It is of course true that the practical formula based on the hypothesis is found in the result to be very similar to some particular formula of the Summation Method, which itself gives results that sufficiently justify Professor Whittaker's description "that every little bit of the graduated curve is to be nearly "a bit of a parabola, in fact we have really a continually varying "parabola, not the same one the whole way along." Thus, in fact, the hypothesis does not lead us astray, but on the contrary leads to a new and brilliant method of graduation. But this does not in itself justify the hypothesis as such; and it still seems to me doubtful how far the method can be regarded as based on the Theory of Probability when it involves a hypothesis that is not true, and a constant that we have no practical means of determining. It is for this reason that it appears to me to be better (as it is simpler) to reach the expression $S+\epsilon F$ by more general and less theoretical considerations, as indicated in my remarks previously referred to. After this the whole of the work of Professor Whittaker and Dr. Aitken proceeds as before, and that being so it may be considered that the question I have raised is purely academic; but it seems important that the real philosophical basis of a new and important method should be clearly understood.

I am, Sir,

Your obedient Servant,

G. J. LIDSTONE.

9 ST. ANDREW SQUARE,
EDINBURGH.

To the Editor of the Transactions of the Faculty of Actuaries.

SIR,

Let me thank you for allowing me to see Mr. Lidstone's letter.

If I may be allowed a few words of comment, it seems to me that he has introduced the term *most probable* without having defined the meaning of these words, and in a context where it is

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difficult to understand what they do mean. Perhaps the best way of making my objection clear will be to give an analogous problem where the same point occurs in a more obvious fashion.

Suppose it is required to find the plane closed curve which satisfies the following two conditions:—

(i) it encloses a given area A.

(ii) it is such that the integral of the square of the curvature, taken along the curve, is a minimum.

The required curve is a circle of radius $\surd(A/\pi)$.

Now, since the integral of the square of the curvature is to be made a minimum, Mr. Lidstone would (I presume) say that the *most probable* curve was that for which the curvature was everywhere zero: that is to say, a straight line. But it seems to me that this is a misuse of the term *most probable*: a straight line has really nothing whatever to do with the problem.

Similarly, I demur to Mr. Lidstone's reference to the simple curve of the second degree as the *most probable* curve in the graduation problem: this curve has, I think, nothing to do with the method.

I am, Sir,

Yours faithfully,

E. T. Whittaker.

Mathematical Institute,
16 Chambers Street,
Edinburgh.

After having seen Professor Whittaker's reply, Mr. Lidstone wrote as follows:—

To the Editor of the Transactions of the Faculty of Actuaries.

Sir,

I used the term "most probable" with the ordinary meaning "having the greatest chance"—in this case the *a priori* chance discussed in Whittaker's hypothesis.

I am sorry that Professor Whittaker has not removed my difficulty by explaining where my argument is erroneous or

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