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EMPIRICAL BAYES WITH A CHANGING PRIOR

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Consider the usual decision model. We are required to make a decision $t(x)$ about an unknown parameter θ on the basis of the observation $X = x$. X is distributed according to $f(x/\theta)$ and θ has the distribution function G . The risk for the decision procedure $t : X \to A$ (the action space) is

(1) $$R(t, G) = \int_X \int_\Omega L(t(x), \theta) f(x/\theta) dG(\theta) d\mu(x) .$$

The Bayes procedure $t_G(x)$ minimises (1) and this minimum is denoted $R(G)$. The empirical Bayes method assumes that G is unknown but that we have information in the form of the outcomes of independent and identical replications of the above component problem. Suppose, however, that the history of the decision problem is such that the component problems change over time. This thesis considers the problem where the *a priori* distribution of θ at stage $n+1$ is $G_{n+1}(\theta)$. In particular we consider the sequence $(\theta_1, X_1), \ldots, (\theta_n, X_n)$ where the X_i are drawn independently, and given $\theta_i = \theta$, X_i is distributed according to the density $f(x/\theta)$ but that for each i , θ_i is drawn from the distribution G_i . For this case an empirical procedure $t_n(x) = t_n(x_1, \ldots, x_n; x)$ is defined to be *asymptotically optimal* if

(2) $$\lim_{n\to\infty} \{R_n(T, G_{n+1}) - R(G_{n+1})\} = 0 ,$$

where $R_n(T, G_{n+1})$ is the overall expected loss, for the sequence

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$T = \{t_n\}$, at stage $n + 1$.

Firstly the usual empirical Bayes problem and associated rates of convergence to optimality is considered. Specific rates are given for certain parametric examples. General results on asymptotic optimality for the modified problem are developed. The problem is then examined for location parameter families in which the *a priori* mean is assumed to change in a linear fashion. Estimators of both the empirical density type and the kernel function type are constructed for the unconditional density of the observations $f_{G_{n+1}}(x)$. These are used to construct asymptotically optimal estimators for both the two-action problem and the estimation problem with squared error loss. Generalised convergence rates are developed for the exponential family of densities. Some examples of parametric *a priori* distributions are also considered.

The problem of the selection of the 'best' of several populations is studied in the modified empirical Bayes framework. Asymptotically optimal procedures for this problem, under a linear loss structure, are developed. The modified empirical Bayes method is then applied to a reliability model. Finally the modified problem is extended to examine the case where the *a priori* means are random variables generated by a martingale process. Asymptotically optimal estimators, with associated rates of convergence, are established here under the assumption that the *a priori* distributions are members of a parametric family.