INFLUENCE OF THE SOLID INNER CORE AND COMPRESSIBILITY OF THE FLUID CORE ON THE EARTH NUTATION

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1 INTRODUCTION

While calculating low frequency oscillations of the Earth liquid core spherical harmonic representation of the deformation field is usually used [1-3]:

$$\mathbf{u} = \sum\sum [S_l^m + T_l^m] \ . \qquad (1)$$

Substitution of (1) into the equations of motion gives an infinite system of differential equations for scalar functions S_l^m and T_l^m. Approximate solutions of such a system are obtained by truncating of the system. But results of [4] show that sometimes such method divergences.

2 NUTATION OF THE EARTH WITH COMPRESSIBLE CORE

2.1 MODEL. Let us calculate a forced nutation amplitude for the Earth model consisting of the rigid mantle and compressible liquid core with simple liquid density distribution:

$$\rho(r_e) = \rho_0(1-\delta r_e^2), \qquad (2)$$

where r_e is dimensionless radius.

Let us investigate behavior of such a system affected by tide-generating potential. The mantle angular velocity may be written in the form:

$$\Omega = \{ \eta(\mathbf{i}-i\mathbf{j})\mathrm{ext}(i\sigma t) + \mathbf{k} \}\omega \quad , \qquad (3)$$

and nutation amplitude η must be found.

2.2 EQUATIONS OF LIQUID OSCILLATIONS. Small oscillations of the fluid core are described by the following equations:

$$\Delta\psi - \frac{4\omega^2}{\sigma^2}\frac{\partial^2\psi}{\partial z^2} = \qquad (4)$$

$$= -(\sigma^2-4\omega^2)\frac{\psi-[V_1+V_t+\eta\omega^2(xz-iyz)\exp(i\sigma t)]}{\alpha^2} - \frac{\sigma^2-4\omega^2}{i\sigma}\mathbf{v}\frac{\nabla\rho}{\rho},$$

$$\Delta V_1 = 4\pi G\rho(1/\alpha^2)(\psi-V_1-V_t-\eta\omega^2(xz-iyz)\exp(i\sigma t)) \qquad (5)$$

$$\psi = P_1/\rho+V_1+V_t+ \eta\omega^2(xz-iyz)\exp(i\sigma t) \ , \qquad (6)$$

Here $\mathbf{v}$, P_1, V_1 are perturbations of liquid velocity, pressure and gravitansional potential correspondingly, α is speed of sound and V_t is tide-generating potential.

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2.3 METHOD OF SOLUTION. Let us represent the solution in the form:

$$\psi = \psi_p + \delta\psi \quad , \qquad (7)$$

where ψ_p is the Poincare solution:

$$\psi_p = -\eta\sigma\omega \frac{1-1/k-\tau^2/(1-\varepsilon_c^2)}{1-1/k+\tau^2/(1-\varepsilon_c^2)} (xz-iyz), \quad k=\sigma/2\omega, \quad \tau^2=1-1/k^2 . \qquad (8)$$

Let represent $\delta\psi$ by expansion on characteristic functions of the Poincare operator:

$$\delta\psi = \sum_m \sum_l \sum_k \left\{a^m_{lk}\Psi^m_{lk}\right\}\exp(i\sigma t) \quad , \qquad (9)$$

and Ψ^m_{lk} functions satisfy to the equation

$$(\Delta - \frac{4\omega^2}{\sigma^2}\frac{\partial^2}{\partial z^2})\Psi^m_{lk} = \lambda^m_{lk}\Psi^m_{lk} \quad . \qquad (10)$$

Substitution of (7) and (9) into the system (4)-(5) gives a system of equations for a^m_{lk}. Nondiagonal elements of the matrix of this system, as numerical results show, are small compared to the diagonal ones. This allows the truncation the expansion (9) in order to obtain an approximate solution.

2.3 MAIN RESULTS AND DISCUSSION. Nutation amplitudes for different maximum numbers of characteristic functions in (9) are given in table 1. For comparison, nutation amplitudes for the rigid and Poincare models are also shoun.

Table 3. Nutation amplitudes η in angular milliseconds (δ=0.2, ε_c=0.0715) for an Earth with rigid mantle.

$\frac{\omega}{\sigma+\omega}$	Solid model	Poincare model	Compressible core model		
			M=2	M=4	M=6
-6800	8051.05	7999.60	8000.851	8000.848	8000.848
-365.3	24.94	-38.633	-25.536	-25.552	-25.554
-182.6	22.60	28.197	28.329	28.329	28.329

The results obtained show that the compressibility of the fluid Earth core may significantly affect theoretical nutation amplitudes and must be taken into account while calculating nutation amplitudes. Comparison of theoretical nutation amplitude with radiointerferometer data may give some information about the Earth core.

3 NUTATION OF EARTH WITH SOLID INNER AND FLUID OUTER CORES

3.1 FORMULATION OF THE PROBLEM. Let us investigate a simplified earth model consisting of: (i) rigid mantle with ellipsoidal cavity; (ii) ideal homogeneous incompressible liquid, filling the cavity; (iii) solid inner core, which is under the influence of the gravitation field in the center of the cavity.

Let the surface of the cavity and the surface of the inner core be discribed by the equations:

$$x^2 + y^2 + z^2/(1-\varepsilon_1^2) = R_1^2 \quad , \tag{11}$$

$$x^2 + y^2 + z^2/(1-\varepsilon_2^2) = R_2^2 \quad . \tag{12}$$

Here R_1 and R_2, ε_1 and ε_2 are equatorial radiuses and eccentricities of cavity and solid core correspondingly.

Let us examine the system behavior under the action of the tide-generating potential. The angular velocity of mantle rotation is of the form:

$$\Omega = \{ \eta(\mathbf{i}-i\mathbf{j})\mathrm{ext}(i\sigma t) + \mathbf{k} \}\omega, \tag{13}$$

Motion of the inner core with respect to the moving coordinate system may be described by instantaneous angular velocity:

$$\delta\omega = \eta_1\omega\ (\mathbf{i} - i\mathbf{j})\exp(i\sigma t) \tag{14}$$

Unknown amplitudes of nutation η and η_1 must be determined from the solution of the problem.

The small oscillations of the fluid core are described by the following equation:

$$\Delta\psi - \frac{4\omega^2}{\sigma^2}\frac{\partial^2\psi}{\partial z^2} = 0 \quad , \tag{15}$$

with boundary conditions of the form:

$$\hat{\mathbb{B}}_o\psi = \frac{ik\sigma\eta}{2(1-k^2)R_1}\left(1 - \frac{1}{k} - \frac{\tau^2}{1-\varepsilon_1^2}\right)(xz - iyz) \;, \qquad \mathbf{r} \in S_1$$

$$\hat{\mathbb{B}}_o\psi = \left[\frac{ik\sigma\eta}{2(1-k^2)R_2}\left(1 - \frac{1}{k} - \frac{\tau^2}{1-\varepsilon_2^2}\right) + \frac{i\eta_1\omega}{R_2}\,\frac{\varepsilon_2^2}{1-\varepsilon_2^2}\right](xz-iyz), \quad \mathbf{r} \in S_2$$

$$\hat{\mathbb{B}}_o\psi = \frac{-ik}{2\omega(1-k^2)R}\left\{\left(x - \frac{y}{ik}\right)\frac{\partial\psi}{\partial x} + \left(y + \frac{x}{ik}\right)\frac{\partial\psi}{\partial y} + \frac{\tau^2 z}{1-\varepsilon^2}\,\frac{\partial\psi}{\partial z}\right\} \tag{16}$$

$$k = \sigma/2\omega \;, \qquad \tau = 1 - 1/k^2 \;.$$

Here S_1 and S_2 are the surfaces of the cavity and the solid core correspondingly.

3.2 RESULTS. Approximate solution of the boundary value problem (7), (9) may be written in the form:

$$\psi \approx (a_1 + a_2)(xz - iyz). \tag{17}$$

$$a_1 = -\eta\sigma\omega \frac{1-1/k-\tau^2/(1-\varepsilon_1^2)}{1-1/k+\tau^2/(1-\varepsilon_1^2)}$$

$$a_2 = -\frac{\eta\sigma\omega}{h_2^*}\left\{\left(1 - \frac{1}{k} - \frac{\tau^2}{1-\varepsilon_2^2}\right) - \left(1 - \frac{1}{k} + \frac{\tau^2}{1-\varepsilon_2^2}\right)\frac{1-1/k-\tau^2/(1-\varepsilon_1^2)}{1-1/k+\tau^2/(1-\varepsilon_1^2)}\right\} - \frac{2\eta_1\omega^2(1-k^2)}{kh_2^*}\frac{\varepsilon_2^2}{1-\varepsilon_2^2}$$

$$-h_2^* R_2^5 = \left\{\left(1 - \frac{1}{k} + \frac{\tau^2}{1-\varepsilon_1^2}\right)R_1^5 - \left(1 - \frac{1}{k} + \frac{\tau^2}{1-\varepsilon_2^2}\right)R_2^5\right\}$$

In table 2 there are represented the values of nutation amplitudes for: (i) the solid Earth model, (ii) the Poincare model, (iii) the examined model. Numbers in Table 2 shows that correction to forced nutation amplitude due to the existance of the inner core, may be greater than the accuracy of observations.

Table 1. Nutation amplitude in angular miliseconds (r_2=0.357r_1, ε_1=0.0715, ε_2=0).

$\frac{\omega}{\sigma+\omega}$	Rigid model	Poincare model	Solid inner core model
-6800	8051.05	7999.92	8000.23
-365.3	24.91	-38.63	-31.24
-182.6	22.60	28.16	28.20
182.6	530.80	571.71	571.63

REFFERENSES

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