,\Omega^X\left(x, k\right)(f'(k) - \delta - \rho) + \frac{f(k)}{k} - \delta - \frac{1}{k}\left(p_C c(x, k) + p_X x\right)\right], \tag{19}$$

$$\frac{\dot{x}}{x} = (1 + x)\left[\Omega^X\left(x, k\right)\left(f'(k) - \delta - \rho\right) + \left(\kappa k - \Phi^X\left(x, k\right)\Delta(k)\,\eta\right)\frac{\dot{k}}{k}\right]. \tag{20}$$

The macroeconomic equilibrium gives rise to a dynamic system in two dimensions: k and x. Indeed, the dynamic system is separable in w, since the dynamic variables k and x affect body

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weight w, but w does not affect k and x, which is to be expected since weight is not a decision variable.[6] The dynamic system in normalized variables is given by (19)–(20): $\dot{k} = \dot{k}(k, x)$; $\dot{x} = \dot{x}(k, x)$.

We separately determine the change in body weight per unit of labor over time and steady-state body weight. The normalized Schofield equation, also expressed with normalized variables, reads (see Section A.5):

$$\frac{\dot{\mathrm{w}}}{\mathrm{w}} = \lambda_C \frac{c(x,k)}{\mathrm{w}} - \left(\lambda_S \bar{s}\,(x) + \lambda_X x + \lambda_N\right) + \Omega^X\left(x,k\right)\left(f'(k) - \delta - \rho\right)$$

$$+ \left(\kappa k - \Phi^X\left(x,k\right)\eta\Delta(k)\right)\frac{\dot{k}}{k}, \tag{21}$$

where consumption $c(x, k)$ is derived from the intratemporal optimality condition (ratio of (A1) and (A2) in the appendix), and solely depends on k and x, and where $\bar{s} = \frac{\bar{S}}{N}$. Since N is endogenous, and depends on X, which depends on x, we have $\bar{s} = \bar{s}\,(x)$.

A steady-state equilibrium is an equilibrium for which $\dot{k} = \dot{x} = 0$. Let k^* denote the steady-state value of k, and $x^* = x\left(k^*\right)$, $w^* = w(k^*)$, $\hat{p}^*_X = p_X + w^*$, and $c^* = c(x^*, k^*)$. Considering the dynamic system (19)–(20), the steady state is described by the following system (see Section A.8):

$$k^* = f'^{-1}(\delta + \rho)\,; \tag{22}$$

$$x^* = \frac{f(k^*) - \delta k^*}{p_X + p_C c(x^*, k^*)/x^*}\,, \tag{23}$$

where $f'^{-1}(.)$ denotes the inverse function of $f'(.)$. Equation (22) follows from our model equivalent of the Keynes–Ramsey rule [equation (20)]. The term $c(x^*, k^*)/x^*$ is implicitly given by dividing (A1) by (A2), which gives $U_C(C, \hat{X})/U_{\hat{X}}(C, \hat{X}) = p_C/\hat{p}_X(k)$. In case $u(C, \hat{X})$ is homothetic, the left-hand side is a function of $(C/\hat{X}) = c/[x(1-\varepsilon(k)]$. By strict concavity, an inverse function exists ($U_C/U_{\hat{X}}$ as a function of $C/\hat{X}$ is one to one). In this case, c^*/x^* is a function of k^*, allowing us to express x^* *explicitly*.[7]

As a consequence, neither our static nor our dynamic positionality effect (DPE) impact the steady-state level of k. Equation (23) determines the steady-state share of output to be devoted to exercise, which increases with the degree of peer influence $\varepsilon(k^*)$. As $f'(k)$ is a strictly monotonous function, there exists only one value k^* satisfying (22). Therefore, the steady state $\left(k^*, x^*\right)$ is unique. Similarly to the standard neoclassical growth model, since there is one predetermined and one jump variable, the unique steady state is a saddle point. As a consequence, for low initial levels of capital $k_0 > 0$, on the stable arm of the saddle, the stock of capital increases monotonically toward the steady state.

Once x^* and c^* are determined, we deduce the steady-state body weight per unit of labor w*:

$$\mathrm{w}^* = \left(\frac{\lambda_C c(x^*, k^*)}{\lambda_S \bar{s}\,(x^*) + \lambda_X x^* + \lambda_N}\right). \tag{24}$$

As the dynamic system is separable in w (that is, w is affected both by x and k but not vice versa), a stationary state (x^*, k^*) does not imply that $\mathrm{w} = \mathrm{w}^*$. However, as can be easily verified, if $\mathrm{w} \gtrless \mathrm{w}^*$ along a stationary state (x^*, k^*), then $\dot{\mathrm{w}} \lesseqgtr 0$, and $\lim_{t\to\infty} \mathrm{w} = \mathrm{w}^*$. Consequently, the pattern of the dynamics of c/x along transitional paths impacts on the weight change $\dot{\mathrm{w}}$, though we may experience periods for which c/x declines, while weight is still increasing, and we may experience periods for which c/x is constant while the weight is declining. That is, w is lagging behind the dynamic pattern of c/x.[8] The behavior of weight, in our model, gives rise to a rich dynamics depending on both the initial weight and the dynamics of the ratio c/x, following the rise in k on the transitional path.

3. Main theoretical results

In the theoretical section, in order to derive straightforward analytical results, we employ a CES utility function [such as (A5)]. Considering that the intratemporal elasticity of substitution plays no qualitative role specific to the model, we assume $\zeta = 0$. In this case, where $1/\gamma$ is the intertemporal elasticity of substitution and $\alpha > 0$ represents the taste for food consumption relative to effective exercise consumption, $\Omega^C = \Omega^X = 1/\gamma$, $\Phi^C = -(1-\alpha)(1-\gamma)/\gamma$, $\Phi^X = (1-\alpha(1-\gamma))/\gamma > 0$ and $\Phi^X + \Phi^C = 1$. Recall that it means that the term ECE equals zero and only the effects specific to our model, DPE and ROC, remain in place. In the steady state, the intertemporal elasticity of substitution plays no role either, so γ can take any value without altering our results. We complement our theoretical results with a numerical section, in which we present dynamic and steady-state quantitative results for a range of intra and intertemporal elasticities of substitution with CES utility (see Section 4).

3.1. Dynamic obesity Kuznets curve

In what follows, we show that the two opposing effects, DPE and ROC, can produce a dynamic Kuznets curve pattern for obesity. We also explain the puzzling fact that high-income earners may increase exercise expenditure despite its rising opportunity cost.

Using the functional form (5), in normalized variables, the DPE and ROC are re-expressed as

$$\text{DPE} = \frac{\dot{\varepsilon}\left(k\right)}{1-\varepsilon\left(k\right)} = \kappa \dot{k}; \tag{25}$$

$$\text{ROC} = \left(\underbrace{\Phi^X + \Phi^C}_{=1}\right) \Delta(k)\frac{\dot{w}}{w} = \frac{1}{\hat{p}_X}\left(-kf''(k)\right)\dot{k}. \tag{26}$$

Considering (15), the proof of Proposition 1, and that $\frac{\dot{X}}{X} - \frac{\dot{C}}{C} = \frac{\dot{x}}{x} - \frac{\dot{c}}{c}$, we have:

$$\frac{\dot{c}}{c} - \frac{\dot{x}}{x} = \underbrace{\frac{-kf''(k)}{p_X + w}\dot{k}}_{\text{ROC}} - \underbrace{\kappa\dot{k}}_{\text{DPE}}. \tag{27}$$

Specifically, for $\dot{k} > 0$,

$$\frac{\dot{c}(t)}{c(t)} - \frac{\dot{x}(t)}{x(t)} \gtreqless 0 \Longleftrightarrow \frac{-kf''(k))}{p_X + w} \gtreqless \kappa \Longleftrightarrow \underbrace{\frac{(1-\eta)\eta}{k^{1-\eta}\left[p_X + (1-\eta)k^\eta\right]}}_{\equiv h(k)} \gtreqless \kappa. \tag{28}$$

We now shed light on the difference between the growth rates of food consumption c and exercise x in equilibrium. To this end, we restrict our study to the case of a growing economy. Recall that k_0 is the initial capital stock at $t = 0$, and $k^* = [\eta/\left(\delta+\rho\right)]^{1/(1-\eta)}$ is the steady-state level of capital (as $t \to \infty$).

Assumption 2. *There is a date $\hat{t} > 0$ for which there is a stock of capital $\hat{k}$ such that $k_0 < \hat{k} < k^*$.*

Proposition 2. *Consider Assumption 2. In a growing economy, $\dot{k} > 0$, $\lim_{t\to 0} h(k) > \kappa$, thus, $\lim_{t\to 0} \frac{\dot{c}}{c} - \frac{\dot{x}}{x} > 0$. As $\hat{t} > 0$, ROC > DPE and $\frac{\dot{c}}{c} - \frac{\dot{x}}{x} > 0$ for $t \in [0, \hat{t})$. Moreover, $\lim_{t\to\infty} h(k) < \kappa$, thus, $\lim_{t\to\infty} \frac{\dot{c}}{c} - \frac{\dot{x}}{x} < 0$. As $\hat{t} < \infty$ (as $\kappa > 0$), ROC < DPE and, as long as $\dot{k} > 0$, $\frac{\dot{c}}{c} - \frac{\dot{x}}{x} < 0$ for $t \in (\hat{t}, \infty)$.*

Proof. See Section A.6. □

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Proposition 2 shows that this assumption implies that, over time, the sign of the difference $\frac{\dot{c}(t)}{c(t)} - \frac{\dot{x}(t)}{x(t)}$ changes from positive to negative. Assumption 2 restricts the parameters of our model $(k_0, p_X, \delta, \eta, \kappa, \rho)$ such that (i) k_0 is "low," i.e., a higher income raises net-calorie intake and (ii) k^* is "high" so that consumption grows by less than effective exercise, i.e., net calorie intake is negative. One way to think about Assumption 2 is that for all points in parameter space $(k_0, p_X, \delta, \eta, \rho)$, there exists a $\kappa > 0$ such that Assumption 2 is satisfied.

Below, we show that the pattern of the difference $\frac{\dot{c}(t)}{c(t)} - \frac{\dot{x}(t)}{x(t)}$ along the transitional path impacts the evolution of weight and gives rise to a dynamic obesity Kuznets curve (unless the initial weight is *very* high).

Assumption 3. *On a transitional path, the change in the ratio (c/x) dominates the changes in x and w on the development of the weight term.*[9]

Assumption 3 poses a natural restriction on the development of weight. It is not the change in exercising, x, (alone) that determines the change in weight, it rather is the change in the ratio of calorie intake to calorie expenditure, (c/x), that impacts the change in weight. In a growing economy $(\dot{k} > 0)$, both c and x increase. In light of Proposition 2, for $t < \hat{t}$, the ratio (c/x) increases, and so does the weight. However, for $t > \hat{t}$, the ratio (c/x) decreases, while x increases. The assumption puts an upper bound on the rise of x. Equipped with Assumptions 2 and 3, we can now establish Proposition 3.

Proposition 3. *Consider Assumptions 2 and 3. In a growing economy ($\dot{k} > 0$), for $W(0)$ not too high, $\dot{W} > 0$ for $t \in [t_0, \hat{t}]$. For $t > \hat{t}$, there exists a date $\hat{t}_w > \hat{t}$ such that $\dot{W} < 0$ for $t > \hat{t}_w$.*

Proof. See Section A.7. □

Proposition 3 helps us explain the evolution of obesity that goes with economic development as coming from a change in preferences connected to social status. The intuition is that peer effects with respect to exercise are higher in the future than in the present. Consequently, ceteris paribus, the marginal utility of X increases over time. In response, individuals shift X from the present to the future, which implies a higher growth rate of exercise than in the absence of peer effects. For low levels of economic development, Proposition 2 predicts that the growth rate of c exceeds the growth rate of x (note that $\frac{\dot{X}}{X} - \frac{\dot{C}}{C} = \frac{\dot{x}}{x} - \frac{\dot{c}}{c}$). As a consequence, the ratio c/x and body weight increase, as shown by Proposition 3. At some point of economic development $\hat{t}$, as the stock of capital per unit of labor exceeds a certain threshold $\hat{k}$, Proposition 2 predicts that the growth rate of x exceeds the growth rate of c. As a consequence, the ratio c/x decreases, as shown in Proposition 3. Since the evolution of body weight is tied to the evolution of c/x, even if lagged, it is easy to show that the growth rate of body weight eventually decreases. Therefore, Propositions 2 and 3 provide an explanation for the empirically estimated dynamic obesity Kuznets curve.

More specifically, Proposition 3 requires $W(0)$ not to be too high. The intuition is obtained from the Schofield equation. If $W(0)$ is too high relative to C and X, there exists an initial period for which $\dot{W} < 0$. Next, suppose that $W(0)$ is less than this level. Then, for $t \in [t_0, \hat{t}]$, $\dot{W} > 0$, as (c/x) increases in this period and so does the weight term under Assumption 3. For $t > \hat{t}$, weight continues to increase until, at $t = \hat{t}_w$, the ratio (c/x) becomes so small that the Schofield equation requires weight to decrease. If $W(t) > W^*$ along a (roughly) stationary state (x^*, k^*), $\dot{W} < 0$, and weight asymptotically converges to its steady-state level. Assuming that the initial weight is not too high is consistent with empirical observations that average body weight is relatively low in counties with low levels of economic development compared to countries with high levels of economic development.

Corollary 1. *In a growing economy, if we ignore the DPE ($\kappa = 0$), $\dot{x}/x < \dot{c}/c$, for all t.*

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Proof. Straight from (27) and Proposition 1, in the absence of the DPE ($\kappa = 0$), the difference $\frac{\dot{x}}{x} - \frac{\dot{c}}{c}$ is always negative, so that exercise expenses grow at a slower pace than food consumption. □

A consequence of Corollary 1 is that without the DPE, the ratio c/x and body weight would never decrease. This would yield two counterfactual results. It would imply that exercise expenses always grow at a lower pace than food consumption, and that obesity always rises and never exhibits a Kuznets curve along a transitional path.[10]

It is not possible to explicitly isolate the effect of the ROC on the results because for two reasons. First, one ingredient of the ROC is the equilibrium wage. The equilibrium wage rises endogenously as capital accumulates. It is therefore not possible in our model to eliminate the rise in the opportunity cost of leisure. Second, the other ingredient of the ROC is the price p_X that individuals pay for leisure. If we were to set $p_X = 0$, this would decrease the relative cost of exercise and the coefficient Δ, directly increasing the ROC ceteris paribus. It would also modify the optimal choice of exercise relative to food consumption and therefore influence capital accumulation, and the marginal product of labor, which indirectly influences both DPE and ROC. Therefore, it is not possible to derive analytical results from setting $p_X = 0$. However, we study the effects of relaxing the assumption of $p_X > 0$ on the dynamic Kuznets curve in the numerical section.

3.2. Static obesity Kuznets curve

In what follows, we draw a parallel between mechanisms operating for the dynamic obesity Kuznets curve and the static obesity Kuznets curve. The following comparative static analysis can be considered a cross-sectional analysis, comparing countries or individuals with different steady-state wealths k^*. It can also be considered a comparative static analysis of a single country for which a change in a technology or preference parameter causes a change in k^*. In contrast to the dynamic Kuznets curve, we study how steady-state weight varies with different steady-state values of k.

To compare steady states, we totally differentiate expression $c^* = \alpha/(1 - \alpha)\left(\hat{p}_X^*/p_C\right)\left(1 - \varepsilon(k^*)\right) x^*$ with respect to k^*, so we study the DPE and ROC associated with $dk^* > 0$ (see Section A.9 for details):

$$ROC^* = \Delta\left(k^*\right)\frac{dw^*}{w^*} = h(k^*),$$

$$DPE^* = \frac{d\varepsilon(k^*)}{\left(1 - \varepsilon(k^*)\right)} = \kappa,$$

where $dw^* = \partial w/\partial k\,|_{k=k^*}\, dk^*$, and $d\varepsilon(k^*) = \partial\varepsilon/\partial k\,|_{k=k^*}\, dk^*$. The difference between the change of c^* and the change of x^* upon a rise in k^* is given by

$$\frac{dc^*/c^* - dx^*/x^*}{dk^*/k^*} = \frac{1}{dk^*/k^*}\left[\underbrace{h(k^*)}_{\text{ROC}^*} - \underbrace{\kappa}_{\text{DPE}^*}\right], \tag{29}$$

where ROC* represents the higher opportunity cost of time spent on exercise that goes with a higher marginal product of labor associated with a higher steady-state capital stock, and DPE* represents the dynamic peer effect that denotes the increase in peer effects associated with a higher steady-state capital stock. The terms ROC* and DPE* are the steady-state equivalents of the terms ROC and DPE presented in the dynamic setting.

For our functional specification, $k^* = [\eta/(\delta + \rho)]^{1/(1-\eta)}$, with $0 < \delta, \eta, \rho < 1$. We make the following assumption:

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Assumption 4. $\eta > \delta + \rho$.

Assumption 4 plausibly implies that $k^*(\delta, \eta, \rho)$ is an increasing function of the output elasticity η. Specifically, $k^*(\delta, 0, \rho) = 0$, $\partial k^*/\partial \eta > 0$, and $\lim_{\eta \uparrow 1} k^* = \infty$. Thus, under Assumption 4, $k^* \in (0, \infty)$ for $\eta \in (0, 1)$. Now, consider $h(k^*)$. We already established that $h'(k^*) < 0$. It is easy to verify that $\lim_{k^* \downarrow 0} h(k^*) = \infty$. Likewise, $\lim_{k^* \uparrow \infty} h(k^*) = 0$. Thus, under Assumption 3, for $k^* \in (0, \infty)$, $h(k^*) \in (0, \infty)$.

As $k^* = k^*(\delta, \eta, \rho)$, we know from (28) that $h(k^*) = h^*(p_X, \delta, \eta, \rho)$. As η varies between (0, 1), h^* varies between $(0, \infty)$. However, h^* also declines in p_X and rises in $(\delta + \rho)$. Thus, for every $\kappa > 0$, there always exist parameter combinations for which $h^*(p_X, \delta, \eta, \rho) = \kappa$. This parametric condition implies a specific steady-state level of capital, $\hat{k}^*$. Clearly, for $k^* < \hat{k}^*$, $h^* > \kappa$, and for $k^* > \hat{k}^*$, $h^* < \kappa$. From (29), we deduce Proposition 4, which is the steady-state equivalent of Proposition 2.

Proposition 4. *Under Assumption 4, let* $\underline{k}^* < \hat{k}^* < \overline{k}^*$ *across sections, with* $\underline{k}^*, \overline{k}^* \in \mathbb{R}_+$. *If* $k^* = \hat{k}^*$,*then* $\frac{dx^*}{x^*} = \frac{dc^*}{c^*}$. *For* $k^* \in (\underline{k}^*, \hat{k}^*)$, $h(k^*) > \kappa$, *DPE* < ROC* and* $\frac{dx^*}{x^*} < \frac{dc^*}{c^*}$, *for* $dk^* > 0$. *For* $k^* \in (\hat{k}^*, \overline{k}^*)$, $h(k^*) < \kappa$, *DPE* > ROC*, and* $\frac{dx^*}{x^*} > \frac{dc^*}{c^*}$, *for* $dk^* > 0$.

Proof. See Section A.10. □

Proposition 4 shows that the sign of ROC* − DPE* becomes negative beyond a certain threshold of steady-state level of capital per worker. The result very much depends on the behavior of ROC*, as DPE* is a constant. We note that ROC* depends negatively on p_X, as the resource opportunity cost (foregone wages times the share of w to $(p_X + w)$) declines in the "fixed cost" p_X.

Corollary 2. *If we ignore the fact that exercise is a type of consumption expenditure, that is,* $p_X = 0$, *ROC* becomes larger. Consequently,* $\hat{k}^*$ *is larger* ($\hat{k}^*|_{p_X=0} > \hat{k}^*|_{p_X>0}$), *and the space* $(k_0, \hat{k}^*)$ *for which ROC* > DPE* and* $\frac{dc^*}{c^*} > \frac{dx^*}{x^*}$ *becomes larger.*

Proof. Straightforward from Proposition 4 and the term ROC* $= h(k^*)$. It is straightforward to show that $\hat{k}^*$, which is obtained from $h(\hat{k}^*) = \kappa$, decreases in p_X: $d\hat{k}^*/dp_X = -\hat{k}^*/\left[\left(\left(\hat{k}^*\right)^{\eta} + p_X\right)(1-\eta)\right] < 0$. □

The corollary shows that the higher p_X the lower $\hat{k}^*$ and the larger the range of k^* for which $\frac{dx^*}{x^*} > \frac{dc^*}{c^*}$. A necessary condition for a static obesity Kuznets curve to occur is $\frac{dx^*}{x^*} > \frac{dc^*}{c^*}$. Therefore, taking into account that exercise is a type of consumption expenditure (i.e., $p_X > 0$ rather than $p_X = 0$) shrinks the space $(k_0, \hat{k}^*)$ for which $\frac{dx^*}{x^*} < \frac{dc^*}{c^*}$ and widens the range of k^* for which a static Kuznets curve is obtained.

The sign of $(h(k^*) - \kappa)$ impacts the steady-state weights across sections, as $\frac{dx^*}{x^*} > \frac{dc^*}{c^*}$ is a necessary condition for the weight to decline.

The comparison of body weight for different steady-state wealth levels is obtained by totally deriving body weight (see (A16) in the appendix) with respect to k^*, which yields:

$$\frac{dw^*}{dk^*} = \frac{\lambda_C c^*/x^*}{z^*}\left[\frac{\frac{d(c^*/x^*)}{c^*/x^*} - \frac{dz^*}{z^*}}{dk^*}\right] \tag{30}$$

where $z\left(k^*\right) = \lambda_S \bar{s}\left(x^*\right)/x^* + \lambda_X + \lambda_N/x^*$ represents total calorie expenditure divided by exercise expenditure. Expressed in terms of elasticities, we obtain:

$$\frac{dw^*/w^*}{dk^*/k^*} = \left[\frac{\frac{d(c^*/x^*)}{c^*/x^*} - \frac{dz^*}{z^*}}{dk^*/k^*}\right],$$

where we consider that $\frac{\lambda_C c^*/x^*}{z^*} = w^*$.

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The first term on the right-hand side in the numerator of (30) represents the percentage change of net calorie intake. The second term is the percentage change of total calorie expenditure relative to exercise expenditure. When exercise time x^* increases, $dz^* < 0$. Thus, a necessary condition for steady-state weight to decrease across sections is $d\left(c^*/x^*\right)/dk^* < 0$.

As we consider $dk^* > 0$, our model implies $dw^*/dk^* > 0$ for $k^* < \hat{k}^*$. However, for $k^* > \hat{k}^*$, our model implies two opposing effects on the steady-state weight. For $k^* > \hat{k}^*$, (c^*/x^*) decreases for $dk^* > 0$. At the same time, z^* also decreases. To gain more insight, we rewrite the above total derivative as follows:

$$\frac{dw^*/w^*}{dk^*/k^*} = \left[\frac{\left(h(k^*) - \kappa\right) - dz^*/z^*}{dk^*/k^*}\right].$$

Assumption 5.

(i) There exists a $\tilde{k}^* > \hat{k}^*$ *such that* $\left(h(\tilde{k}^*) - \kappa\right) = dz^*/z^*$.

(ii) For $k^* > \tilde{k}^*$, *the decline in* $\left(h(k^*) - \kappa\right)$ *is larger than the change (decline) in* dz^*/z^*:

$$\left|\frac{d(c/x)^*/(c/x)^*}{dk^*}\right| = \left|\frac{h(k^*) - \kappa}{dk^*}\right| > \left|\frac{dz^*/z^*}{dk^*}\right|.$$

Proposition 5. *Under Assumption 5, a static Kuznets curve exists.*

Proof. See Section A.11. □

Intuitively, recall that the steady state $k^* \in \mathbb{R}_+$ (by, e.g., varying parameter η). As $\left(h(k^*) - \kappa\right) > 0$ for $k^* < \hat{k}^*$ by definition, and $dz^*/z^* < 0$, condition (i) of the assumption can only be satisfied for a $\tilde{k}^* > \hat{k}^*$. This condition also excludes the case for which $-dz^*/z^* > \kappa$ for all k^*. It requires that the degree of peer influence is large enough. Condition (ii) of the assumption imposes a monotonicity condition: $h(k^*) - \kappa$ is required to decline by more than dz^*/z^* for $k^* > \tilde{k}^*$. In this case $dw^*/dk^* < 0$ for all $k^* > \tilde{k}^*$. While Assumption 5 might sound restrictive at first sight, it is always satisfied for the standard utility specifications employed in the numerical simulations (see below).

Corollary 3. *If we ignore the DPE* ($\kappa = 0$), as $h(k^*) > 0$, it follows that* $\frac{dw^*}{dk^*} > 0$.

Proof. Straightforward from Proposition 4, $h(k^*) \underbrace{-\frac{dz^*}{z^*}}_{(+)} > 0 \Rightarrow \frac{dw^*}{dk^*} > 0$. □

If $\kappa = 0$, then no static obesity Kuznets occurs. The DPE* generates substitution toward exercise and drives the negative correlation between the stock of capital and body weight for high values of the steady-state capital stock. In the absence of the DPE*, for high values of the steady-state capital stock, the main effect is the ROC* that unambiguously results in a decrease of exercise and a higher steady-state body weight. Therefore, the introduction of dynamic peer effects helps explain why, despite a higher opportunity cost of exercise, rich individuals may exercise more than poor individuals.

With Proposition 3, we showed that the evolution of the stock of capital over time generates ROC and DPE, eventually yielding a dynamic obesity Kuznets curve. With Proposition 4, we explain how differences in ROC* and DPE* are tied to different steady-state capital stocks between poor and rich countries (or individuals). Propositions 4 and 5 are consistent with evidence presented in the introduction that $dw^*/dk^* > 0$ for poor countries (or individuals) and $dw^*/dk^* < 0$ for rich countries (or individuals) and suggests that dynamic peer effects associated

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with the consumption of time consuming goods plays a larger role for rich than poor countries (or individuals).

3.3. Calorie consciousness

In this section, we study the behavior of individuals who are both weight conscious and calorie conscious. Individuals internalize the net effect of calorie intake on weight gain [equation (7)], accounting for the fact that labor and exercise choices are endogenous [equation (1)] and that exercise choices are influenced by peer effects [equation (4)]. The formulation of the problem is presented in Section A.12.

Proposition 6. *Assuming* $\frac{1}{p_C}\lambda_C > \frac{1}{\hat{p}_X}\left[\frac{W}{N}(\lambda_N - \lambda_X)\right]$, *when overweight individuals are calorie conscious, the ratio* $C/\hat{X}$ *is lower than if they are calorie unconscious.*

Proof. See Section A.12. □

As expected, calorie consciousness reinforces the choice of exercise over consumption and results in lower equilibrium body weight. To understand the assumption and gain intuition, consider an individual for whom $W > W^I$, that is, for whom a gain in weight reduces utility (parallel arguments apply for the opposite case). Notice that the term $\frac{W}{N}(\lambda_N - \lambda_X)$ describes the weight change caused by an increase of $\hat{X}$ by one unit, as seen in (A18). While we typically expect this term to be negative, in principal it can take on either positive or negative values. The term λ_C describes the weight gain caused by an increase of C by one unit. Shifting expenses by reducing C by one dollar and increasing $\hat{X}$ by one dollar results in a reduction of C by $1/p_C$ units and an increase of $\hat{X}$ by $1/\hat{p}_X$ units. Thus, the assumption in Proposition 6 necessitates that the weight loss resulting from reducing C by one dollar must surpass the weight change incurred by increasing $\hat{X}$ by one dollar.[11] Weight consciousness leads to an increase in utility through this shift in expenses, consequently lowering the value of $C/\hat{X}$. With calorie consciousness $C/\hat{X}$ is consistently lower for every k compared to when calorie consciousness is absent. This also holds true for the steady state: since calorie consciousness does not impact the steady-state capital stock [as seen in equation (A21)], and $C/\hat{X}$ is lower, the steady-state weight is consequently reduced under calorie consciousness.

The Kuznets curve continues to reflect the competing rise in opportunity cost of exercise and peer effects. Calorie consciousness may expedite the transition from a positive to a negative income–obesity relationship or push the Kuznets curve downward.

4. Numerical results

We supplement the qualitative analysis with a quantitative analysis relying on numerical simulations. Our numerical simulations illustrate our theoretical results and give a sense of magnitudes regarding the role of peer influenced on exercise expenditures for the steady state and dynamic Kuznets curves.[12] We use data from the American time use survey by the Bureau of Labor Statistics, the U.S. Department of Health and Human Services, the Consumer Expenditure Survey, and the work by Valero-Elizondo et al. (2016), Turnovsky (2000), Barro and Sala-i-Martin (2003), and Burda and Wyplosz (2017) to calibrate the model. We simulate steady-state body weight for different steady-state levels of the stock of capital per unit of labor to explore the possibility of a static Kuznets curve for obesity in the USA. We also study the evolution of body weight toward its steady state as the stock of capital increases over time to study the possibility of a dynamic obesity Kuznets curve in the USA. Additionally, we conduct sensitivity analysis for the static and dynamic Kuznets curves.

4.1. Calibration

First, we set parameters for the Schofield equation. We use data on time use, calories spent exercising, and BMR to calibrate the parameters of the Schofield equation.

An individual spends about 2000 hours per year working (40 hours times 50 weeks) out of 8736 total hours in a year (24 hours time 7 days times 52 weeks). The time that is not spent working represents 6736 hours of leisure time ($8736 - 2000 = 6736$) and is split between sedentary leisure and exercise. With exercise time representing only 122 hours (20 minutes times 365 days divided by 60), sedentary leisure represents 6614 hours ($6736 - 122 = 6614$). As a result, sedentary leisure represents a fraction equal to $\bar{S} = 6614/8736 = 75.7\%$ of total time, and time that is not spent on sedentary leisure represents a fraction of $\bar{L} = 1 - 0.757 = 24.3\%$ of total time. This time is split between exercise, which represents a fraction equal to $122/8736 = 1.4\%$ of total time, and work, which represents a fraction $N = 2000/8736 = 22.9\%$ of total time. The average of men and women average weight in the USA is 185 pounds. On average, an individual at rest spends $\lambda_S \bar{S} W = 1577.5$ calories per day. As a result, $\lambda_S = 1577.5/(0.757 * 185) = 11.26$.

Based on the yearly American time use survey by the Bureau of Labor Statistics (2023), time spent "participating in sports, exercise and recreation," measured since 2003, represents 0.33 hours (20 minutes) a day for the average individual. How many calories do those activities burn? The 2008 Bureau of Labor Statistics Spotlight on Statistics indicates that the three most popular types of exercise are walking, weightlifting, and using cardiovascular equipment (see Figure A.13.2 in the appendix). For a 185 pound individual, we estimate that these activities, respectively, burn 2.0, 1.4, and 4.7 calories per pound per hour. The calories burnt are taken from the chart provided by Harvard Medical School (2021) and activities specifically correspond to walking 4 miles per hour, general weight lifting, and high impact step (or vigorous rowing). We estimate that calories burnt by an individual splitting their exercise time among those three activities, weighted by their popularity, equals 2.5 calories per pound per hour. The details of our calculation are provided in Table A.13.1 in the appendix. Thus, an average individual of 185 pounds spends $2.5 * 185 = 462.5$ calories per hour. Since individuals exercise 20 minutes a day, they spend $\lambda_X X W = 154$ calories per day. As a result, $\lambda_X = 154/(0.014 * 185) = 59.5$.

Based on the U.S. Department of Health and Human Services and U.S. Department of Agriculture (2015), we use the average of daily caloric expenditure of men and women, which equals $((1600 + 2400)/2 + (2000 + 3000)/2)/2 = 2250$. With 1577.5 calories spent in sedentary leisure and 154 calories spent exercising, calories spent at work are $\lambda_N N W = 2250 - 1577.5 - 154 = 518.5$. As a result, $\lambda_N = 518.5/(0.23 * 185) = 12.2$.

In order to estimate parameter λ_C (energy density of food), we use the Schofield equation, written for a stationary body weight: $\lambda_C = \frac{(\lambda_S \bar{S} + \lambda_X X + \lambda_N N) W/N}{C} = \frac{2250}{4} = 562.5$ per unit of food expenditure per day. As explained above, the numerator represents total calorie expenditure per day. The denominator represents the quantity of food consumed per day, which is between 3 and 5 pounds. We take 4 pounds for the baseline calibration. As a result, parameter λ_C represents energy intake per unit of food expenditure per day. Note that there is no distinction between men and women or individuals of different ages within households in the per quintile food consumption data.

We need a consistent estimate for food and exercise, so we use cost per day. To estimate the price of exercise, we use the work by Valero-Elizondo et al. (2016), who estimate the marginal benefit of exercising (which, in equilibrium, equals the marginal cost) equal to \$2500 per year. We divide it by 365 days and by the time a person spends exercising daily to obtain p_X, the price per day. Note that the marginal benefit (or cost) of exercising is high as it represents more than the sole expenditure on exercise-related activities or goods in that case. It also encompasses additional benefits in the form of savings on health care expenditure due to better health outcomes. So it overstates the direct cost of goods and services connected to calorie expenditure.

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Table 2. Parameters

λ_X	λ_N	λ_S	λ_C	$\bar{S}$	p_X	p_C	ρ	δ	η	α	κ	ζ	γ
59.5	12.2	11.26	562.5	0.757	20.76	1.54	0.05	0.05	0.3	0.6	0.1	0	1

Table 3. Actual and simulated economies

	k/y	$(\hat{p_X}\, X)/(p_C\, C)$	DOP	C/Y
Actual US economy	3	1.1	0.4	Between 0.10 and 0.65
Simulated US economy	3	1	0.4	0.28

The average amount spent on food since 2003 is \$2,251 per person year. We obtain this number as follows. Based on the Consumer Expenditure Survey, aggregate food consumption expenditure per year equals 776,647 millions of dollars. The share of aggregate food expenditure coming from middle income households is 17.8%. The average number of consumer units in the middle-income category is 24,560 thousands and the number of person per consumer unit is 2.5. As a result, 776,647,000,000 * 17.8%/24,560,000/2.5 = \$2,251 per year per person. We divide this number by 365 days and by the quantity of food a person consumes in a day (4 pounds) to obtain p_C, the price per pounds.

The model is simulated using a production function with a unit elasticity of substitution between consumption and effective exercise ($\zeta = 0$) and a unit intertemporal elasticity of substitution ($\gamma = 1$). We estimate the remaining yearly parameters. They are consistent with the standard range of parameters of theoretical growth models [cf. Turnovsky (2000); Barro and Sala-i-Martin (2003)] and are adjusted to reproduce some essential features of the US economy. We set the production elasticity of capital η, the rate of depreciation δ, and the rate of time preference ρ to obtain a capital output ratio of 3 [e.g. Burda and Wyplosz (2017, p. 64)].

Parameter α, which represents the taste for consumption relative to leisure, and parameter κ, which represents the exogenous component of the degree of peer influence modify steady-state peer effects, consumption, calorie expenditure, and body weight. However, neither are directly observable. We set those parameters to reflect the following empirical observations. Our calibration generates a degree of peer influence close to 0.4, which is consistent with experimental estimates,[13] and a ratio of exercise related expenditure over food expenditure close to one.

Additionally, the food consumption-income ratio is 10% in the USA, and total consumption in output is about 65%. Because our model has only one good, which is food consumption, if we set parameters to reproduce a consumption output ratio of 10%, it would mean that most resources are allocated to investment. The economy and food consumption would grow too fast compared to reality.

At the same time, if we set parameters to reproduce a consumption output ratio of 65%, food consumption and its effect on body weight would largely be overstated. With our parameters, we obtain a ratio of for food consumption in total output between 10 and 65%. Parameter values are summarized in Table 2.

Table 3 shows the fit between actual and simulated steady-state economies.

4.2. Baseline scenario

First, we build the curve that connects steady-state body weight to the steady-state capital stock. Recall that in our model, peer effects have an endogenous component tied to the level of capital per unit of labor. In the steady state, the stock of capital is given by exogenous parameters, which are the rates of time preference, depreciation, and the production elasticity of capital. These

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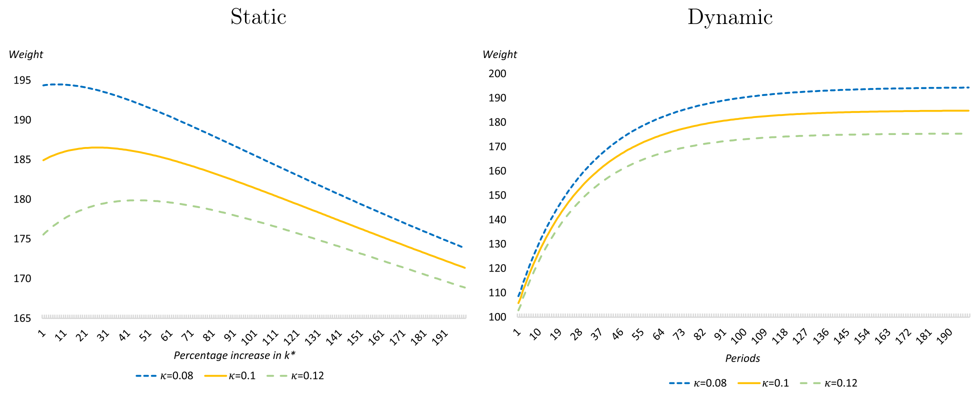

Figure 1. Body weight and capital stock with different degrees of peer influence.

parameters have an impact on the steady-state capital stock k^* and steady-state weight. We simulate the steady-state weights by directly changing k^*, going from the baseline steady-state capital stock to two times its value.[14] Since peer effects also have an exogenous component κ related to idiosyncratic differences between individuals or countries, we provide simulations for three different levels of κ, at the baseline value of 0.10 and close to it. The static relation between obesity and the stock of capital per unit of labor is presented on the left graph in Figure 1. Second, we build the curve showing the evolution of body weight over time toward the current steady state. The dynamic relation between obesity and the stock of capital per unit of labor is presented on the right graph in Figure 1.

The current steady-state body weight for the USA is 185 pounds, which is the starting point for the static relation between body weight and the stock of capital per unit of labor in our baseline scenario. We find the existence of a static Kuznets curve: for the baseline level of κ, the steady-state level of average body weight increases with the average stock of capital up to a level of 186.5 pounds, corresponding to a stock of capital per unit of labor 25% higher than its the baseline and decreases thereafter. The first interpretation of this finding is that the US economy's tipping point of the Kuznets curve is not yet reached. It will be reached once the stock of capital per unit of labor is 25% higher than its baseline in the simulations.

The second interpretation of this result is that the steady-state average body weight starts being inversely related to wealth when individual wealth is 25% above the average wealth in the USA and is positively related to wealth below this threshold. Furthermore, when the parameter κ is higher than in the baseline, DPE* is relatively stronger than ROC*. As a consequence, as expected, steady-state body weight is lower, and the obesity Kuznets curve is even more concave as individuals choose to exercise more. Thus, our simulations also show that exogenous differences in the degree of peer influence between individuals or countries may yield different Kuznets curves.

By contrast, the dynamic evolution of body weight towards its steady-state value of 185 points does not show a dynamic Kuznets curve pattern for body weight in our baseline scenario. It means that given the level of κ, the model shows that the increase in the capital stock has so far generated a DPE that has resulted in slowing down the growth rate of body weight but that it has not been sufficient to produce a dynamic obesity Kuznets curve. For both the static and the dynamic relations, the higher κ, the more prevalent peer effects, the lower the level of steady-state body weight.

The existence of a static Kuznets curve is consistent with results of empirical studies for the USA presented in the introduction. Given the exogenous degree of peer influence κ, the stock of capital per unit of labor would need to be 25% higher for the DPE* to be large enough and steady-state body weight to decrease. The absence of the Kuznets curve to this date is also consistent with

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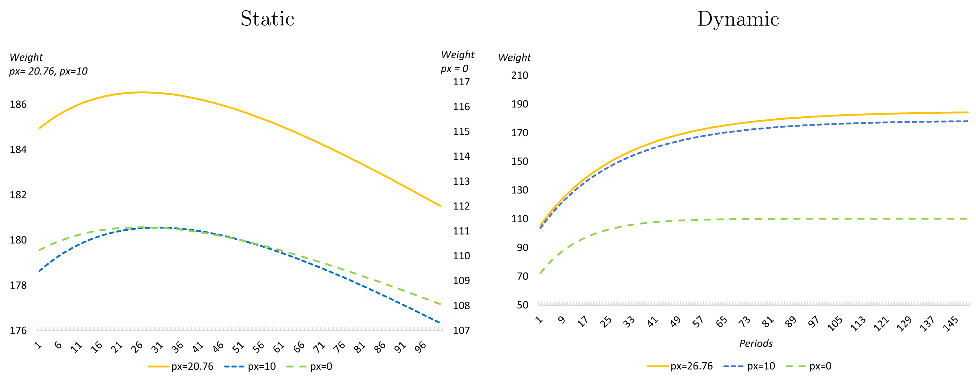

Figure 2. Body weight and capital stock with different consumer prices for exercise.

US data showing that obesity has increased over time and its growth rate has slowed down (see Figure A.13.3 in the appendix). The DPE may explain the slowdown of the evolution of obesity in the USA as individuals who are influenced by their peers have allocated more time toward exercise as the economy developed. However, this influence has so far been insufficient on its own to produce a decrease in average weight gain.

4.3. Sensitivity analysis

In this subsection, we first show the effect of considering exercise as a consumption expenditure on the static and dynamic Kuznets curves by varying the cost of exercise expenditure, the elasticity of substitution between consumption and effective exercise, the intertemporal elasticity of substitution, and the degree of peer influence.

4.3.1. Exercise as a consumption expenditure

We simulate the static and dynamic relations between body weight and the stock of capital per unit of labor with the price of consumption expenditure ranging from 0 to 20.76. The results are presented in Figure 2.

When the price of exercise expenditure decreases, the static and dynamic Kuznets curves shift down. The reason is that a lower price of exercise lowers ROC* thereby encouraging exercise. The increase in calorie expenditure results in lower body weight for all levels of the stock of capital.

4.3.2. Elasticity of substitution between consumption and effective exercise

Recall that the baseline scenario corresponds to a unit elasticity, with parameter $\zeta = 0$. We simulate the static and dynamic relations between body weight and the stock of capital per unit of labor with elasticities of substitution between consumption and effective leisure ranging from 0.66 to 2 (corresponding to ζ ranging from -0.5 to 0.5). The results are presented in Figure 3.

When the elasticity of substitution between food consumption and effective exercise becomes larger than in the baseline scenario and equal to 2, with $\zeta = 0.5$, then, as the stock of capital becomes higher in the steady state, individuals substitute food consumption for exercise more easily than in the baseline with unit elasticity of substitution. As a consequence, the relation between weight and the capital stock is decreasing as the steady-state stock of capital becomes higher. In this case, the relation between steady-state weight and the capital stock is not a Kuznets curve. In other words, the DPE* dominates the ROC* for all steady-state levels of capital. In contrast, when the elasticity of substitution between food consumption and effective exercise becomes smaller

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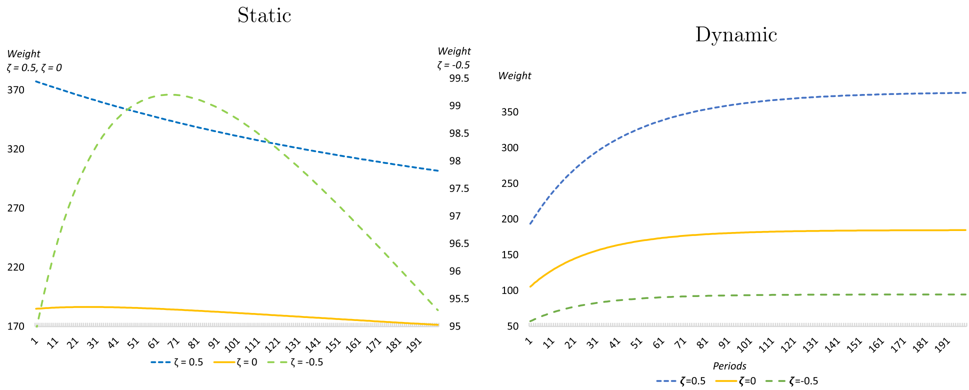

Figure 3. Body weight and capital stock with different elasticities of substitution between food consumption and effective exercise.

than in the baseline scenario and equal to 0.66, with $\zeta = -0.5$, individuals substitute food consumption for exercise less easily as the stock of capital becomes higher in the steady state. The relation between steady-state weight and the capital stock is a Kuznets curve. In this case, until a certain level of steady-state stock of capital per unit of labor, the ROC* dominates and weight increases. Beyond this level, the DPE* dominates, and weight decreases as the steady-state stock of capital becomes higher. The static Kuznets curve is also present in the baseline case (solid curve) but less pronounced than for the lower CES with $\zeta = -0.5$.

The dynamic relation between weight and the capital stock does not exhibit a Kuznets curve pattern for a wide range of elasticities of substitution between food consumption and effective exercise. For both the static and dynamic relations, the more food consumption and effective exercise are complements, the more prevalent peer effects, the lower the level of steady-state body weight.

To our knowledge, there is no empirical estimate of the elasticity of substitution between food consumption and exercise. If food consumption and exercise are substitutes, we should see a decrease in steady-state weight happening much earlier that in our baseline. If we consider that when people exercise, they also eat more, then food consumption and effective exercise may be complements and the relation between steady-state weight and stock of capital per unit of labor could exhibit a Kuznets curve. However, the model predicts that with an elasticity of substitution of 0.66, the tipping point of the Kuznets curve would happen at a steady-state stock of capital 67% higher and a body weight of 344 pounds, which is a more pessimistic scenario than the baseline scenario.

4.3.3. Intertemporal elasticity of substitution

Recall that the baseline scenario corresponds to an intertemporal elasticity of substitution equal to one with parameter $\gamma = 1$. We simulate the relation between body weight and the stock of capital per unit of labor with intertemporal elasticities of substitution ranging from 0.33 to 1 (corresponding to γ ranging from 3 to one). By definition, the intertemporal elasticity of substitution does not affect the steady state, but it modifies the dynamic evolution of weight as the stock of capital builds up. The results are presented in Figure 4.

When the intertemporal elasticity of substitution becomes smaller, from 1 to 0.33 (as γ goes from 1 to 3), the dynamic relation between weight and the stock of capital per unit of labor becomes flatter. As γ increases, the effect of the ROC on the difference between the growth rate of exercise and the growth rate of food consumption, $\frac{\dot{x}}{x} - \frac{\dot{c}}{c}$, becomes smaller, and the effect of

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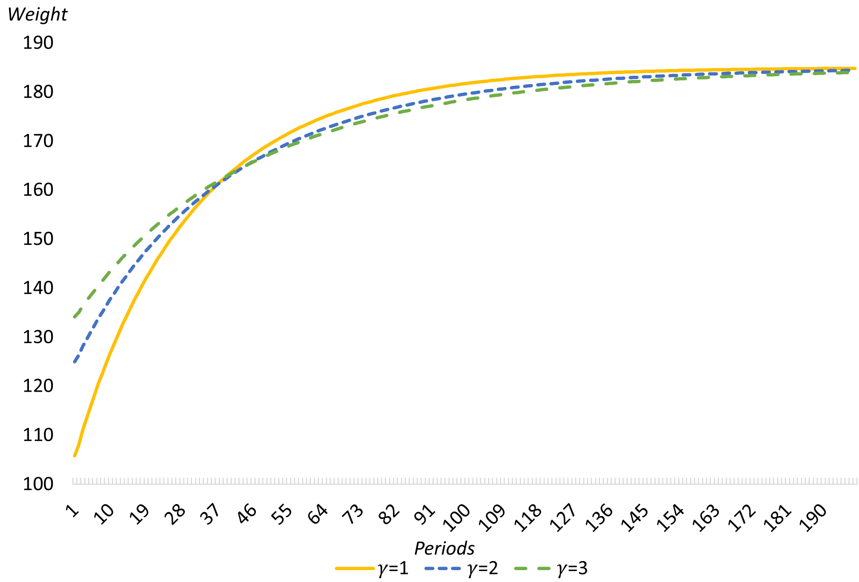

Figure 4. Dynamic of body weight with different intertemporal elasticities of substitution.

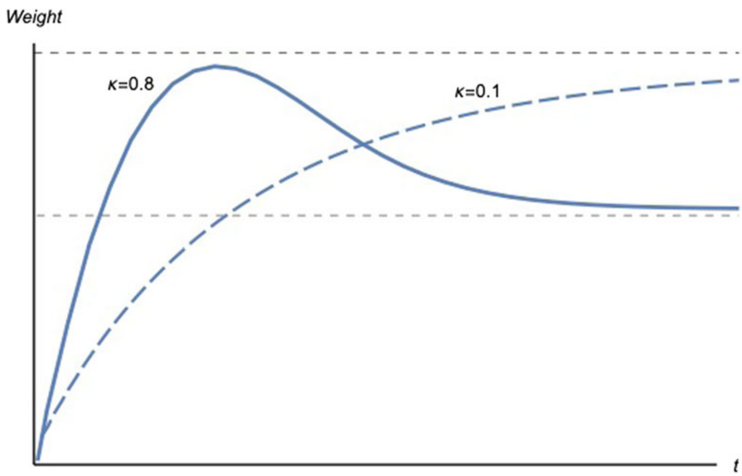

Figure 5. Dynamic of body weight gain with low and high degrees of peer influence.

the DPE becomes dominant, explaining that the relation between obesity and capital becomes less and less positive. As the DPE become dominant, individuals postpone net calorie intake to the future, which flattens the evolution of body weight.

4.3.4. High degree of peer influence

While in the baseline scenario, the Kuznets curve is generated for low values of κ matching empirical estimates of peer effects, we further explore the role of a higher degree of peer influence in generating a dynamic Kuznets curve and present it in Figure 5. The reason for this is that there is significant uncertainty regarding empirical estimates of peer effects. We simulate the evolution of body weight toward its steady state as the stock of capital increases over time for parameter

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values of κ equal to 0.1 and 0.8. In that range of values for κ, there are very large differences in the level of body weight: the higher κ, the lower the body weight. Thus, we use two different scales corresponding to the two different κ to present the level of body weight. With high values of κ, we obtain obesity Kuznets curves: After a number of periods, body weight overshoots its steady-state value and then decreases to its steady- state value.

Therefore, the most striking results of our sensitivity analysis are as follows. The dynamic relation between weight and the capital stock does not exhibit a Kuznets curve pattern for a wide range of elasticities of substitution between food consumption and effective exercise. For both the static and dynamic relations, the more food consumption and effective exercise are complements, the more prevalent peer effects, the lower the level of steady-state body weight. Peer effects would need to be larger than empirically observed for a dynamic Kuznets curve to occur.

5. Conclusion

Our model expands the theoretical literature to acknowledge the importance of calorie expenditure in maintaining healthy body weights and the fact that preferences change over time. We build and simulate the first theoretical growth model that combines Becker's (1965) theory of the allocation of time and Veblen's (1899) theory of conspicuous leisure and focus on exercise choices and changing preferences to explain obesity-income patterns.

We show that both a dynamic and a static Kuznets curve for obesity can arise and provide a novel explanation of the mechanisms generating the dynamic and static Kuznets curves. Our dynamic model shows that the difference between the growth rate of consumption and the growth rate of exercise reflects the difference between the rise in the opportunity cost of exercise and the change in peer-influence with respect to exercise. We formally demonstrate the existence of a level of capital per worker for which the growth rate of exercise starts exceeding the growth rate of food consumption. These mechanisms apply to a dynamic environment in which the stock of capital per worker builds up over time and a static environment used for cross-sectional analysis of countries or individuals with different income levels, explored through comparative statics.

Furthermore, we show that ignoring dynamic peer effects with respect to exercise choices would yield a positive correlation between economic development and weight gain at all levels of economic development. By contrast, in the presence of dynamic peer effects with respect to exercise, for high levels of economic development, we show that body weight gain becomes negative as the economy develops over time. The static analysis indicates that for high levels of the steady-state stock of capital per worker, the link between body weight and the steady-state capital stock is negative.

We supplement the qualitative analysis with numerical simulations. The simulated US economy shows that peer effects on exercise choices are not sufficient to generate a dynamic Kuznets curve pattern for obesity. However, they are sufficient to generate a static Kuznets curve for the USA: the steady-state level of average body weight increases with the average stock of capital up to a level of 187 pounds, corresponding to a stock of capital per worker 25% higher than its current steady-state level, and decreases thereafter.

We acknowledge a few caveats in our work. Neither our behavioral model nor our simulations distinguish between men and women, while the data show differences in the income obesity relation between men and women. This task is limited by the lack of empirical data on behavioral differences with respect to food consumption and exercise and specific to men and women. Notwithstanding these limitations, this study clarifies the impact of peer effects on the relation between income and obesity.

A natural extension of this work is to use our model to study the effect of exercise subsidies. While calorie expenditure is an important factor in weight gain, in the USA, government policies aiming at encouraging exercise have been limited. The most recent federal level initiative is the Let's Move program led by former First Lady Michelle Obama. Additionally, while a larger and

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larger number of employers are offering financial incentives for heathy behavior, small businesses who employ more than half of the private sector workforce do not. Less than 5% of worksites with 50–99 employees offer comprehensive workplace health programs. However, Cawley's (2015) literature review, Mukhopadhyay and Wendel (2013), and Goetzel (2016) indicate that physical exercise programs have the potential in preventing youth obesity and improving employees' health. It is therefore important to understand the effect of exercise subsidies on obesity, distinguishing between subsidies to consumers as opposed to employers. Our model provides a starting framework for this analysis since it accounts for the fact that the cost of exercise is both an expenditure and an opportunity cost and provides a vehicle to study various types of exercise subsidies. It also suggests that subsidies targeting the price of exercise expenditure (p_X) may have a different quantitative effect from those targeting wages (w) because they affect the ROC in different ways. By lowering p_X to show the role of exercise as an expenditure, we already showed that a subsidy lowering the cost of exercise would result in lower body weight. Exploring the effects of subsidies should be the focus of future investigations.

Notes

1 See the extensive literature review at https://www.hsph.harvard.edu/obesity-prevention-source/obesity-causes/.

2 In our model, economic development is captured by the stock of capital per worker (identical to the per capita stock of capital) which, given technology, fully determines income per worker.

3 This specification of status preferences is prevalent throughout the literature. Formulating it as a multiplicative function [Gali (1994)] is also possible and yields essentially equivalent results.

4 We provide a discussion on the role of calorie consciousness in Section 3.3.

5 To understand this, observe that:

$$\int_0^\infty u(C,\hat{X})e^{-\rho t}dt \le \int_0^\infty q\,\rho\, e^{-(1-\beta)\rho t}dt = \frac{q}{1-\beta}\,.$$

and:

$$\int_0^\infty v\left[\left(W-W^I\right)^2\right]e^{-\rho t}dt \le \int_0^\infty q_w\,\rho\, e^{-(1-\beta_w)\rho t}dt = \frac{q_w}{1-\beta_w}\,.$$

6 Notice the different fonts for weight (w) and wage (w).

7 For example, with constant elasticities of substitution in (A5), $c^*/x^* = \alpha/(1-\alpha)\left(\hat{p}_X\left(k^*\right)/p_C\right)\left(1-\varepsilon(k^*)\right)$, and the steady-state value of x becomes: $x^* = \frac{f(k^*)-\delta k^*}{p_X+\frac{\alpha}{1-\alpha}\hat{p}_X(k^*)(1-\varepsilon(k^*))}$.

8 Notice that w* does not necessarily represent the ideal body weight. It simply is the stationary weight, consistent with a steady state (x^*, k^*).

9 This change is given by $\dot{W}/X = \lambda_C\left(\frac{c}{x}\right) - \left[\lambda_S\frac{\bar{s}}{x} + \lambda_X + \frac{\lambda_N}{x}\right]$ w. If, on the transitional path, c/x declines, x increases, and w possibly declines, then the impact of the decline in c/x lowers weight change, while the changes in x and w tend to increase weight change. Assumption 3 requires the decline in c/x to dominate the effects on weight change induced by changes in x and w.

10 Suppose the economy reaches a stationary state at date t^*. In this case, a dynamic Kuznets curve is obtained only for $t > t^*$ and if $W(t^*) > W^*$, where the latter is the steady-state value of weight ($W^* = \mathrm{w}^*N^*$).

11 Of course, if $\lambda_N < \lambda_X$, this assumption is always satisfied.

12 We rely on Mathematica and use the relaxation algorithm to obtain dynamic results.

13 Quasi-experimental research provides estimates in the 0.2–0.6 range [see, e.g., Johansson-Stenman et al. (2002); Clark and Senik (2010); Carlsson et al. (2007); and the overview by Wendner and Goulder (2008)].

14 To obtain meaningful body weight levels, we normalize the steady-state body weight at 185 pounds in the initial steady state.

15 The variations applied to $(C, \hat{X})$ affect the weight, but any change in weight does not impact the decisions of $(C, \hat{X})$. To streamline the presentation, instead of representing utility as $u(C,\hat{X}) - v\left[\left(W-W^I\right)^2\right]$, we disregard $v\left[\left(W-W^I\right)^2\right]$.

16 Therefore, $\lim_{T\to\infty}\mu^*(T)e^{-\rho T}K^*(T) = 0$ is not a necessary condition, very much in contrast to (economics) textbooks, though less in contradiction to insights from major mathematicians, including Ekeland (cf. https://www.youtube.com/watch?v=0upVJC39hCw).

17 The same considerations concerning Ekeland's variational principle, as discussed in Section A.1, are applicable in this context as well.

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Appendix A

A.1. Optimality conditions

$$\mathcal{H} = U(C, \hat{X}, W) + \mu \left[rK + w\bar{L} - \hat{p}_X \varepsilon \left(\bar{k} \right) \bar{X} - p_C C - \hat{p}_X \hat{X} \right],$$

where μ is the shadow value of saving expressed in utility units. An interior solution regarding the control variables implies:

$$\frac{\partial \mathcal{H}}{\partial C} = U_C - \mu\, p_C = 0\,, \tag{A1}$$

$$\frac{\partial \mathcal{H}}{\partial \hat{X}} = U_{\hat{X}} - \mu\, \hat{p}_X = 0\,. \tag{A2}$$

The canonical equations regarding the state variable K are

$$\frac{\partial \mathcal{H}}{\partial K} = \mu r = \rho\mu - \dot{\mu}\,, \tag{A3}$$

which yields:

$$\frac{\dot{\mu}}{\mu} = -(r - \rho)\,. \tag{A4}$$

As the transversality condition (TVC) is not a settled matter for an infinite horizon problem [Halkin (1974)], we find the transversality condition by applying Ekeland's variational principle as in Bosi et al. (2023). Notice that the utility function U is separable in weight and that weight is not a choice variable in our base framework (we choose food and exercise, and weight is the outcome that we like or dislike). Thus, we consider instantaneous utility function u, which by Assumption 1, is C^2, strictly increasing in both arguments and strictly concave:[15]

$$u(C, \hat{X})\,.$$

We apply Ekeland's variational principle. Suppose an optimal path $(C^*, \hat{X}^*, K^*)$ exists, i.e., $K(0) = K_0$ and $\dot{K} = rK + w\bar{L} - \hat{p}_X \varepsilon \left(\bar{k} \right) \bar{X} - p_C C - \hat{p}_X \hat{X}$. Let λ^* be the path of the usual multiplier associated with the optimal path. The necessary conditions for an optimum can be obtained from the following value function:

$$V(C, \hat{X}, \lambda^*) = \int_0^\infty u(C, \hat{X}) e^{-\rho\tau} d\tau + \int_0^\infty \lambda^* \left[rK + w\bar{L} - \hat{p}_X \varepsilon \left(\bar{k} \right) \bar{X} - p_C C - \hat{p}_X \hat{X} - \dot{K} \right] d\tau\,.$$

Here, $\lambda^* = \mu^* e^{-\rho\tau}$, where μ^* is the current value costate variable in the Hamiltonian function. Thus, $(\dot{\lambda}/\lambda)^* = (\dot{\mu}/\mu)^* - \rho$. Integration by parts yields:

$$\int_0^\infty \lambda^* \dot{K}\, d\tau = \lambda^* K \,|_0^\infty - \int_0^\infty \dot{\lambda}^* K\, d\tau = -\lambda^*(0)K(0) + \lim_{T\to\infty} \lambda^*(T)K(T) - \int_0^\infty \dot{\lambda}^* K\, d\tau\,.$$

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Thus, we re-write the value function as:

$$V(C,\hat{X},\lambda^*) = \int_0^\infty u(C,\hat{X})e^{-\rho\tau}d\tau + \int_0^\infty \lambda^* \left[rK + w\bar{L} - \hat{p}_X\varepsilon\left(\bar{k}\right)\bar{X} - p_C C - \hat{p}_X\hat{X}\right]d\tau$$

$$+ \lambda^*(0)K(0) - \lim_{T\to\infty}\lambda^*(T)K(T) + \int_0^\infty \dot{\lambda}^* K\, d\tau\,,$$

or, equivalently, as

$$V(C,\hat{X},\lambda^*) = \int_0^\infty u(C,\hat{X})e^{-\rho\tau}d\tau + \int_0^\infty \dot{\lambda}^*K + \lambda^* \left[rK + w\bar{L} - \hat{p}_X\varepsilon\left(\bar{k}\right)\bar{X} - p_C C - \hat{p}_X\hat{X}\right]d\tau$$

$$+ \lambda^*(0)K(0) - \lim_{T\to\infty}\lambda^*(T)K(T)\,.$$

We introduce perturbations $(\tilde{C},\tilde{\hat{X}},\tilde{K})$ as follows. Let $(C^*,\hat{X}^*,K^*)$ denote the optimal paths of $(C,\hat{X},K)$. For $\epsilon\neq 0$, define: $\tilde{C}\equiv[C-C^*]/\epsilon$; $\tilde{\hat{X}}\equiv[\hat{X}-\hat{X}^*]/\epsilon$; $\tilde{K}\equiv[K-K^*]/\epsilon$; with $(\tilde{C},\tilde{\hat{X}},\tilde{K}):\mathbb{R}_+\to\mathbb{R}$. Thus, we write any feasible path as a deviation from the optimal path, e.g., $C(\epsilon)=C^*+\epsilon\tilde{C}$:

$$V(\epsilon) = \int_0^\infty u(C(\epsilon),\hat{X}(\epsilon))e^{-\rho\tau}d\tau$$

$$+ \int_0^\infty \dot{\lambda}^*K(\epsilon) + \lambda^*\left[rK(\epsilon) + w\bar{L} - \hat{p}_X\varepsilon\left(\bar{k}\right)\bar{X} - p_C C(\epsilon) - \hat{p}_X\hat{X}(\epsilon)\right]d\tau$$

$$+\lambda^*(0)K_0 - \lim_{T\to\infty}\lambda^*(T)[K^*(T)+\epsilon\tilde{K}(T)]\,,$$

noting that $K(0)=K_0$ is exogenously given (i.e., fixed). As a result, the functional V was transformed into a function of ϵ. The optimum then implies $V'(\epsilon)=0$ [see, e.g., Chiang (1992, pp. 177ff; pp. 242f)]:

$$V'(\epsilon) = \int_0^\infty e^{-\rho\tau}\left(u_C\tilde{C} + u_{\hat{X}}\tilde{\hat{X}}\right)d\tau$$

$$+ \int_0^\infty \dot{\lambda}^*\tilde{K} + \lambda^*\left[r\tilde{K} - p_C\tilde{C} - \hat{p}_X\tilde{\hat{X}}\right]d\tau - \lim_{T\to\infty}\lambda^*(T)\tilde{K}(T) = 0\,.$$

Grouping terms yields:

$$V'(\epsilon) = \int_0^\infty \tilde{C}\left(e^{-\rho\tau}u_C - \lambda^*p_C\right)d\tau + \int_0^\infty \tilde{\hat{X}}\left(e^{-\rho\tau}u_{\hat{X}} - \lambda^*\hat{p}_X\right)d\tau$$

$$+ \int_0^\infty \tilde{K}\left(\dot{\lambda}^* + r\lambda^*\right)d\tau - \lim_{T\to\infty}\lambda^*(T)\tilde{K}(T) = 0\,.$$

Consider the subclass $\mathcal{K}$ of functions $\tilde{K}$ for which $\lim_{T\to\infty}\lambda^*(T)\tilde{K}(T)=0$. Thus, for $\tilde{K}\in\mathcal{K}$, $\lim_{T\to\infty}\lambda^*(T)K(T)=\lim_{T\to\infty}\lambda^*(T)K^*(T)\in\mathbb{R}$. For every such $\tilde{K}$, the previous first-order condition $V'(\epsilon)=0$ implies:

$$\int_0^\infty \tilde{C}\left(e^{-\rho\tau}u_C-\lambda^* p_C\right)d\tau+\int_0^\infty \tilde{\hat{X}}\left(e^{-\rho\tau}u_{\hat{X}}-\lambda^*\hat{p}_X\right)d\tau+\int_0^\infty \tilde{K}\left(\dot{\lambda}^*+r\lambda^*\right)d\tau=0\,.$$

As $\tilde{C}$, $\tilde{\hat{X}}$, and $\tilde{K}$ are arbitrary functions, necessary conditions for the first-order condition above to be satisfied are $\left(e^{-\rho\tau}u_C-\lambda^* p_C\right)=0$, $\left(e^{-\rho\tau}u_{\hat{X}}-\lambda^*\hat{p}_X\right)=0$, and $\left(\dot{\lambda}^*+r\lambda^*\right)=0$ for all $\tau\geq 0$. Noting that $\mu^*=\lambda^* e^{\rho\tau}$, we can rewrite these conditions in present-value terms: $\left(u_C-\mu^* p_C\right)=0$, $\left(u_{\hat{X}}-\mu^*\hat{p}_X\right)=0$, and $\left(\dot{\lambda}^*+r\lambda^*\right)=\lambda^*\left(\frac{\dot{\lambda}^*}{\lambda^*}+r\right)=\lambda^*\left(\frac{\dot{\mu}^*}{\mu^*}+(r-\rho)\right)=0$, for all $\tau\geq 0$, which are the necessary first-order conditions appearing in Section A.1 above.

Next, consider these necessary first-order conditions. Then, optimality requires that $\lim_{T\to\infty}\lambda^*(T)\tilde{K}(T)=0$. As a consequence (and as argued above), this condition implies $\lim_{T\to\infty}\lambda^*(T)K(T)\in\mathbb{R}$ for any feasible K, and in particular for K^*:

$$\lim_{T\to\infty}\lambda^*(T)K^*(T)\in\mathbb{R}\,,$$

which is a necessary transversality condition for our optimal control problem, as in Bosi et al. (2023), though in a different context. Again noting that $\mu^*=\lambda^* e^{\rho\tau}$, Ekeland's variational principle yields the following necessary transversality condition for our optimization problem:[16]

$$\lim_{T\to\infty}\mu^*(T)e^{-\rho T}K^*(T)\in\mathbb{R}\,.$$

A.2. Solution to the individual's optimization problem [equations (13) and (14)]

Based on the first-order conditions (A1), (A2), and (A3), the intratemporal optimality condition becomes

$$\frac{U_{\hat{X}}(C,\hat{X},W)}{U_C(C,\hat{X},W)}=\frac{u_{\hat{X}}(C,\hat{X})}{u_C(C,\hat{X})}=\frac{p_{\hat{X}}}{p_C},$$

due to separability of the utility function in W. Differentiating the first-order conditions with respect to time yields

$$\frac{\dot{U}_C(C,\hat{X},W)}{U_C(C,\hat{X},W)}=\frac{u_{CC}(C,\hat{X})C}{u_C(C,\hat{X})}\frac{\dot{C}}{C}+\frac{u_{C\hat{X}}(C,\hat{X})\hat{X}}{u_C(C,\hat{X})}\frac{\dot{\hat{X}}}{\hat{X}}=\frac{\dot{\mu}}{\mu}=-(r-\rho),$$

and

$$\frac{\dot{U}_{\hat{X}}(C,\hat{X},W)}{U_{\hat{X}}(C,\hat{X},W)}=\frac{u_{\hat{X}C}(C,\hat{X})C}{u_{\hat{X}}(C,\hat{X})}\frac{\dot{C}}{C}+\frac{u_{\hat{X}\hat{X}}(C,\hat{X})\hat{X}}{u_{\hat{X}}(C,\hat{X})}\frac{\dot{\hat{X}}}{\hat{X}}=\frac{\dot{\mu}}{\mu}+\frac{\dot{\hat{p}}_X}{\hat{p}_X}=-(r-\rho)+\Delta\frac{\dot{w}}{w}.$$

Let e_{ij}, $i,j\in\left\{C,\hat{X}\right\}$ define the elasticities $u_{ij}(i,j)j/u_i$. Then, the above growth rates can be written as (suppressing the arguments of the elasticity functions):

$$e_{CC}\frac{\dot{C}}{C}+e_{C\hat{X}}\frac{\dot{\hat{X}}}{\hat{X}}=-(r-\rho),$$

and

$$e_{\hat{X}C}\frac{\dot{C}}{C}+e_{\hat{X}\hat{X}}\frac{\dot{\hat{X}}}{\hat{X}}=-(r-\rho)+\Delta\frac{\dot{w}}{w}.$$

Considering both equations and collecting terms yields:

$$\frac{\dot{C}}{C}=\left[\frac{e_{\hat{X}\hat{X}}-e_{C\hat{X}}}{-e_{CC}e_{\hat{X}\hat{X}}+e_{C\hat{X}}e_{\hat{X}C}}\right](r-\rho)+\left[\frac{e_{C\hat{X}}}{-e_{CC}e_{\hat{X}\hat{X}}+e_{C\hat{X}}e_{\hat{X}C}}\right]\Delta\frac{\dot{w}}{w},$$

and

$$\frac{\dot{\hat{X}}}{\hat{X}}=\left[\frac{e_{CC}-e_{\hat{X}C}}{-e_{CC}e_{\hat{X}\hat{X}}+e_{C\hat{X}}e_{\hat{X}C}}\right](r-\rho)-\left[\frac{e_{CC}}{-e_{CC}e_{\hat{X}\hat{X}}+e_{C\hat{X}}e_{\hat{X}C}}\right]\Delta\frac{\dot{w}}{w}.$$

The function Δ is such that $0<\Delta\equiv\frac{w}{w+p_X}<1$ (implying that $\frac{\partial\Delta}{\partial w}>0$, and $\frac{\partial\Delta}{\partial p_X}<0$). We define the following elasticities: $\Omega^C\left(C,\hat{X}\right)=\frac{e_{\hat{X}\hat{X}}-e_{C\hat{X}}}{-e_{CC}e_{\hat{X}\hat{X}}+e_{C\hat{X}}e_{\hat{X}C}}$, $\Omega^X\left(C,\hat{X}\right)=\frac{e_{CC}-e_{\hat{X}C}}{-e_{CC}e_{\hat{X}\hat{X}}+e_{C\hat{X}}e_{\hat{X}C}}$, $\Phi^C\left(C,\hat{X}\right)=\frac{e_{C\hat{X}}}{-e_{CC}e_{\hat{X}\hat{X}}+e_{C\hat{X}}e_{\hat{X}C}}$, and $\Phi^X\left(C,\hat{X}\right)=\frac{e_{CC}}{-e_{CC}e_{\hat{X}\hat{X}}+e_{C\hat{X}}e_{\hat{X}C}}$, with $e_{CC}=(U_{CC}C/U_C)$, $e_{C\hat{X}}=\left(U_{C\hat{X}}\hat{X}/U_C\right)$, $e_{\hat{X}C}=\left(U_{\hat{X}C}C/U_{\hat{X}}\right)$, $e_{\hat{X}\hat{X}}=\left(U_{\hat{X}\hat{X}}\hat{X}/U_{\hat{X}}\right)$. Considering the definitions Ω^C, Ω^X, Φ^C, and Φ^X we can re-write the above equations as

$$\frac{\dot{C}}{C}=\Omega^C\left(C,\hat{X}\right)(r-\rho)+\Phi^C\left(C,\hat{X}\right)\Delta\frac{\dot{w}}{w},$$

and

$$\frac{\dot{\hat{X}}}{\hat{X}}=\Omega^X\left(C,\hat{X}\right)(r-\rho)-\Phi^X\left(C,\hat{X}\right)\Delta\frac{\dot{w}}{w}.$$

In equilibrium, $\hat{X}=X(1-\varepsilon(k))$, that is:

$$\frac{\dot{\hat{X}}}{\hat{X}}=\frac{\dot{X}}{X}-\frac{\dot{\varepsilon}(k)}{1-\varepsilon(k)},$$

so:

$$\frac{\dot{X}}{X}=\Omega^X\left(C,\hat{X}\right)(r-\rho)+\frac{\dot{\varepsilon}(k)}{1-\varepsilon(k)}-\Phi^X\left(C,\hat{X}\right)\Delta\frac{\dot{w}}{w}.$$

A.3. Proof of Proposition 1

Straightforward from (15). The ECE includes two elasticities, respectively $\Omega^C\left(C,\hat{X}\right)$ and $\Omega^X\left(C,\hat{X}\right)$, which are different unless preferences are CES, in which case this term disappears. To illustrate the terms Ω^C, Ω^X, Φ^C, Φ^X, we can consider a constant elasticities of substitution specification for the utility function, such that:

$$U(C,\hat{X},W)=\frac{\left[\left(\alpha C^{\zeta}+(1-\alpha)\hat{X}^{\zeta}\right)^{1/\zeta}\right]^{1-\gamma}}{1-\gamma}-\nu\left[\left(W-W^I\right)^2\right],$$
$$0<\alpha<1,\ \gamma>0,\ \zeta\leq 1. \tag{A5}$$

where α represents the taste for food consumption relative to effective exercise consumption. The intratemporal elasticity of substitution between C and $\hat{X}$ is given by $1/(1-\zeta)$, while the intertemporal elasticity of substitution (IES) is given by $1/\gamma$.

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For $\gamma = 1$, *the sub-utility function* $u(C, \hat{X})$ *equals* $\ln\left(\alpha C^{\zeta} + (1-\alpha)\hat{X}^{\zeta}\right)^{1/\zeta}$. *It is increasing in both arguments as* $0 < \alpha < 1$, *twice continuously differentiable and strictly concave (as follows from negative definiteness of the Hessian due to* $0 < \alpha < 1$, $\zeta \leq 1$*). Moreover, for our parameterization, it can easily be checked that* $\lim_{C\downarrow 0} u_C = \lim_{X\downarrow 0} u_{\hat{X}} = \infty$ *and that* $\lim_{C\to\infty} u_C = \lim_{X\to\infty} u_{\hat{X}} = 0$. *Additionally, recall that from Assumption 1, the sub-utility function* $v: \mathbb{R}_+ \to \mathbb{R}$ *is twice continuously differentiable, increasing and strictly convex in its argument* $(W - W^I)^2$ *and* $v\left[\left(W - W^I\right)^2\right] \leq q_w\, \rho\, e^{\beta_w \rho t}$, *where* $q_w \in \mathbb{R}_+$ *and* $\beta_w < 1$.

Our utility integral is bounded. Along the transitional path starting from below the stationary state, both C and $\hat{X}$ *rise and converge to some stationary state* $(C^*, \hat{X}^*)$. *Let y denote* $y = \alpha C^{\zeta} + (1-\alpha)\hat{X}^{\zeta}$. *y converges to* y^*. *As u is increasing in both arguments, u increases in y. With some abuse of notation, let* $u(y)$ *denote the instantaneous utility function. Then boundedness of the utility integral, as shown above, requires* $u(y) \leq q\, \rho\, e^{\beta\rho t}$. *That is, there must be a* $q \in \mathbb{R}_+$ *and a* $\beta < 1$ *such that for all t,* $u(y) \leq q\, \rho\, e^{\beta\rho t}$. *As our* $y(t)$ *path is not diverging and u is a real valued function, for every* $y(t)$ *there exist* $q \in \mathbb{R}_+$ *and* $\beta < 1$ *such that for all t,* $u(y) \leq q\, \rho\, e^{\beta\rho t}$. *Moreover, as* $\lim_{t\to\infty} y(t) = y^*$, $u(y)$ *is bounded, and for* $\rho > 0$, *the integral* $\int_0^{\infty} u(y) e^{-\rho\tau} d\tau$ *is bounded [cf. Chiang (1992, p. 101)]. Additionally, by Assumption 1, the integral* $\int_0^{\infty} v\left[\left(W - W^I\right)^2\right] e^{-\rho\tau} d\tau$ *is bounded, thus, the utility integral (difference of the integrals of sub-utilities) is also bounded.*

For the whole class of constant elasticities of substitution utility functions, the term $\left[\Phi^X + \Phi^C\right] = 1/(1-\zeta) > 0$ (with $\Phi^C = -(1-\alpha)(1-\gamma)/\gamma < 0$, when $\zeta = 0$) and $\Omega^C = \Omega^X = 1/\gamma$. That is, in (15), the term ECE equals zero. Therefore, equation (15) indicates that the sign of the rate of change of exercise expenditure is ambiguous, solely depending on the difference between DPE and ROC, which are two positive terms. □

A.4. Derivation of the dynamic system in k and x [equations (19) and (20)]

Noting that $x = X/N = X/(\bar{L} - X)$, we can re-write $X = x(\bar{L} - X)$, hence, $X = [x/(1+x)]\, \bar{L}$. Thus, $X = X(x)$. Next, consider that $\hat{X} = (1 - \varepsilon(k))X(x)$ in equilibrium. Thus, $\hat{X} = \hat{X}(x, k)$. Finally, dividing (A1) by (A2), the resulting intratemporal first-order condition $U_C(C, \hat{X})/U_{\hat{X}}(C, \hat{X}) = p_C/\hat{p}_X(k)$ implicitly defines a relationship $C(\hat{X}(x, k), k)$. Since the arguments $\left(C, \hat{X}\right)$ can also be written in terms of (x, k), $\Omega^i(C, \hat{X}) = \Omega^i(x, k)$ and $\Phi^i(C, \hat{X}) = \Phi^i(x, k)$, $i = C, X$. Recall that: $\Delta = w/(p_X + w)$ and $\hat{p}_X = p_X + w$. Since w depends on k, we write $\Delta = \Delta\left(k\right)$. The comparison term $\varepsilon(k)$ is specified according to (5). Considering our functional specification of $\varepsilon(k)$,

$$\frac{\dot{\varepsilon}(k)}{1 - \varepsilon(k)} = \frac{\varepsilon'(k)k}{1 - \varepsilon(k)}\frac{\dot{k}}{k} = (\kappa k)\frac{\dot{k}}{k}.$$

As $\dot{w}/w = \eta\, \dot{k}/k$, and considering that $\hat{X} = \hat{X}(x, k)$ and $C = C(x, k)$—see Section 2.3—we re-write the growth rate of X:

$$\frac{\dot{X}}{X} = \Omega^X\left(x, k\right)(r - \rho) + \left[\kappa k - \Phi^X\left(x, k\right)\Delta\, \eta\right]\frac{\dot{k}}{k}.$$

As $x = X/N$, $\dot{x}/x = \dot{X}/X - \dot{N}/N$. We note that $\dot{N}/N = -X/(\bar{L} - X)\left(\dot{X}/X\right) = -x\left(\dot{X}/X\right)$. Thus, $\dot{x}/x = (1+x)\dot{X}/X$:

$$\frac{\dot{x}}{x} = (1+x)\left[\Omega^X\left(x, k\right)\left(f'(k) - \delta - \rho\right) + \left[\kappa k - \Phi^X\left(x, k\right)\Delta\, \eta\right]\frac{\dot{k}}{k}\right]. \tag{A6}$$

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Next, $\left(\dot{k}/k\right) = \left(\dot{K}/K\right) - \left(\dot{N}/N\right) = \left(\dot{K}/K\right) + x\left(\dot{X}/X\right)$. We consider the resource constraint $\dot{K} = f(k)N - \delta K - p_C C - p_X X$:

$$\frac{\dot{k}}{k} = \left(\frac{\dot{K}}{K}\right) + x\frac{\dot{X}}{X}$$
$$= \left(\frac{f(k)}{k} - \delta - p_C\frac{c}{k} - p_X\frac{x}{k}\right) + x\left\{\Omega^X\left(x, k\right)(r-\rho) + \left[\kappa k - \Phi^X\left(x, k\right)\Delta\,\eta\right]\frac{\dot{k}}{k}\right\},$$

thus,

$$\frac{\dot{k}}{k}\left\{1 - x\left[\kappa k - \Phi^X\left(x, k\right)\Delta\,\eta\right]\right\} = \left(\frac{f(k)}{k} - \delta - p_C\frac{c}{k} - p_X\frac{x}{k}\right) + x\,\Omega^X\left(x, k\right)(r-\rho),$$

and as a result:

$$\frac{\dot{k}}{k} = \left\{1 - x\left[\kappa k - \Phi^X\left(x, k\right)\Delta\,\eta\right]\right\}^{-1}$$
$$\left[x\,\Omega^X\left(x, k\right)\left(f'(k) - \delta - \rho\right) + \frac{f(k)}{k} - \delta - p_C\frac{c(x, k)}{k} - p_X\frac{x}{k}\right]. \quad \text{(A7)}$$

A.5. Derivation of the Schofield equation with normalized variables [equation (21)]

We now derive (21). We rewrite the Schofield equation (A8) as

$$\dot{W} = \lambda_C C - \frac{\left(\lambda_S\bar{S} + \lambda_X X + \lambda_N N\right)}{N}W, \quad \text{(A8)}$$

$$\frac{\dot{W}}{W} = \lambda_C\frac{C}{W} - \frac{\left(\lambda_S\bar{S} + \lambda_X X + \lambda_N N\right)}{N} = \frac{\dot{\mathrm{w}}}{\mathrm{w}} + \frac{\dot{N}}{N}. \quad \text{(A9)}$$

Since:

$$\frac{\dot{N}}{N} = -x\frac{\dot{X}}{X} = -\frac{1}{1+x}\frac{\dot{x}}{x},$$

$$\frac{\dot{\mathrm{w}}}{\mathrm{w}} = \lambda_C\frac{C}{N}\frac{N}{W} - \frac{\left(\lambda_S\bar{S} + \lambda_X X + \lambda_N N\right)}{N} + \frac{1}{1+x}\frac{\dot{x}}{x}.$$

Substituting (A6) in this expression yields:

$$\frac{\dot{\mathrm{w}}}{\mathrm{w}} = \lambda_C\frac{c}{\mathrm{w}} - \frac{\left(\lambda_S\bar{S} + \lambda_X X + \lambda_N N\right)}{N}$$
$$+\Omega^X\left(x, k\right)\left(f'(k) - \delta - \rho\right) + \left(\kappa k - \Phi^X\left(x, k\right)\eta(k)\,\Delta(k)\right)\frac{\dot{k}}{k},$$

which is equivalent to:

$$\frac{\dot{\mathrm{w}}}{\mathrm{w}} = \lambda_C\frac{c}{\mathrm{w}} - \left(\lambda_S\bar{s} + \lambda_X x + \lambda_N\right)$$
$$+\Omega^X\left(x, k\right)\left(f'(k) - \delta - \rho\right) + \left(\kappa k - \Phi^X\left(x, k\right)\eta(k)\,\Delta(k)\right)\frac{\dot{k}}{k}.$$

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A.6. Proof of Proposition 2

Notice that we study a Cobb—Douglas production function: $f(k) = k^{\eta}$, $0 < \eta < 1$, and that prices $p_X > 0$ and $p_C > 0$ are exogenous. Throughout, we consider a growing economy, $\dot{k} > 0$. From 27:

$$\frac{\dot{c}}{c} - \frac{\dot{x}}{x} = \underbrace{\frac{-kf''(k)}{p_X + w}\dot{k}}_{\text{ROC}} - \underbrace{\kappa\dot{k}}_{\text{DPE}}.$$

Step 1: The sign of the difference ROC − DPE is determined by

$$\frac{\frac{\dot{c}}{c} - \frac{\dot{x}}{x}}{\dot{k}} = \frac{\text{ROC} - \text{DPE}}{\dot{k}} = \frac{-kf''(k)}{p_X + w} - \kappa\,.$$

Thus, for $\dot{k} > 0$:

$$\frac{\dot{c}}{c} - \frac{\dot{x}}{x} \gtreqless 0 \Longleftrightarrow \frac{-kf''(k))}{p_X + w} \gtreqless \kappa \Longleftrightarrow \underbrace{\frac{(1-\eta)\eta}{k^{1-\eta}\left[p_X + (1-\eta)k^{\eta}\right]}}_{\equiv h(k)} \gtreqless \kappa, \quad 0 < \eta < 1, \quad \kappa > 0,$$

where the latter is an implicit inequality of k. Function $h(k)$ has the following properties: (i) $h(k)$ is a continuous function, (ii) $\lim_{k \to 0} h(k) = \infty$ and $\lim_{k \to \infty} h(k) = 0$, and (iii) $h'(k) < 0$. By the intermediate value theorem, for any $\kappa > 0$, there exists a unique $\hat{k}\left(\kappa, \eta, p_X\right)$ such that $h(\hat{k}) = \kappa$. Formally,

$$h(\hat{k}) = \frac{(1-\eta)\eta}{\hat{k}^{1-\eta}\left[p_X + (1-\eta)\hat{k}^{\eta}\right]} = \kappa.$$

As a consequence, for $k < \hat{k}$, $\frac{\dot{c}(t)}{c(t)} - \frac{\dot{x}(t)}{x(t)} > 0$, that is, ROC > DPE. Likewise, for $k > \hat{k}$, $\frac{\dot{c}(t)}{c(t)} - \frac{\dot{x}(t)}{x(t)} < 0$, that is, ROC < DPE.

Notice that $\hat{k}\left(0, \eta, p_X\right) \to \infty$ and $d\hat{k}/d\kappa = 1/h'(\hat{k}) < 0$. For $\kappa > 0$, if k assumed every value in $\mathbb{R}_+$ during the transition from its initial value k_0 toward its steady-state value $k^*(\delta, \eta, \rho) = [\eta/(\delta + \rho)]^{1/(1-\eta)}$, then there would always be a period for which $k < \hat{k}$, that is, $\frac{\dot{c}(t)}{c(t)} - \frac{\dot{x}(t)}{x(t)} > 0$, and a period for which $k > \hat{k}$, that is, $\frac{\dot{c}(t)}{c(t)} - \frac{\dot{x}(t)}{x(t)} < 0$. However, for a growing economy, in our model $k \in \left[k_0, k^*\right]$ rather than $\mathbb{R}_+$, where $k_0 > 0$ and $k^* < \infty$.

Step 2: Suppose during transition, k assumed every value in $\mathbb{R}_+$. Then, our assumptions of a growing economy together with continuity of $h(k)$ would imply the existence of a date $\hat{t}$ such that for $t < \hat{t}$, $\frac{\dot{c}(t)}{c(t)} - \frac{\dot{x}(t)}{x(t)} > 0$, and for $t > \hat{t}$, $\frac{\dot{c}(t)}{c(t)} - \frac{\dot{x}(t)}{x(t)} < 0$. In the following, we impose parametric restrictions on k_0 and $(\delta, \eta, \kappa, \rho)$ such that there exists such a $\hat{t}$ in $\mathbb{R}_{++}$ even when $k \in \left[k_0, k^*\right]$ rather than $\mathbb{R}_+$:

Let $k_0 < \hat{k}\left(\kappa, \eta, p_X\right) < k^*(\delta, \eta, \rho)$. Notice that for $k \in [k_0, k^*)$, $\dot{k} > 0$. Then, $\lim_{t \to 0} h(k) > \kappa$, thus, $\lim_{t \to 0} \frac{\dot{c}}{c} - \frac{\dot{x}}{x} > 0$. As $\hat{t} > 0$, for $t \in [0, \hat{t})$, $\frac{\dot{c}}{c} - \frac{\dot{x}}{x} > 0$ and ROC > DPE. Moreover, $\lim_{t \to \infty} h(k) < \kappa$, thus, $\lim_{t \to \infty} \frac{\dot{c}}{c} - \frac{\dot{x}}{x} < 0$. As $\hat{t} < \infty$ (because $\kappa > 0$), $\frac{\dot{c}}{c} - \frac{\dot{x}}{x} < 0$ for $t \in (\hat{t}, \infty)$. Specifically, for $t \in (\hat{t}, \infty)$, ROC < DPE for $\dot{k} > 0$ (and ROC = DPE = 0 for $\dot{k} = 0$). □

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A.7. Proof of Proposition 3

We consider a growing economy ($\dot{k} > 0$). To prove Proposition 3, we rewrite the Schofield equation as

$$\frac{\dot{W}}{X} = \lambda_C \frac{c}{x} - \left(\frac{\lambda_S}{x} \bar{s} + \lambda_X + \frac{\lambda_N}{x} \right) w. \tag{A10}$$

The sign of $\dot{W}$ depends on how the term $\left(\frac{\lambda_C \frac{c}{x}}{\left(\frac{\lambda_S}{x}\bar{s}+\lambda_X+\frac{\lambda_N}{x}\right)}\right)$ compares to weight:

$$\dot{W} \gtreqless 0 \Longleftrightarrow \frac{\lambda_C \frac{c}{x}}{\left(\frac{\lambda_S}{x}\bar{s} + \lambda_X + \frac{\lambda_N}{x}\right)} \gtreqless w.$$

At t_0, if $W(0) < \frac{\lambda_C \frac{c(0)}{x(0)}}{\left(\frac{\lambda_S}{x(0)}\bar{s}(0)+\lambda_X+\frac{\lambda_N}{x(0)}\right)} N(0)$, $\dot{W}(0) > 0$. Otherwise, $\dot{W}(0) < 0$ during an initial period.

Suppose $\dot{W}(0) > 0$. Then, for $t \in \left[t_0, \hat{t}\right]$, $\dot{W}(t) > 0$. This follows from the fact that (c/x) increases during this time period (see Proposition 2), and from Assumption 2, according to which $\frac{\lambda_C \frac{c}{x}}{\left(\frac{\lambda_S}{x}\bar{s}+\lambda_X+\frac{\lambda_N}{x}\right)}$ increases in (c/x), regardless of the development of x.

For $t > \hat{t}$, the ratio (c/x) decreases. This decrease lowers the weight growth, though it does not immediately lead to *negative* weight growth. However, the ratio (c/x) decreases toward its stationary value by Proposition 2. As a consequence, if $W(\hat{t}) > W^*$, there exists $\hat{t}_w > \hat{t}$, so that $\dot{W} < 0$ for $t > \hat{t}_w$. To see this, first consider a date t^* as of which (x, k) is (roughly) stationary. If $W(t^*) > W^*$, then the Schofield equation becomes a very simple differential equation implying that $\dot{W}(t) < 0$ for $t > t^*$. Second, consider a date before (but close to) t^*. For such dates, in addition, (c/x) declines, which further reduces $\dot{W}$ (i.e., makes it more negative). □

A.8. Steady-State equilibrium x* and k*

In the steady state $f'(k^*) = \delta + \rho$. As $f(k)$ is an increasing and strictly concave function, $f'(k)$ is monotone, and there exists an inverse function $f'^{-1}(.)$. An asterisk denotes a steady-state value. Then:

$$k^* = f'^{-1}(\delta + \rho).$$

We express the intratemporal trade-off between exercise and food consumption as:

$$p_C c^* = (p_X + w^*) \frac{\alpha}{1-\alpha} (1 - \varepsilon(k^*)) x^* .$$

From (19), we know that:

$$f(k^*) - \delta k^* = p_C c^* + p_X x^* = x^* \left(p_X + p_C \frac{c^*}{x^*} \right) .$$

Combining both equations yields an implicit steady-state relationship between x and k:

$$f(k^*) - \delta k^* = x^* \left[p_X + \frac{\alpha}{1-\alpha} \left(p_X + w^*\right) \left(1 - \varepsilon(k^*)\right) \right] ,$$

equivalent to:

$$x^* = \frac{f(k^*) - \delta k^*}{p_X + \frac{\alpha}{1-\alpha} \left(p_X + w^*\right) \left(1 - \varepsilon(k^*)\right)} = \frac{f(k^*) - \delta k^*}{p_X + p_C \frac{c^*}{x^*}} . \tag{A11}$$

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Furthermore, c^* follows from substituting x^* in the optimality condition:

$$c^* = \frac{f(k^*) - \delta k^* - p_X x^*}{p_C}. \tag{A12}$$

A.9. Change of c* with respect to k*

The ratio of first-order conditions (A1) by (A2) yields:

$$c^* = \alpha/(1-\alpha)\left(\hat{p}_X^*/p_C\right)\left(1-\varepsilon(k^*)\right)x^*$$

Recalling that $\hat{p}_X^* = p_X + w^*$ and that w^* also depends on k^*, we derive c^* with respect to k^* and obtain:

$$\frac{dc^*}{dk^*} = \frac{\alpha}{1-\alpha}\left[\frac{dw^*/dk^*}{p_C}\left(1-\varepsilon(k^*)\right)x^* - \frac{p_X + w}{p_C}\frac{d\varepsilon(k^*)}{dk^*}x^* + \frac{p_X+w}{p_C}\left(1-\varepsilon(k^*)\right)\frac{dx^*}{dk^*}\right].$$

Dividing $\frac{dc^*}{dk^*}$ by $\frac{c^*}{k^*}$ and simplifying, we obtain the elasticity of $c^*$with respect to k^*:

$$\frac{dc^*/c^*}{dk^*/k^*} = \frac{1}{dk^*/k^*}\left[\Delta\left(k^*\right)\frac{dw^*}{w^*} - \frac{d\varepsilon(k^*)}{(1-\varepsilon(k^*))} + \frac{dx^*}{x^*}\right], \tag{A13}$$

As a consequence,

$$\frac{dc^*/c^* - dx^*/x^*}{dk^*/k^*} = \frac{1}{dk^*/k^*}\left[\underbrace{\Delta\left(k^*\right)\frac{dw^*}{w^*}}_{\text{ROC}^*} - \underbrace{\frac{d\varepsilon(k^*)}{\left(1-\varepsilon(k^*)\right)}}_{\text{DPE}^*}\right]. \tag{A14}$$

Using the functional forms for utility and peer effects, we get:

$$\text{ROC}^* = \frac{\eta(1-\eta)k^{*\eta-1}}{(1-\eta)k^{*\eta} + p_X} = h(k^*),$$

$$\text{DPE}^* = \frac{d\varepsilon(k^*)}{\left(1-\varepsilon(k^*)\right)} = \kappa.$$

which produces equation (29).

A.10. Proof of Proposition 4

We consider $dk^* > 0$ (across sections). The sign of the difference ROC* − DPE* is determined by

$$\frac{\frac{dc^*}{c^*} - \frac{dx^*}{x^*}}{dk^*} = \frac{\underbrace{h(k^*)}_{\text{ROC}^*} - \underbrace{\kappa}_{\text{DPE}^*}}{dk^*}. \tag{A15}$$

The sign of this expression is ambiguous, as both terms DPE* and ROC* are positive. However, while coefficient κ, related to the term DPE*, is constant, the term related to ROC* changes as k^* increases. Assumption 3 implies that $k^*(\delta, \eta, \rho)$ is an increasing function of the output elasticity η and a decreasing function of $(\delta + \rho)$. Specifically, $k^*(\delta, 0, \rho) = 0$, $\partial k^*/\partial\eta > 0$, and $\lim_{\eta\uparrow 1} = \infty$. Thus, under Assumption 3, $k^* \in (0, \infty)$ for $\eta \in (0, 1)$. Now, consider $h(k^*)$. We already established that $h'(k^*) < 0$. It is easy to verify that $\lim_{k^*\downarrow 0} h(k^*) = \infty$. Likewise, $\lim_{k^*\uparrow\infty} h(k^*) = 0$. Thus, under Assumption 3, for $k^* \in (0, \infty)$, $h(k^*) \in (\infty, 0)$. By the Intermediate value theorem, there exist parameter combinations for which $h^*(p_X, \delta, \eta, \rho) = \kappa$. We denote the corresponding steady-state values of capital by $\hat{k}^*(p_X)$. Clearly, for $k^* < \hat{k}^*$, $h^* > \kappa$, and for $k^* > \hat{k}^*$, $h^* < \kappa$.

Let $\underline{k}^* < \hat{k}^*$. Then $h(k^*) > \kappa > 0$ for all $k^* \in (\underline{k}^*, \hat{k}^*)$. Let $\overline{k}^* > \hat{k}^*$. Then $h(k^*) < \kappa$ for all $k^* \in (\hat{k}^*, \overline{k}^*)$. □

A.11. Proof of Proposition 5

We express steady-state weight as

$$\mathrm{w}^* = \left(\frac{\lambda_C \left(c^*/x^*\right)}{z\left(k^*\right)} \right), \tag{A16}$$

where $z\left(k^*\right) = \lambda_S \bar{s}\left(x^*\right)/x^* + \lambda_X + \lambda_N/x^*$ represents total calorie expenditure divided by x^*. Comparing body weight for different wealth levels is obtained by totally deriving body weight with respect to k^*,

$$\frac{d\mathrm{w}^*}{dk^*} = \frac{\lambda_C \left(\frac{c}{x}\right)^*}{z^*} \left[\frac{d(c/x)^*/(c/x)^* - dz^*/z^*}{dk^*} \right],$$

thus, the sign of $d\mathrm{w}^*/dk^*$ is determined by the signs of $\frac{d(c^*/x^*)}{dk^*}$ and $\frac{dz^*}{dk^*}$. First, based on Proposition 3:

$$\frac{d(c^*/x^*)}{c^*/x^*} = \frac{dc^*}{c^*} - \frac{dx^*}{x^*} = \left[\underbrace{h\left(k^*\right)}_{\mathrm{ROC}^*} - \underbrace{\kappa}_{\mathrm{DPE}^*} \right]. \tag{A17}$$

We know from Proposition 3 that this term is positive all $k^* \in (\underline{k}^*, \hat{k}^*)$, and it is negative for all $k^* \in (\hat{k}^*, \bar{k}^*)$.

Second, we consider the impact of dk^* on z^*:

$$\frac{dz^*}{dk^*} = \frac{dz^*}{dx^*}\frac{dx^*}{dk^*}, \quad \text{where } x^* = \frac{f(k^*) - \delta k^*}{p_X + p_C\left(\frac{c^*}{x^*}\right)}.$$

We first examine dx^*/dk^*. The numerator of x^* can be rewritten as $\left[f(k^*) - (\delta+\rho)k^*\right] + \rho k^*$. A rise in k^* raises the numerator by $\left[f'(k^*) - (\delta+\rho)\right] + \rho$ units. As $\left[f'(k^*) - (\delta+\rho)\right] = 0$ in a steady state, the numerator raises by $\rho\, dk^* > 0$ units. For $k^* > \hat{k}^*$, the ratio (c^*/x^*) declines in k^*. Thus, for $k^* > \hat{k}^*$, $(dx^*/k^*) > 0$.

Second, we examine dz^*/dx^*:

$$\frac{dz^*}{dx^*} = \lambda_S \frac{\partial\left(\bar{s}\left(x^*\right)/x^*\right)}{\partial x^*} - \frac{\lambda_N}{(x^*)^2}.$$

The term $\frac{\partial(\bar{s}(x^*)/x^*)}{\partial x^*}$ can be positive or negative in principle. Based on the definition of $\bar{S}$, we obtain:

$$\bar{s}^* = \frac{1}{N} - 1 - \frac{X^*}{N} = \frac{1}{N} - 1 - x^*.$$

Because N is endogenous, we need to make a reasonable assumption: we consider an increase in x^*, representing an increase in X^* given N, or assume that it is not offset by a decrease in N. In that scenario, an increase in x^* produces a decrease in $\bar{s}^*$. In other words, an increase in per worker exercise leads to a decrease in per worker sedentary leisure. Therefore, $\frac{d[\bar{s}(x^*)/x^*]}{dx^*} < 0$, and $\frac{dz^*}{dx^*} < 0$. Thus, for $k^* > \hat{k}^*$, we have $\frac{dz^*}{dk^*} < 0$.

As we consider $dk^* > 0$, our model implies $d\mathrm{w}^*/dk^* > 0$ for $k^* < \hat{k}^*$. However, for $k^* > \hat{k}^*$, our model implies two opposing effects on the steady-state weight. For $k^* > \hat{k}^*$, (c^*/x^*) decreases for

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$dk^* > 0$. At the same time, also z^* decreases under the assumption we adopted. with respect to k^*. First, we rewrite the above total derivative as follows:

$$\frac{dw^*}{dk^*} = \frac{\lambda_C \left(\frac{c}{x}\right)^*}{z^*}\left[\frac{\left(h(k^*) - \kappa\right) - dz^*/z^*}{dk^*}\right].$$

Recall that the steady state $k^* \in \mathbb{R}_+$ (by, e.g., varying parameter η). A static Kuznets curve exists under two conditions. (i) There exists a $\tilde{k}^* > \hat{k}^*$ such that $\left(h(\tilde{k}^*) - \kappa\right) = dz^*/z^*$. As $\left(h(\tilde{k}^*) - \kappa\right) > 0$ for $k^* < \hat{k}^*$ by definition, and $dz^*/z^* < 0$, condition (i) can only be satisfied for a $\tilde{k}^* > \hat{k}^*$. This condition also excludes the case for which $-dz^*/z^* > \kappa$ for all k^*. (ii) For $k^* > \tilde{k}^*$, the decline in $\left(h(k^*) - \kappa\right)$ is larger than the change (decline) in dz^*/z^*:

$$\left|\frac{d(c/x)^*/(c/x)^*}{dk^*}\right| = \left|\frac{h(k^*) - \kappa}{dk^*}\right| > \left|\frac{dz^*/z^*}{dk^*}\right|,$$

in which case $dw^*/dk^* < 0$ for all $k^* > \tilde{k}^*$.

A.12. Formulation of the problem with calorie consciousness and Proof of Proposition 6

With calorie consciousness, weight becomes a costate variable, so the dynamic constraint for $\dot{W}$ must be included in the Hamiltonian. The choices of C and $\hat{X}$ not only directly affect utility but also do so via their respective effects on weight, which is considered by an individual.

$$\dot{W} = \lambda_C C - \frac{\left(\lambda_S \bar{S} + \lambda_X X + \lambda_N N\right)}{N} W.$$

We can write the numerator as $(\lambda_S - \lambda_N)\bar{S} + \lambda_N \bar{S} + \lambda_X X + \lambda_N N$. Considering that $\bar{S} = 1 - N - X$ and $X = \hat{X} + \varepsilon\left(\bar{k}\right)\bar{X}$ we re-write the weight change as

$$\dot{W} = \lambda_C C - \frac{\left((\lambda_S - \lambda_N)\bar{S} + \lambda_N + (\lambda_X - \lambda_N)\left(\hat{X} + \varepsilon\left(\bar{k}\right)\bar{X}\right)\right)}{N} W. \tag{A18}$$

We focus on an individual for whom $W > W^I$, that is, for whom a gain in weight reduces utility (the same arguments apply for the opposite case). The current-value Hamiltonian becomes:

$$\mathcal{H} = U(C, \hat{X}, W) + \mu\left[rK + w\bar{L} - \hat{p}_X \varepsilon\left(\bar{k}\right)\bar{X} - p_C C - \hat{p}_X \hat{X}\right]$$
$$- \xi\left[\lambda_C C - \frac{\left((\lambda_S - \lambda_N)\bar{S} + \lambda_N + (\lambda_X - \lambda_N)\left(\hat{X} + \varepsilon\left(\bar{k}\right)\bar{X}\right)\right)}{N} W\right],$$

where $\xi > 0$ is the shadow value of weight gain expressed in utility units. An interior solution satisfies:[17]

$$\frac{\partial \mathcal{H}}{\partial C} = U_C - \mu\, p_C - \xi \lambda_C = 0\,, \tag{A19}$$

$$\frac{\partial \mathcal{H}}{\partial \hat{X}} = U_{\hat{X}} - \mu\, \hat{p}_X - \xi \frac{W}{N}\left(\lambda_N - \lambda_X\right) = 0\,, \tag{A20}$$

$$\frac{\partial \mathcal{H}}{\partial K} = \mu r = \mu \rho - \dot{\mu}\,, \tag{A21}$$

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$$\frac{\partial \mathcal{H}}{\partial W} = -\xi\rho + \dot{\xi}\,, \tag{A22}$$

$$\lim_{\tau \to \infty} \xi(\tau) e^{-\rho\tau} W(\tau) = 0. \tag{A23}$$

The necessary first-order conditions (A19) and (A20) imply:

$$\frac{U_{\hat{X}}}{U_C} = \frac{\hat{p}_X + \frac{\xi}{\mu}\frac{W}{N}(\lambda_N - \lambda_X)}{p_C + \frac{\xi}{\mu}\lambda_C}\,. \tag{A24}$$

Without weight consciousness ($\xi = 0$), the marginal rate of substitution $U_{\hat{X}}/U_C = \hat{p}_X/p_C$. With weight consciousness ($\xi > 0$), the additional terms are interpreted as follows. As shown by (A18), the term $\frac{W}{N}(\lambda_N - \lambda_X)$ describes the weight change caused by an increase of $\hat{X}$ by one unit. In principal, this term can be positive or negative (while we typically expect it to be negative). The term λ_C describes the weight change caused by an increase of C by one unit. The multiplier ξ/μ converts these weight changes into utility units.

Under the assumption stated in Proposition 6,

$$\frac{1}{p_C}\lambda_C > \frac{1}{\hat{p}_X}\left[\frac{W}{N}(\lambda_N - \lambda_X)\right],$$

$$\frac{U_{\hat{X}}}{U_C}\,|_{\xi=0} > \frac{U_{\hat{X}}}{U_C}\,|_{\xi>0}\,.$$

By strict concavity of the utility function, the indifference curves are strictly convex. As a consequence,

$$\frac{C}{\hat{X}}\,|_{\xi=0} > \frac{C}{\hat{X}}\,|_{\xi>0}\,.$$

□

A.13. Figures and Tables

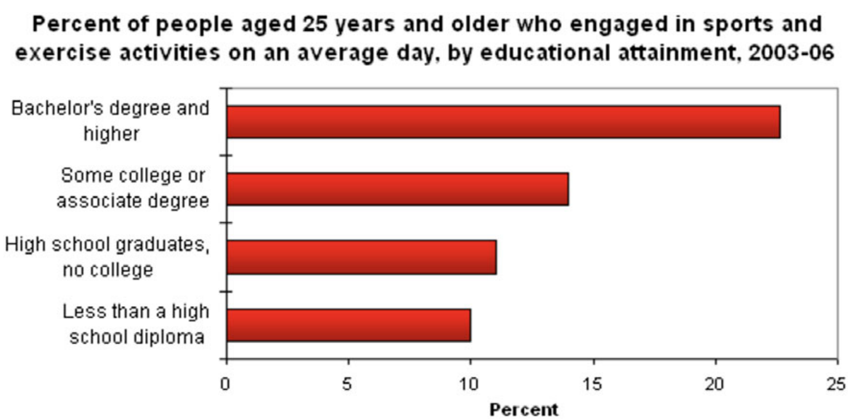

Figure A.13.1 Engagement in sports and educational attainment (Source: Bureau of Labor Statistics, 2008).

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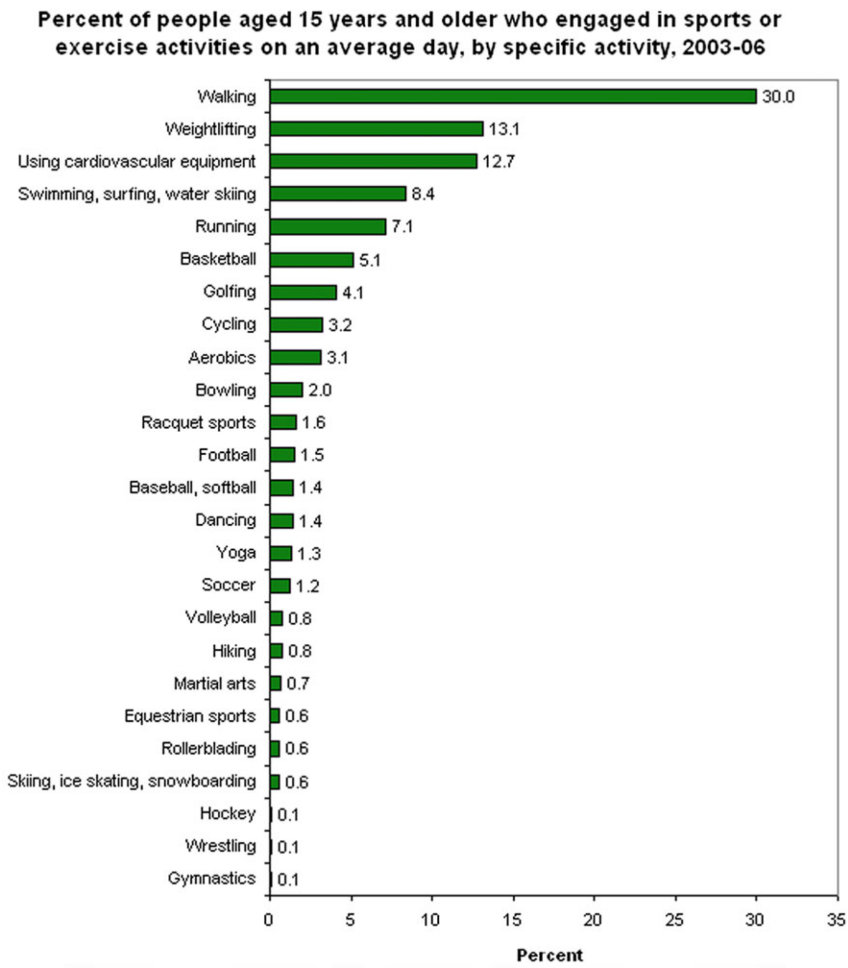

Figure A.13.2 Engagement in sports per type of exercise (Source: Bureau of Labor Statistics, 2008).

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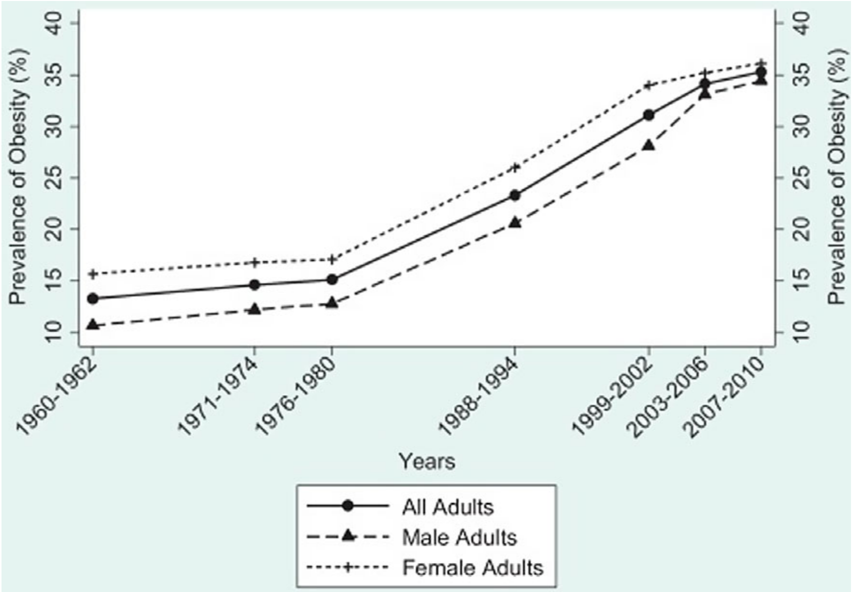

Figure A.13.3 Obesity prevalence over time in the USA [Source: Cawley (2015)].

Table A.13.1 Calorie spent per activity

Activity	Percentage of total time	Calorie expenditure per pound per hour
Walking	$= 30/(30 + 13.1 + 12.7) = 0.54$	$378/185 = 2.0$
Weight lifting	$= 13.1/(30 + 13.1 + 12.7) = 0.23$	$252/185 = 1.4$
Cardio	$= 12.7/(30 + 13.1 + 12.7) = 0.23$	$880/185 = 4.7$
Weighted average		2.5

Sources: Bureau of Labor Statistics (2008) and Harvard Medical School (2021).

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