J. Austral. Math. Soc. (*Series A*) **29** (1980), 52–60

SOME MULTIPLIERS ON $H_P^1(G)$

HIROSHI YAMAGUCHI

(Received 14 August 1978; revised 15 February 1979)

Communicated by E. Strzelecki

Abstract

In this paper, we define the function space $H_P^1(G)$ on a LCA group G with the algebraically ordered dual, and construct a multiplier on $H_P^1(G)$ similar to the one given by Gaudry (1968).

1980 Mathematics subject classification (*Amer. Math. Soc.*): 43 A 22

1. Introduction

Let G be a LCA group with dual $\hat{G}$. For $1 \leqslant p < \infty$, $L^p(G)$ denotes the usual Lebesgue space with respect to a Haar measure on G. For a subset E of $\hat{G}$, let $L_E^1(G)$ be the subspace of $L^1(G)$ consisting of those functions whose Fourier transforms vanish off E. Let $M(G)$ be the Banach algebra of all complex valued bounded regular measures on G. In Gaudry (1968), he showed the following interesting example.

Let $H^1(T)$ be the Hardy space. That is,

$$H^1(T) = \{f \in L^1(T); \hat{f}(n) = 0 \text{ for } n < 0\}.$$

We define a function ψ on Z^+ (subsemigroup of Z consisting of nonnegative integers) by $\psi = \chi_E$, where $E = \{a_n \in Z^+ \setminus \{0\}; a_{n+1}/a_n > 3\}$ and χ_E denotes the characteristic function of E. By Paley's theorem, for each $f \in H^1(T)$, there exists a function $g \in H^2(T)$ such that $\psi(n)\hat{f}(n) = \hat{g}(n)$ for every $n \in Z$ ($\psi(n)\hat{f}(n)$ is 0 if n does not belong to Z^+).

Therefore, ψ determines a multiplier on $H^1(T)$. But, by Rudin's F. and M. Riesz theorem, ψ does not belong to $M(T)\hat{}|_{Z^+}$.

Since $E = \operatorname{supp}(\psi)$ is a lacunary sequence, we note that

$$L^1_{\operatorname{supp}(\psi)}(T) \subset \bigcap_{1 \leqslant p < \infty} L^p(T).$$

https://doi.org/10.1017/S1446788700020929 Published online by Cambridge University Press

DEFINITION 1. Let Γ be a LCA group. Γ is called an *algebraically ordered group* if there exists a subsemigroup P of Γ which satisfies the (AO)-condition, namely, (i) $P \cup (-P) = \Gamma$ and (ii) $P \cap (-P) = \{0\}$.

Γ is an algebraically ordered group if and only if it is torsion-free (Rudin (1962a), p. 194).

Let G be a LCA group and P be a subsemigroup of $\hat{G}$ satisfying the (AO)-condition. Suppose P is not dense in $\hat{G}$. Now, we define $H^1_P(G)$ as follows:

$$H^1_P(G) = \{f \in L^1(G); \hat{f}(\gamma) = 0 \text{ on } P^c\}.$$

REMARK 1. If P is dense in $\hat{G}$, then $H^1_P(G) = \{0\}$. If $G = R$ and $P = R^+$, then $H^1_P(G) = H^1(R)$, where R^+ is a subsemigroup of R consisting of nonnegative real numbers.

Our purpose in this paper is to construct a multiplier on $H^1_P(G)$ similar to the one given by Gaudry.

2. Multipliers on $H^1_P(G)$

Let G be a LCA group with the dual group $\hat{G}$, and let P be a subsemigroup of $\hat{G}$ with the (AO)-condition such that P is not dense in $\hat{G}$.

DEFINITION 2. Let S be a bounded linear operator on $H^1_P(G)$. S is called a *multiplier* if S commutes with every translation operator τ_x on G.

The following two lemmas are due to Meyer (1968).

LEMMA 1. *Suppose S is a bounded linear operator on $H^1_P(G)$. Then, the following are equivalent:*

(1) *S is a multiplier,*

(2) *there exists a function $\psi \in L^\infty(P^\circ)$ such that $\widehat{Sf}(\gamma) = \psi(\gamma)\hat{f}(\gamma)$ on $\hat{G}$ for every $f \in H^1_P(G)$, where $\psi(\gamma)\hat{f}(\gamma)$ is 0 if γ does not belong to P°.*

DEFINITION 3. $\Phi \in L^\infty(P^\circ)$ is also called a *multiplier* on $H^1_P(G)$ if there exists a multiplier S_Φ on $H^1_P(G)$ such that $\widehat{S_\Phi f}(\gamma) = \Phi(\gamma)\hat{f}(\gamma)$ on $\hat{G}$ for every $f \in H^1_P(G)$. We define a norm $\|\Phi\|$ by $\|\Phi\| = \|S_\Phi\|$.

LEMMA 2. (Meyer (1968), § 1.1 Corollary 3). *Suppose G is a LCA group and E is a closed subset of $\hat{G}$. Let Λ be a closed subgroup of $\hat{G}$ and H be the annihilator of Λ.*

Let ψ be in $L^\infty(E^c)$. If ψ is a multiplier on $L^1_{E^c}(G)$, then ψ_Λ (restriction of ψ to $\Lambda \cap E^c$) is also a multiplier on $L^1_{\Lambda\cap E^c}(G/H)$ such that $\|\psi_\Lambda\| \leqslant \|\psi\|$.

DEFINITION 4. For $b \in Z$, define $\mathbf{b} \in Z^n$ by $\mathbf{b} = (b, 0, \ldots, 0)$. For a subset $F \subset Z$, we define $\mathbf{F}$ by $\mathbf{F} = \{\mathbf{b};\ b \in F\}$.

PROPOSITION 3. *Let P be a subsemigroup of R^n with the (AO)-condition such that P° (the interior of P) $= \{x = (x_1, x_2, \ldots, x_n) \in R^n;\ x_1 > 0\}$. Let F be a sequence $\{a_m\}$ in $Z^+ \backslash \{0\}$ such that $a_{m+1}/a_m > 3$ $(m = 1, 2, 3, \ldots)$. Define a function ψ on $\{y = (y_1, y_2, \ldots, y_n) \in Z^n;\ y_1 > 0\}$ by*

$$\psi(y) = \begin{cases} 1 & \text{if } y \in \mathbf{F} = \{\mathbf{a}_m;\ m = 1, 2, \ldots\}, \\ 0 & \text{otherwise.} \end{cases}$$

Let $\Phi(z) = \sum \psi(l)\Delta(z - l)$ for $z \in P^\circ$, where sum is taken over the set

$$\{l = (l_1, l_2, \ldots, l_n) \in Z^n\colon l_1 \geqslant 1\},$$

and where $\Delta(x) = \prod_{i=1}^n \max(1 - 3|x_i|, 0)$, $x = (x_1, x_2, \ldots, x_n) \in R^n$. Then, Φ is a multiplier on $H^1_P(R^n)$ with the following property:

$$\text{(I)} \qquad \Phi \notin M(R^n)^\wedge|_{P^\circ}.$$

PROOF. Let us denote by $\leqslant_*$ the order on Z^n induced by a semigroup $P \cap Z^n$. For $l \in P \cap Z^n$, let $G_{\mathbf{F}}(l) = \operatorname{Card}\{\{\gamma \in Z^n;\ l \leqslant_* \gamma \leqslant_* 2l\} \cap \mathbf{F}\}$. Then, $G_{\mathbf{F}}(l)$ is a bounded function on $P \cap Z^n$. Hence, by Rudin (1962a), Theorem 8.6, p. 213, $\psi\hat{g} \in H^1_{P\cap Z^n}(T^n)^\wedge$ for every $g \in H^1_{P\cap Z^n}(T^n)$. By Meyer (1968), Theorem 3, p. 510, Φ is a multiplier on $H^1_P(R^n)$. Since $M(R^n)^\wedge|_Z = M(T)^\wedge$, Φ does not belong to $M(R^n)^\wedge|_{P^\circ}$.

The following two lemmas are due to Otaki (1977) (Lemma 1, Lemma 3).

LEMMA 4. *Let F be a compact abelian group and P a subsemigroup of F satisfying the (AO)-condition. Then, P is dense in F.*

LEMMA 5. *Let P be a subsemigroup of R^n with the (AO)-condition such that it is not dense in R^n. Then, there exists a unitary transformation τ on R^n such that $\tau(P^\circ) = \{x = (x_1, x_2, \ldots, x_n) \in R^n\colon x_1 > 0\}$.*

PROPOSITION 6. *Let P be a subsemigroup of R^n satisfying the (AO)-condition. Suppose P is not dense in R^n. Then, there exists a multiplier S_Φ on $H^1_P(R^n)$ with the following property:*

$$\text{(I)} \qquad \Phi \in M(R^n)^\wedge|_{P^\circ},$$

where Φ is a bounded measurable function on P° corresponding to S_Φ.

PROOF. By Lemma 5, there exists a unitary transformation τ on R^n such that $\tau(P^\circ) = \{x = (x_1, x_2, \ldots, x_n) \in R^n;\ x_1 > 0\}$. Hence, by Proposition 3, there exists a multiplier Φ' on $H^1_{\tau(P)}(R^n)$ such that $\Phi' \notin M(R^n)^\wedge|_{\tau(P^\circ)}$. Define a function Φ on P° by $\Phi(x) = \Phi'(\tau^{-1}(x))$. Then, Φ is a multiplier on $H^1_P(R^n)$ such that $\Phi \notin M(R^n)^\wedge|_{P^\circ}$.

LEMMA 7. *Let F be a LCA group and Z the usual integer group. Let P be a subsemigroup of $Z \oplus F$ with the (AO)-condition such that it is not dense in $Z \oplus F$. If P is dense in F, then we have*

$$P = \{(n,f) \in Z \oplus F;\ n > 0,\ \textit{or}\ n = 0\ \textit{and}\ f \geqslant_P 0\}, \quad \textit{or}$$

$$= \{(n,f) \in Z \oplus F;\ n < 0,\ \textit{or}\ n = 0\ \textit{and}\ f \geqslant_P 0\},$$

where $>$ denotes the usual order on Z and $\geqslant_P$ denotes the order on F induced by P. In particular, by Lemma 4, the conclusion holds when F is compact.

PROOF. We consider only the case $P \cap Z = \{n \in Z;\ n \geqslant 0\}$. Suppose $(1, f_0) \in (-P)$ for some $f_0 \in F$. Since $F \subset (-P)^-$ and $(-P)^-$ is a semigroup, we have $Z \oplus F \subset (-P)^-$. This is a contradiction. Hence, we have

$$P = \{(n,f) \in Z \oplus F;\ n > 0, \text{ or } n = 0 \text{ and } f \geqslant_P 0\}.$$

PROPOSITION 8. *Let G be a LCA group such that the dual group $\hat{G}$ has an open compact subgroup F_0. Let P be a subsemigroup of $\hat{G}$ with the (AO)-condition such that it is not dense in $\hat{G}$. Then, there exists a multiplier S_Φ on $H^1_P(G)$ with the following property*:

$$\text{(I)} \qquad \Phi \notin M(G)^\wedge|_{P^\circ},$$

where Φ is a function in $L^\infty(P^\circ)$ corresponding to S_Φ.

PROOF. Let $I_1 = \{\gamma \in \hat{G};\ O(\gamma + F_0) < \infty\}$ and $I_2 = \{\gamma \in \hat{G};\ O(\gamma + F_0) = \infty\}$, where $O(\gamma + F_0)$ denotes the order of $\gamma + F_0$ in $\hat{G}/F_0$. Let $[\gamma + F_0]$ be an open subgroup of $\hat{G}$ generated by $\gamma + F_0$. Then, $\hat{G} = \bigcup_{\gamma \in I_1} [\gamma + F_0] \cup \bigcup_{\gamma \in I_2} [\gamma + F_0]$. By Lemma 4, P is necessarily dense in $\bigcup_{\gamma \in I_1} [\gamma + F_0]$. Hence by the hypothesis, there exists some $\gamma_0 \in I_2$ such that P is not dense in $[\gamma_0 + F_0] \cong Z \oplus F_0$. Hence, by Lemma 7, $P \cap [\gamma_0 + F_0] = \{(n,f) \in Z \oplus F_0;\ n > 0, \text{ or } n = 0 \text{ and } f \geqslant_P 0\}$, where $\geqslant_P$ denotes the order on F_0 induced by P. Let H be an annihilator of $[\gamma_0 + F_0]$. Then, $G/H \cong [\gamma_0 + F_0]^\wedge = T \oplus \hat{F}_0$. Let F be a sequence $\{a_m\}$ in $Z^+ \backslash \{0\}$ such that

https://doi.org/10.1017/S1446788700020929 Published online by Cambridge University Press

$a_{m+1}/a_m > 3$ $(m = 1, 2, \ldots)$. We define a function ψ on $Z^+ \setminus \{0\}$ by

$$\psi(l) = \begin{cases} 1 & \text{if } l \in F, \\ 0 & \text{otherwise.} \end{cases}$$

For $f \in H^1_{P\cap[\gamma_0+F_0]}(T \oplus \hat{F}_0)$, f can be represented as follows:

$$f = \sum_{m=1}^{\infty} f_m \times d\delta_{-x_m},$$

where

$$f_m \in H^1_0(T) = \{g \in H^1(T);\ \hat{g}(0) = 0\}$$

and δ_{-x_m} are Dirac measures at $-x_m \in \hat{F}_0$. Moreover,

$$\|f\|_1 = \sum_{m=1}^{\infty} \|f_m\|_1.$$

Let S_ψ be a multiplier on $H^1_0(T)$ such that $S_\psi f(l) = \psi(l)\hat{f}(l)$ $(l \in Z)$ (see Section 1). For

$$f = \sum_{m=1}^{\infty} f_m \times d\delta_{-x_m} \in H^1_{P\cap[\gamma_0+F_0]}(T \oplus F_0),$$

we define an operator S_1 on $H^1_{P\cap[\gamma_0+F_0]}(T \oplus \hat{F}_0)$ by

$$S_1(f) = \sum_{m=1}^{\infty} S_\psi(f_m) \times d\delta_{-x_m}.$$

For $(l, s) \in (Z^+ \setminus \{0\}) \oplus F_0$, put $\Phi_1(l, s) = \psi(l)$. Then,

$$\begin{aligned} \widehat{S_1(f)}(l, s) &= \sum_{m=1}^{\infty} \widehat{S_\psi(f_m)}(l)(x_m, s) \\ &= \psi(l) \sum_{m=1}^{\infty} \hat{f}_m(l)(x_m, s) \\ &= \psi(l)\hat{f}(l, s) \\ &= \Phi_1(l, s)\hat{f}(l, s) \quad \text{for } (l, s) \in Z \oplus F_0. \end{aligned}$$

Hence, S_1 is a multiplier on $H^1_{P\cap[\gamma_0+F_0]}(T \oplus \hat{F}_0)$. Define a bounded linear operator A_1 from $H^1_P(G)$ to $H^1_{P\cap[\gamma_0+F_0]}(G/H)$ as follows:

$$\widehat{A_1(f)} = \hat{f}|_{[\gamma_0+F_0]} \quad \text{for } f \in H^1_P(G).$$

Next, we define a bounded linear operator A_2 from $H^1_{P\cap[\gamma_0+F_0]}(G/H)$ to $H^1_P(G)$ by

$$\widehat{A_2(g)}(\gamma) = \begin{cases} \hat{g}(\gamma) & \text{if } \gamma \in [\gamma_0 + F_0], \\ 0 & \text{otherwise.} \end{cases}$$

Furthermore, define a bounded linear operator S_2 on $H_P^1(G)$ by $S_2 = A_2 \circ S_1 \circ A_1$ (see Figure 1).

$$\begin{array}{ccc} H^1_{P\cap[\gamma_0+F_0]}(G/H) & \xrightarrow{\ S_1\ } & H^1_{P\cap[\gamma_0+F_0]}(G/H) \\ {\scriptstyle A_1}\big\uparrow & & \big\downarrow{\scriptstyle A_2} \\ H^1_P(G) & \xrightarrow{\ S_2\ } & H^1_P(G) \end{array}$$

FIGURE 1.

Let Φ be a function on P° defined by

$$\Phi(\gamma) = \begin{cases} \Phi_1(\gamma) & \text{if } \gamma \in [\gamma_0+F_0] \cap P^\circ \cong (Z^+ \setminus \{0\}) \oplus F_0, \\ 0 & \text{if } \gamma \in P^\circ \cap (\hat{G} \setminus [\gamma_0+F_0]). \end{cases}$$

Then, S_2 is a multiplier on $H_P^1(G)$ and $\widehat{S_2(f)}(\gamma) = \Phi(\gamma)\hat{f}(\gamma)$. Since

$$\Phi_1 \notin M(G/H)^\wedge|_{[\gamma_0+F_0]\cap P^\circ},$$

we have $\Phi \notin M(G)^\wedge|_{P^\circ}$. This completes the proof of Proposition 8.

We need the following lemmas in order to prove the main theorem.

LEMMA 9. *Let Γ be a LCA group and P a subsemigroup of Γ satisfying the (AO)-condition. Let F be an open subgroup of Γ such that P is dense in it. If there exists an element $\gamma_0 \in \Gamma$ such that $-P$ is not dense in γ_0+F, then we have $P \supset \gamma_0+F$.*

PROOF. Since $-P$ is not dense in γ_0+F, there exists an open subset V of F such that $(\gamma_0+V)\cap(-P)^- = \varphi$. Hence, we have $\gamma_0+V \subset P$. Since P is dense in F, we have $V+P \supset F$. Therefore, we have $\gamma_0+F \subset \gamma_0+V+P \subset P$.

LEMMA 10. *Suppose F is a compact abelian torsion-free group. Let P be a subsemigroup of $R^n \oplus F$ with the (AO)-condition. If P is not dense in $R^n \oplus F$, then P includes $(P_{R^n})^\circ + F$, where $P_{R^n} = P \cap R^n$.*

PROOF. Since P is necessarily dense in F, P is not dense in R^n. Hence, by Lemma 5, there exists an unitary transformation τ on R^n such that

$$\tau((P_{R^n})^\circ) = \{x = (x_1, x_2, \dots, x_n) \in R^n;\ x_1 > 0\}.$$

Define an automorphism $\tilde{\tau}$ on $R^n \oplus F$ by $\tilde{\tau}(z,t) = (\tau(z), t)$ for $(z,t) \in R^n \oplus F$. Then, $\tilde{\tau}(P)$ is a subsemigroup of $R^n \oplus F$ with the (AO)-condition such that it is not dense in $R^n \oplus F$. $\leqslant_P$ and $\leqslant_{\tilde{\tau}(P)}$ denote the orders on $R^n \oplus F$ induced by P and $\tilde{\tau}(P)$

respectively. Suppose there exist $y=(y_1,y_2,\ldots,y_n)\in(P_{R^n})^\circ$ and $s\in F$ such that $y\not\leqslant_P s$. Then, $\tau(y)=\tilde{\tau}(y)\not\leqslant_{\tilde{\tau}(P)}\tilde{\tau}(s)=s$. Let $\tau(y)=(x_1,x_2,\ldots,x_n)$ $(x_1>0)$. Then, for $z=(z_1,z_2,\ldots,z_n)\in R^n$ with $z_1<x_1$, we obtain that

$$z=(0,s)+(z-\tau(y),0)+(\tau(y),-s)\in(-\tilde{\tau}(P))+(-\tilde{\tau}(P))^-+(-\tilde{\tau}(P))^-=(-\tilde{\tau}(P)).$$

Since $(-\tilde{\tau}(P))^-$ is a semigroup, R^n is contained in $(-\tilde{\tau}(P))^-$. This is a contradiction. Hence, we have $P\supset(P_{R^n})^\circ+F$.

THEOREM 11. *Let G be a nondiscrete LCA group with the dual group $\hat{G}$. Suppose there exists a subsemigroup P of $\hat{G}$ with the (AO)-condition such that it is not dense in $\hat{G}$. Then, there exists a multiplier S_Φ on $H^1_P(G)$ with the following property:*

$$\text{(I)} \qquad \Phi\notin M(G)\hat{}\,|_{P^\circ},$$

where Φ is a function in $L^\infty(P^\circ)$ which corresponds to S_Φ.

PROOF. By the structure theorem, $\hat{G}\cong R^n\oplus F$, where n is a nonnegative integer and F is a LCA group containing an open compact subgroup F_0.

(Case i): If $n=0$, it has been proved in Proposition 8.

(Case ii): Suppose $n\geqslant 1$. Let $\Lambda=R^n\oplus F_0$. Then, Λ is an open subgroup of $\hat{G}$.

(Case ii)$_I$: Suppose P is not dense in Λ. Then, by Lemma 10, $P\cap\Lambda$ contains $(P_{R^n})^\circ+F_0$. Note $G/\Lambda^\perp\cong\hat{\Lambda}\cong R^n\oplus F_0$. For each $g\in H^1_{P\cap\Lambda}(G/\Lambda^\perp)$, g can be represented by

$$g(x,y)=\sum_{m=1}^{\infty}g_m(x)\times d\delta_{-z_m}(y)$$

with $\|g\|_1=\sum_{m=1}^\infty\|g_m\|_1$, where $g_m\in H^1_{P_{R^n}}(R^n)$ and δ_{-z_m} are Dirac measures at $-z_m\in\hat{F}_0$ $(m=1,2,\ldots)$. By Proposition 6, there exists a multiplier $S^{(1)}$ on $H^1_{P_{R^n}}(R^n)$ with the following property:

$$\text{(a)} \qquad \psi^{(1)}\notin M(R^n)\hat{}\,|_{P_{R^n}},$$

where $\psi^{(1)}$ is a function in $L^\infty((P_{R^n})^\circ)$ corresponding to $S^{(1)}$. Define a bounded linear operator $S^{(2)}$ on $H^1_{P\cap\Lambda}(G/\Lambda^\perp)$ by

$$S^{(2)}(g)=\sum_{m=1}^{\infty}S^{(1)}(g_m)\times d\delta_{-z_m}$$

for $g(x,y)=\sum_{m=1}^\infty g_m(x)\times d\delta_{-z_m}(y)\in H^1_{P\cap\Lambda}(G/\Lambda^\perp)$. For $(s,t)\in(P_{R^n})^\circ\oplus F_0$, put $\Phi^{(2)}(s,t)=\psi^{(1)}(s)$. Then, $\widehat{S^{(2)}(g)}(s,t)=\Phi^{(2)}(s,t)\hat{g}(s,t)$ for $(s,t)\in R^n\oplus F_0$.

Hence, $S^{(2)}$ is a multiplier on $H^1_{P\cap\Lambda}(G/\Lambda^\perp)$ corresponding to $\Phi^{(2)}$. Next, we define a bounded linear operator from $H^1_P(G)$ to $H^1_{P\cap\Lambda}(G/\Lambda^\perp)$ such that

https://doi.org/10.1017/S1446788700020929 Published online by Cambridge University Press

$A_1(h)^\wedge = \hat{h}|_\Lambda$, and A_2 is a bounded linear operator from $H^1_{P\cap\Lambda}(G/\Lambda^\perp)$ to $H^1_P(G)$ such that

$$\widehat{A_2(k)}(\gamma) = \begin{cases} \hat{k}(\gamma) & \text{if } \gamma\in\Lambda, \\ 0 & \text{otherwise.} \end{cases}$$

Let Φ be a function on P° defined by

$$\Phi(\gamma) = \begin{cases} \Phi^{(2)}(\gamma) & \text{if } \gamma\in P^\circ\cap\Lambda, \\ 0 & \text{if } \gamma\in P^\circ\cap(\hat{G}\setminus\Lambda). \end{cases}$$

Then, S is a multiplier on $H^1_P(G)$ corresponding to Φ. Since $\Phi^{(1)}\notin M(R^n)^\wedge|_{(P_{R^n})^\circ}$, $\Phi\notin M(G)^\wedge|_{P^\circ}$.

(Case ii)$_{II}$: Suppose P is dense in Λ. Since P is not dense in $\hat{G}$, there exists $\gamma_0\in\Lambda$ such that P is not dense in $\gamma_0+\Lambda$. Suppose the order of $\gamma_0+\Lambda$ in $\hat{G}/\Lambda$ is finite. By Lemma 9, $\gamma_0+\Lambda$ is contained in $-P$. Hence, we have $[\gamma_0+\Lambda]\subset -P$. This is a contradiction. Therefore, the order of $\gamma_0+\Lambda$ in $\hat{G}/\Lambda$ is infinite. Hence, $[\gamma_0+\Lambda]\cong R^n\oplus F_0\oplus Z$. Put $\Lambda_0 = [\gamma_0+\Lambda]$. By Lemma 7, we have

$$P\cap\Lambda_0\cong\{(x,t,l)\in R^n\oplus F_0\oplus Z;\ l>0, \text{ or } l=0 \text{ and } (x,t)=(x,t,0)\geqslant_P 0\}.$$

Hence, $P\cap R^n\oplus Z\cong\{(x,l)\in R^n\oplus Z;\ l>0, \text{ or } l=0 \text{ and } x\geqslant_{P_{R^n}}0\}$. We put

$$Q = P\cap R^n\oplus Z.$$

Then, by Proposition 6 and Lemma 2, there exists a multiplier $S^{(1)}$ on $H^1_Q(R^n\oplus T)$ such that $\psi^{(1)}\notin M(R^n\oplus T)^\wedge|_{Q^\circ}$, where $\psi^{(1)}$ is a function in $L^\infty(Q^\circ)$ corresponding to $S^{(1)}$.

Let $\Phi^{(2)}$ be a function on $P^\circ\cap\Lambda_0$ such that $\Phi^{(2)}(x,t,l)=\psi^{(1)}(x,l)$ for (x,l) $(=(x,0,l))\in Q^\circ$ and $t\in F_0$. Let Φ be a function on P° defined by

$$\Phi(\gamma) = \begin{cases} \Phi^{(2)}(\gamma) & \text{if } \gamma\in P^\circ\cap\Lambda_0, \\ 0 & \text{if } \gamma\in P^\circ\cap(\hat{G}\setminus\Lambda_0). \end{cases}$$

Then, evidently, $\Phi\notin M((G)^\wedge|_P$. By the same method as in (Case ii)$_I$, we can show that there exists a multiplier S on $H^1_P(G)$ corresponding to Φ. This completes the proof of Theorem 11.

REMARK 1. By the construction of Φ in Theorem 11, we note that the following is established:

$$L^p_{\mathrm{supp}(\Phi)}(G)\subset\bigcap_{p\leqslant q<\infty}L^q(G)\quad(1\leqslant p\leqslant 2),$$

where

$$L^p_{\mathrm{supp}(\Phi)}(G)=\{f\in L^p(G);\ \hat{f}(\gamma)=0 \text{ a.e. on } (\mathrm{supp}(\Phi))^c\}.$$

https://doi.org/10.1017/S1446788700020929 Published online by Cambridge University Press

Finally, I wish to express my thanks to Professor S. Yamamuro and referees for their valuable advice and suggestions.

References

R. E. Edwards and G. I. Gauudry (1977), *Littlewood–Paley and multipliers theorem* (Springer-Verlag, Berlin–Heidelberg–New York).

G. I. Gaudry (1968). 'H^p multipliers and inequality of Hardy and Littlewood', *J. Austral. Math. Soc.* **10**, 23–32.

E. Hewitt and K. Ross (1971), *Abstract harmonic analysis*, Vol. II (Springer-Verlag, Berlin–Heidelberg–New York).

R. Larsen (1971). *An introduction to the theory of multipliers* (Springer-Verlag, Berlin–Heidelberg–New York).

Y. Meyer (1968), 'Endomorphismes de inéaux fermés de $L^1(G)$, classes de Haerdy, et séries de Fourier lacunaires, *Ann. Sci. École Norm. Sup.* (4) **1**, 499–580.

H. Otaki (1977), 'A relation between the F. and M. Riesz theorem and the structures of LCA groups,' *Hokkaido Math. J.* **VI**, 306–312.

W. Rudin (1962a), *Fourier analysis on groups* (New York), Interscience).

W. Rudin (1962b), 'Trigonometric series with gaps', *J. Math. Mech.* **9**, 203–228.

Department of Mathematics
Hokkaido University
Sapporo
Japan

Current address:
Department of Mathematics
Josai University
Sakado, Saitama
Japan