(A LETTER TO PROFESSOR J. LAMBEK)

30th December, 1968

Dear Professor Lambek:

In your review of A.P. Morse's, A Theory of Sets, [Canadian Mathematical Bulletin, II (1968) 354] you state in effect that

$$\{x\} = 0$$

would seem to follow from the definition of singleton on page 42 and axiom 2.5.0., namely

$$x \longleftrightarrow (0 \in x) .$$

Presumably here you mean 'sng x' instead of '$\{x\}$' which is not defined until page 60. This quibble aside, you perhaps argue as follows

$$\begin{aligned} x \neq 0 \longrightarrow \text{sng } x &= \wedge y(y \rightarrow (x \in y)) \\ &= \wedge y(0 \in y \rightarrow x \in y) \\ &= 0 . \end{aligned}$$

The error appears in the second equality. Although

$$\wedge y(y \rightarrow x \in y) \longleftrightarrow \wedge y(0 \in y \rightarrow x \in y)$$

follows from 2.5.0.,

$$\wedge y(y \rightarrow x \in y) = \wedge y(0 \in y \rightarrow x \in y)$$

does not, no more than does

$$y = (0 \in y).$$

More generally (see 2.9)

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$$(x = y) \longrightarrow (x \longleftrightarrow y).$$

However, the single arrow does not always reverse. Sometimes it does, as for instance in

$$(*) \qquad ((p \rightarrow q) \longleftrightarrow (\sim p \vee q)) \rightarrow ((p \rightarrow q) = (\sim p \vee q)).$$

Intuitively sng x is the intersection of all sets of the form $\sim y$ where x does not belong to y. In contrast $\{x\}$ is intuitively the intersection of all sets y where x belongs to y. Here we concede that an empty intersection is the universe.

Intuition may be fortified by observing via (*) that

$$\text{sng } x = \wedge y(\sim y \vee (x \in y)), \quad \{x\} = \wedge y(\sim (x \in y) \vee y)$$

and realizing that

$$(x \in y) = U$$

if in fact

$$x \in y$$

and that

$$(x \in y) = 0$$

otherwise.

Yours truly,

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