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# ON A BROWN-HALMOS-PEARCY LEMMA

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To begin with, we say that an operator (bounded linear transformation) $A$ on a separable complex Hilbert space $H$ is a commutator if there exist operators $B$ and $C$ such that $A = BC - CB$. The following lemma is due to Brown, Halmos and Pearcy [**3**, p. 698].

LEMMA. *If the matrix* $(a_{ij})_{i,j=0}^{\infty}$ *of an operator* $A$ *on* $H$ *with respect to an orthonormal basis* $\{x_0, x_1, \ldots\}$ *is such that* $a_{ij} = 0$ *whenever* $i + j$ *is an even integer, then* $A$ *is a commutator.*

The authors proceed to give a proof of this lemma by noting that $A = (\frac{1}{2}A)D_\alpha - D_\alpha(\frac{1}{2}A)$, where $D_\alpha$ is a diagonal operator with diagonal $\{\alpha_0, \alpha_1, \ldots, \alpha_n, \ldots\}$ and $\alpha_n = (-1)^n$. This proof is obviously incorrect.

Although, the works of Brown and Pearcy [**2**] and that of Anderson and Stampfli [**1**] have superseded these results, nevertheless, this Lemma has several important consequences. To name a few, every weighted unilateral shift and weighted bilateral shift are commutators and analogos results are also valid with operator weights (consult [**3**]). Thus, for the benefit of the reader it may be helpful to have a correct proof of this lemma.

*Proof of the Lemma.* Observe that $D_\alpha A D_\alpha = -A$ and $D_\alpha^2 = 1$. Thus, it follows immediately that

$$(-\tfrac{1}{2}D_\alpha A)D_\alpha - D_\alpha(-\tfrac{1}{2}D_\alpha A) = A.$$

This proves the lemma.

## REFERENCES


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