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OBITUARY

F. BATH, O.B.E., Ph.D., F.R.S.E.

Frederick Bath took his first degree of B.Sc., with first class honours in mathematics, at Bristol and proceeded to King's College, Cambridge, for research in geometry under H. W. Richmond, F.R.S. He took the Cambridge Ph.D. in 1927, being already then an assistant lecturer at King's College, London. In 1928 he took the decisive step of moving to Scotland, being appointed to a post at what was then University College Dundee from April that year. The staff, by present day standards, was tiny; to teach both pure and applied mathematics there were the professor, a single lecturer, and at most two assistants. The professor had occupied the chair since the foundation of the college in 1883 and Bath proved to be his right-hand man in the fullest sense, gaining his complete confidence. He at once joined our Society and was an active member, attending its jubilee dinner in Edinburgh in 1933 and later that year becoming an editor of our Proceedings—a post he held until his secondment to wartime administration. In January 1936 we had the privilege of acquiring him as a colleague in Edinburgh. He was elected President of the Society in 1938.

Bath was encouraged by Richmond to catalogue and describe the geometry of algebraic curves on del Pezzo's quintic surface—the only non-ruled quintic surface in [5]; the resulting work is in (**1**). But there was also a strong impulse to endeavour to generalise the classical geometry of the 28 bitangents of a non-singular plane quartic Γ^3 and study the $2^{p-1}(2^p-1)$ contact primes of Γ^p, the canonical curve of genus p. Hesse had labelled the 28 bitangents by the 28 unordered pairs of eight letters, and shown that there were 36 essentially different ways of doing this. Cayley observed that to pass from any one of these 36 ways to any other one could use a bifid substitution—bifid meaning the splitting of an octad into complementary tetrads. In (**2**) Bath used E. Pascal's labelling of the 120 tritangent planes of Γ^4 by the 120 unordered triads of ten letters and proferred a bifid substitution splitting the decad into a tetrad and complementary hexad. This substitution was called in aid by Coxeter in his celebrated early work (**C**, p. 173).

Bath perceived that the notation of Hesse and Pascal could be extended to canonical curves of any genus, and it was the realisation of this that triggered his joint work (**4, 5**) with Richmond. Γ^p has order $2p-2$ and lies in a projective space $[p-1]$; it possesses $2^{p-1}(2^p-1)$ contact primes, each touching it at $p-1$ points, and $2^{2p}-1$ systems of contact quadrics, each touching it at $2p-2$ points and not containing it entirely. These primes and systems of quadrics can be labelled by what Clifford called marks: arrays

$$\begin{pmatrix}\alpha_1, \alpha_2, \ldots, \alpha_p\\ \beta_1, \beta_2, \ldots, \beta_p\end{pmatrix}$$

of two rows and p columns with every entry either 0 or 1, the mark being even or odd with $\sum \alpha_i \beta_i$. When the marks serve as period characteristics they label the $2^{2p}-1$ systems of contact quadrics; the zero mark is exceptional, the corresponding quadrics collapsing as repeated primes. When the marks serve as theta characteristics the $2^{p-1}(2^p-1)$ odd marks label the contact primes, and here the zero mark is not exceptional.

Bath and Richmond assert that this notation is too complicated; that it does not conduce to clarity to use the same label for both kinds of characteristic; that the laborious proofs in treatises on Abelian functions are unnecessary; that no interpretation is given for the $2^{p-1}(2^p+1)$ even theta characteristics. They produce a notation using $2p+2$ letters, residual sets labelling the same geometric object, the period characteristics being replaced by sets with an even number of letters. Their notation shows to best advantage when p is even; then sets of 1, 5, 9, 13,... letters replace theta characteristics of the same parity as $\frac{1}{2}p$, sets of 3, 7, 11,... letters replace theta characteristics of opposite parity to $\frac{1}{2}p$. If, for example, $p=4$ the even theta characteristics are replaced by sets of 1, 5, 9 letters; their number, since complementary sets label the same object, is

$$10+\tfrac{1}{2}{}^{10}C_5=2^3(2^4+1).$$

They describe a method for passing to genus p from genus $p+1$; it applies instantly both to the geometry and to the new notation. As Γ^p has order less by 2 than has Γ^{p+1} and lies in a space of dimension less by 1 it will be a projection of a Γ^{p+1} specialised to acquire a double point (its genus dropping thereby to p). But for present purposes the double point should be a cusp, the reason being that contact primes would then coalesce in threes through the cusp and not just in twos through a node. Indeed

$$2^p(2^{p+1}-1)-3.2^{p-1}(2^p-1)=2^{p-1}(2^p+1).$$

The number, on the right, of contact primes of the cusped curve that do not pass through the cusp proffers the even theta characteristics for geometrical interpretation.

One must now speak about *Bath's Theorem*. A plane quadrilateral affords, by omitting each of its sides in turn, four triangles; their four circumcircles have a common point P (Wallace's Theorem) and their four circumcentres are on a circle C (Steiner). Five coplanar lines, no three concurrent, afford, by omitting each in turn, five quadrilaterals; the five points P are on a circle Γ (the Miquel circle) and the five circles C have a common point Q (de Longchamps). Bath's Theorem is that Q *is on* Γ.

To recognise a hitherto unsuspected object in a field of geometry inaugurated by a Scottish professor not later than 1804 and assiduously tilled by geometers over the intervening years is surely an achievement, and it received its accolade from H. F. Baker—one of the more assiduous among the tillers—when (**B**, p. 344) he wrote of "Dr. Bath's capital result". I happen to know, as the informant, that Baker was startled when informed of it a month or two after its discovery. Bath *proved* it (**7**) by projection from four dimensions, but he had *discovered* it when, preparing in the spring of 1938 an informal talk requested by a group of Edinburgh students, he drew some diagrams: a mode of serendipity happily still with us ($\mathbf{R}_1$, Fig. 2A; $\mathbf{R}_2$, Figs. 4, 7). Thirty years later (**L-H**, p. 394) the result was still being called remarkable.

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It was planned that (6) would have supplements, but none ever appeared. The exigencies of war constrained the higher powers to look to the universities for administrative assistance, and when the appeal reached Edinburgh Bath was recommended to become, as E. T. Whittaker put it, one of our rulers. Though he neither knew it nor intended it he was never to return to academic life.

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W. L. Edge