Chapter 8

How do we put it together?

HOW OUGHT ONE PARAMETERIZE THE MILKY WAY?

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1. The parameter glut

In a well known paper written a quarter century ago, cosmology was described as a search for two numbers (Sandage 1970). As those of you who have attended any of the parallel sessions of Symposium 167 surely appreciate, today's cosmologists find more than just two numbers interesting. It is likewise clear from the present symposium that the number of interesting parameters associated with the Milky Way is very much greater than two.

To illustrate the levels of complexity with which we must deal, I've constructed an incomplete chart of Milky Way structure which is something of a caricature but will still serve to make my point.

TABLE 1. Milky Way Structure

components	sub-components	sub-sub-	sub-sub-sub-
disk →	stellar →	thin →	old
	neutral ↘	thick ↘	young
	molecular ↘		
	dust ↘		
bulge →	metal rich		
	metal poor		
halo →	visible →	stellar	
	dark ↘	gaseous	
center			
ring			

Each of these components and sub-components has its associated scale lengths and multiple shape parameters. As bad as this may seem, we know

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that the situation is worse yet in other galaxies. There is a class of galaxies, the polar rings, which exhibit two disks with roughly perpendicular angular momentum axes (Schweizer *et al.* 1983). And there is a new class of galaxies (Rubin *et al.* 1992; Merrifield and Kuijken 1994) with *counter*-rotating stellar disks.

The proliferation of components is daunting. It is tempting to restrict one's attention to a single component, invoking the principle of "divide and conquer." This is a popular strategy, judging from the presentations at this symposium. But while the components may have very different shapes and enrichment histories, their dynamics are governed by a single gravitational potential. It therefore behooves us to look for a simple parameterized model of the potential.

2. Oort's parameters

In Oort's youth, when astronomy was carried out only at visible wavelengths, most of what one knew about the Milky Way was inferred from a relatively small neighborhood centered on the Sun. It made sense to approximate the mean patterns of observed proper motions and radial velocities as truncated power and Fourier series,

$$\bar{\mu} = (B + A\cos 2l - C\sin 2l) + \mathcal{O}\left(\frac{r}{R_0}\right) \quad , \quad \text{and} \qquad (1.a)$$

$$\bar{v}_{los} = r(K + A\sin 2l + C\cos 2l) + \mathcal{O}\left(\frac{r^2}{{R_0}^2}\right) \quad . \qquad (1.b)$$

This is first order in (r/R_0), where r is distance from the Sun, and second order in galactic longitude l. The four coefficients are the Oort parameters,

B: rotation about the Sun's position,
A: differential rotation,
K: Hubble-like expansion, and
C: differential expansion.

Oort's C and K are less familiar than B and A because they are usually taken to be zero on the assumption of circular orbits. The fundamentally local nature of Oort's parameterization is evident from the fact that only with the addition of higher order terms (e.g. Pont *et al.* 1994) can one derive a distance to the galactic center, R_0. Units of length are completely absent from Oort's four constants.

The addition of R_0, a global parameter, to Oort's four local parameters is esthetically unsatisfying. Moreover, as one observes stars and gas at distances r approaching R_0, one needs higher order terms in Oort's expansion (again, see Pont *et al.* 1994). Many authors (e.g. Caldwell and Coulson 1987) have used global rather than local models. Kuijken and Tremaine (1992, 1994; henceforth KT92 and KT94) introduced a non-axisymmetric global parameterization which strikes a balance between flexibility and complexity. In abandoning Oort we note that his parameterization has proven so durable precisely because local data have until now been very much better than global data.

3. Kuijken and Tremaine's parameters

The Kuijken and Tremaine model, in its full glory, involves six parameters, but for the sake of comparison with the Oort parameterization is useful to suppress one of these. It then includes

R_0	Sun's distance from the galactic center,
v_0	average circular velocity at the Sun's distance from the galactic center,
α	logarithmic derivative of the dependence of circular velocity on radius, $v \sim (R/R_0)^\alpha$,
ϵ_Ψ	ellipticity of the potential at R_0, and
ϕ_Ψ	angle between major axis of the potential and Sun-center line, positive in rotation direction.

In practice one finds that ϵ_Ψ and ϕ_Ψ usually occur in the combinations

$$c_\Psi = \epsilon_\Psi \cos 2\phi_\Psi \quad \text{and} \tag{2.a}$$

$$s_\Psi = \epsilon_\Psi \sin 2\phi_\Psi \quad . \tag{2.b}$$

These correspond to deviations from axisymmetry which are, respectively, symmetric and antisymmetric about the line from the Sun through the center of the galaxy. It is often the case that measurements of some particular kind are much more sensitive to one of these than to the other. If c_Ψ is positive, the LSR is presently at apocenter. If s_Ψ is positive, the apocenter of the LSR is in the second quadrant.

Kuijken and Tremaine adopt a "standard model" with $\alpha = 0$. This model gives a flat (constant) rotation curve and has the pleasing property that the equipotentials and closed orbits have (to first order) the same ellipticity but lie perpendicular to each other. For this model the connection with Oort's parameters is particularly simple (KT94),

$$A = -B = \left(\frac{v_0}{2R_0}\right)(1 + c_\Psi) \quad \text{and} \tag{3.a}$$

$$C = -K = \left(\frac{v_0}{2R_0}\right) s_\Psi \quad . \tag{3.b}$$

We note that having dropped the assumption of circular orbits, it is no longer true that $A - B = \Omega$, the angular velocity of the LSR. Were we to drop the assumption that $\alpha = 0$, it would no longer be true that $A = -B$.

The interpretation of the local velocity ellipsoid is particularly straightforward using the KT parameterization. Taking ℓ_v to be the galactic longitude (in radians) of its long axis and X^2 as the squared ratio the azimuthal velocity dispersion to to the radial disperson, Kuijken and Tremaine find

$$\ell_v = -2s_\Psi \quad \text{and} \tag{4.a}$$

$$X^2 \approx \frac{1}{2} - \frac{3}{2}c_\Psi \quad , \tag{4.b}$$

where in the second equation the first term on the right hand side represents the limit of small velocity dispersion. As the velocity dispersion grows, Kuijken and Tremaine (1992) and Cuddeford and Binney (1994) find that this term is significantly greater than 1/2, particularly when the ratio of the distance to the galactic center to the scale length for the disk is greater than 2, which it would appear to be.

The sixth parameter in the KT formulation characterizes the variation of ellipticity of the potential with radius. Its effect on the observable quantities is smaller than than that of α (although of the same order). For the sake of simplicity we shall assume here that the equipotentials are similar ellipses.

4. Nature abhors a power law

The KT "standard model" has a rotation curve which remains flat as R increases. The dark and luminous components of the Milky Way, whatever they might be, surely end somewhere between here and Andromeda. It therefore makes sense to truncate our assumed power law potential at some radius, R_{cutoff}. There has been considerable discussion in the literature about where that cutoff might be (e.g. Norris and Hawkins 1991, Leonard and Tremaine 1989, Little and Tremaine 1987, Zaritsky *et al.* 1989).

Three circumstances combine to make the determination of this cutoff radius difficult. First, unlike the case of the disk of our galaxy (where even a single star can give information on the assumption that the orbit is closed or nearly closed), orbits of stars and star clusters at large galactocentric radii are likely to be far more random. Second, in the absence of proper motion data, we can only measure one of a distant object's three velocity components, the component which is approximately radial with respect to the center of the galaxy. Finally, the numbers of objects at large galactocentric radii are small. At this point, one might reasonably ask, "If it is so hard to measure, why do we care?" We just do.

5. The acceleration perpendicular ,o the galactic plane

The Oort and KT parameterizations are two dimensional, and tell us nothing about the structure of the galaxy perpendicular to the plane. Just as there is a radial distribution of mass, there is also a vertical distribution. Unfortunately for our understanding, but fortunately from the point of view of constructing a simple model, it may suffice to specify only one parameter, a local surface density, Σ_{local}. If one makes the simplifying assumption that disk is infinitely thin, then its surface mass density is obtained by observing the scale height, h, and vertical velocity dispersion, σ^2_{zz} of a tracer population with an assumed Maxwellian velocity distribution,

$$\Sigma_{local} = \frac{\sigma^2_{zz}}{2\pi G h} \quad ,$$

where we have ignored higher order terms which come into play with large dispersions perpendicular to the plane. It is instructive to compare this local determination with the surface density which would be needed if the disk were solely responsible for the radial gradient in the potential. This is particularly simple for a Mestel disk, for which

$$\Sigma_{Mestel} = \frac{{v_0}^2}{2\pi G R_0} \quad .$$

Taking the ratio of these two surface densities we have

$$\frac{\Sigma_{local}}{\Sigma_{Mestel}} = \frac{\sigma^2_{zz}}{{v_0}^2}\frac{R_0}{h} \quad .$$

Adopting values typical for stars like the Sun, $\sqrt{\sigma^2_{zz}} = 20$ km/s and $h = 300$ pc, and canonical values for v_0 and R_0 (Kerr and Lynden-Bell 1986) we get a ratio of 1/4. More realistic models give slightly larger ratios (Schechter 1992), but the the conclusion that much of the mass in the Milky Way lies elsewhere than in the disk would seem hard to escape.

6. Nuisance parameters

Sometimes, when a young radio astronomer talks about radial velocity measurements and I'm feeling mischievous, I'll ask how these have been corrected for the Sun's motion with respect to the Local Standard of Rest. In most cases the answer is "Oh the computer corrects for it automatically." The Sun's peculiar velocity is typical of stars of its type, and of no special interest except that measurements of the parameters which *are* interesting require that we know it. The three components of the Sun's velocity therefore qualify as "nuisance" parameters. In my own experience values for these derived variously from Cepheids and carbon stars agree less well with the standard values than one would like. The Sun's distance from the galactic plane, which becomes important when considering dust and young stars at large distances, would likewise qualify as a "nuisance" parameter.

7. Bar parameters

The evidence for a barlike structure inside the solar circle is very persuasive. It is is seen photometrically in IR balloon data (Blitz and Spergel 1991), IRAS point sources (Weinberg 1993), Miras (Whitelock *et al.* 1991), COBE DIRBE maps (Arendt *et al.* 1994, and most recently in RGB clump stars (Stanek *et al.* 1994). It is seen dynamically in effects which are attributed to the rotation of a bar, including the gas motions at the center of the galaxy (Gerhard and Vietri, 1986; Binney *et al.* 1991) and the molecular (and perhaps stellar) ring at 4 kpc. Doubters may point to the significant differences in the photometrically and dynamically derived orientations for the bar, but I suspect that this discrepancy will quickly be resolved.

Such a bar adds a minimum of three parameters to our model: the bar's corotation radius, an orientation angle, ϕ_b (different from the ϕ_Ψ measured at roughly the Sun's distance from the galactic center) and a measure of the bar's strength at some fiducial radius. A fourth parameter, describing how that strength varies with radius, may also be needed.

Binney (1993) has argued that if the bar's corotation radius is at the position determined from the dynamics of galactic center gas, of order 2.4 kpc, its effect near the solar neighborhood would be negligible. We might then use the KT parameterization, since the bar can be treated as a dynamical subsystem which at large distances looks like a point mass. Conversely, at radii typical of the bar, the outer part of the Galaxy would, at worst, make itself felt through its quadrupole contributions to the potential. Not all parameters are important at all radii – they often decouple.

However Weinberg and Combes have each argued at this meeting that the bar may have a corotation radius as large as 5 kpc, in which case the the potential of the bar strongly influences the motions of stars in the Sun's neighborhood. There is always a transition region where the bar and the outer parts of the galaxy make comparable contributions to the orbits of stars. It may be our misfortune that this happens closer to the Sun's position than we would have preferred.

8. Top Ten Parameters

Ignoring our four nuisance parameters and the parameters which govern the rates at which the bar and halo quadrupole terms vary with radius, we are left with ten parameters,

R_0	Sun's distance from center,
v_0	circular velocity,
c_Ψ	symmetric ellipticity,
s_Ψ	antisymmetric ellipticity,
α	rotation curve logarithmic slope,
Σ_{local}	local surface density,
R_{cutoff}	edge of galaxy,
R_{corot}	corotation radius,
ϕ_b	bar position angle, and
b/a	bar strength.

9. Recent Results

The axis ratio of the local velocity ellipsoid as has long been a puzzle. If our galaxy had a flat rotation curve and closed orbits near the LSR were circular, the axis ratio, as measured in the plane, ought to be greater than 1/2 (KT92). It isn't. When Kuijken and Tremaine (1994) interpret the axis ratio as a deviation from axisymmetry, they find $c_\Psi = +0.12 \pm 0.04$. They find additional support for this interpretation using Merrifield's (1992) method for measuring the rotation curve.

The consequences of this conclusion are far reaching. It would imply that the circular velocity at the Sun's distance from the galactic center is smaller, by a factor of order $1 + 2c_\Psi$, than has heretofore been thought. It would also imply that distances to the galactic center, as estimated from the kinematics of stars assumed to be on circular orbits, (e.g. Caldwell and Coulson 1987; Gwinn *et al.* 1992), would be overestimated, again by a factor of order $1 + 2c_\Psi$.

Metzger (1994), in a recently completed Ph.D. thesis, has measured velocities for 8 newly discovered Cepheids toward $\ell = 300°$ (Caldwell, Keane and Schechter, 1991; Schechter *et al.* 1992) and used these to obtain a new determination of the distance to the galactic center. These eight Cepheids, *taken by themselves*, give $R_0 = 7.8 \pm 0.4$ kpc. The agreement with the "best" value given in Reid's (1993) recent review, 8.0 ± 0.5 kpc, is excellent. This would argue against a large positive value for c_Ψ.

Metzger (1994) has himself searched for new Cepheids at a complementary galactic longitude, $\ell = 60°$. Their positions, projected onto the galactic

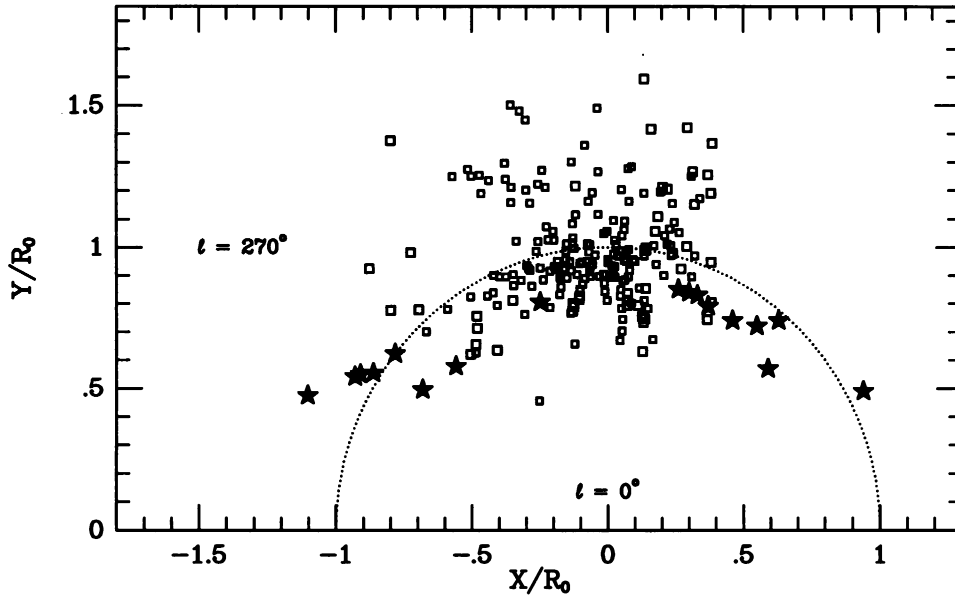

Figure 1. Newly discovered Cepheids toward $\ell = 300°$ by Caldwell, Keane and Schechter (1991) and toward $\ell = 60°$ by Metzger (1994) are shown projected onto the plane of the Galaxy as filled symbols. Previously known Cepheids are shown as open symbols.

plane, are shown in Figure 1. These are among the most heavily obscured Cepheids known, with $V-I$ colors of 4 and 5. If velocities can be measured they will provide considerable leverage on s_Ψ.

A longstanding mystery in the study of Cepheid kinematics has been their apparent net blueshift (e.g. Kraft and Schmidt 1963). The possibility that this was due to a non-zero value of Oort's K had been appreciated, but one could not rule out the possibility of a systematic error in the center of mass, "γ" velocities. Pont *et al.* (1994) have compared velocities for five Cepheids with those of their parent open clusters and find stunningly good agreement with the cluster means. Pont *et al.* find a net blueshift of 2.3 km/s for a sample of 266 Cepheids. Fitting their data to a model which allows for non-zero antisymmetric ellipticity, I find $s_\Psi = +0.037 \pm 0.017$. One can see the sense of the effect either by following the signs in equations (1b) and (3b) or by examinining Figure 2.

One of the most exciting recent results has only just been reported at this meeting, Backer and Sramek's proper motion for Sgr A* (see also Backer and Sramek 1987) , the radio source identified with the galactic center. They find a longitudinal component, $\mu_\ell = -5.83 \pm 0.4$ mas/yr. Taking R_0 to be 8.0 ± 0.5 kpc, and letting $A - B = 26.4 \pm 0.5$ km/sec/kpc (Kerr and Lynden-Bell 1986) gives $c_\Psi = +0.01 \pm 0.04$ in Kuijken and Tremaine's

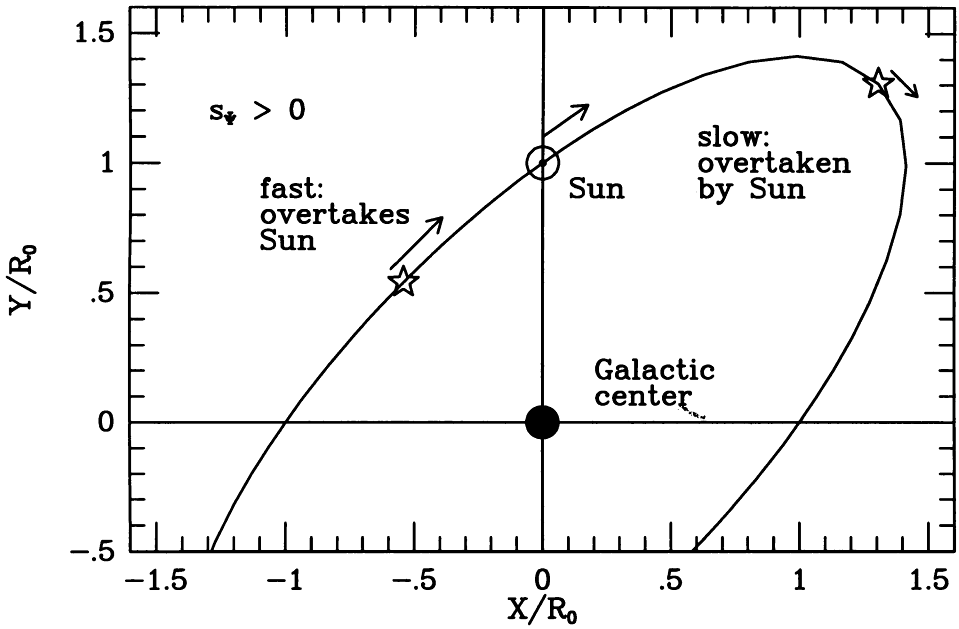

Figure 2. Closed elliptical orbits passing close to the LSR produce a net blueshift for stars in the solar neighborhood when $s_\Psi > 0$.

standard model, on the assumption that Sgr A* is at rest with respect to the center of the galaxy.

TABLE 2. Recent Results

$c_\Psi = 0.12 \pm 0.04$	velocity ellipsoid
	Kuijken and Tremaine (1994)
$R_0 = 7.8 \pm 0.4$ kpc	Cepheid velocities
	Metzger (1994)
$s_\Psi = 0.04 \pm 0.02$	Cepheid velocities
	Pont *et al.* (1994)
$\mu_\ell = -5.83 \pm 0.4$ mas/yr	Sgr A* / VLA
	Backer and Sramek (this symposium)

10. Prospects for improved parameter estimates

Both the proper motion of the Galactic center and the distance to the Galactic center would now appeaar to be measured to better than one part in 20. One of the beauties of proper motion measurements is that uncertainties continue to grow smaller until the investigators lose interest

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in the problem. We are likely to see improvements soon both in the Cepheid derived value of R_0 and in the RR Lyrae value. Some of the eclipsing binaries in the bulge found by the MACHO and OGLE collaborations may ultimately yield yet better values of R_0.

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DISCUSSION

K. Stanek: How do the deviations from circular orbits change the distance to the galactic center as measured using the Cepheids?

Schechter: Kuijken & Tremaine tell us that with positive c_Ψ the Cepheids overestimate the distance to the center by ~20%. The apparent agreement between the Cepheid kinematic distance & other photometric distances may challenge the large positive value of c_Ψ.

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