different altitudes above the ground, and some observers assert that in the centre of the disturbance ascending vertical currents often exist. In connection with this point it may be of interest to refer to Buys Ballot's law for cyclonic storms. In these there is a central area where the barometer is low and the wind blows round this area. According to the law in question the wind does not blow perpendicularly to the line joining the observer to the point where the barometer is lowest, but is more or less directed towards the centre of the depression. Now, in accordance with the results we have obtained, if the motion were in two dimensions this law would be true only if the section of the vortex were contracting, in which case the density would be increasing and the barometer rising at the centre of the depression. Further, the magnitude of the radial velocity would be proportional to the rate of variation with the time in the height of the barometer. If the barometer were falling throughout the area of the disturbance the direction of the wind would be on the whole outwards from the centre. Thus, supposing Buys Ballot's law well founded, we must conclude either that vertical currents do exist in the centre of cyclonic storms, or else that cyclonic depressions fill up in much less time than they take to form. It should also be noticed that the rate of fluctuation of the barometer at any one station affords no clue to the law of fluctuation of the density at the centre of the disturbance. A rapid fall, for instance, might mean merely that the storm had a rapid motion of translation, or that the density diminished rapidly in approaching the centre of the depression.

A Theorem in Algebra.

By J. L. Mackenzie.

If we have given two equations $\phi(x)=0$ and $\psi(y)=0$, it is possible to express in the form of a determinant the equation whose roots are $f(x, y)$, where f is any given rational integral function.

Let α_r, β_r, be the sums of the r^{th} powers of the roots of the given equations, and s_r of the required equation. Then

$$\begin{aligned} s_r = \{f(a_1, b_1)\}^r + \{f(a_2, b_1)\}^r + \ldots\ldots \\ + \{f(a_1, b_2)\}^r + \{f(a_2, b_2)\}^r + \ldots\ldots \ \&c. \end{aligned}$$

https://doi.org/10.1017/S0013091500001401 Published online by Cambridge University Press

Thus to any term $\lambda x^m y^n$ in the expansion of $\{f(x, y)\}^r$, there will correspond in s_r the sum

$$\lambda a_1^m b_1^n + \lambda a_2^m b_1^n + \ldots\ldots\ldots + \lambda a_1^m b_2^n + \lambda a_2^m b_2^n + \ldots\ldots \text{ \&c.}$$
$$= \lambda a_m \beta_n.$$

Hence s_r may be found by expanding $\{f(x, y)\}^r$, and substituting a_m, β_n, for x^m, y^n in every term of the expansion.

It is not difficult to extend this to many cases where f is not a rational nor an integral function, and where there are more than two equations $\phi(x)$, $\psi(y)$, &c.

When s_r is known, the required equation is obtained in the form of a determinant by eliminating 1, p_1, $p_2 \ldots\ldots\ldots p_n$ between

$$u^n + p_1 u^{n-1} + p_2 u^{n-2} + \ldots\ldots\ldots + p_n = 0$$

and any n of Newton's equations

$$s_1 + p_1 = 0$$
$$s_2 + p_1 s_1 + 2p_2 = 0 \qquad \text{\&c.}$$

Taking the first n of these we get

$$\begin{vmatrix} u^n & u^{n-1} & u^{n-2} \ldots\ldots 1 \\ s_1 & 1 & 0 \quad \ldots\ldots 0 \\ s_2 & s_1 & 2 \quad \ldots\ldots 0 \\ \ldots\ldots & \ldots\ldots & \ldots\ldots \\ \ldots\ldots & \ldots\ldots & \ldots\ldots \\ s_n & s_{n-1} & s_{n-2} \ldots\ldots n \end{vmatrix} = 0.$$

It is evident that the only extraneous factor in this equation is $n\,!$

The calculations required for finding and reducing this determinant are usually very laborious; but I have applied this method to calculate the equations of certain loci derived from two conics. Write the equations of the conics in the form

$$u_2 + u_1 + u_0 = 0,$$
$$v_2 + v_1 + v_0 = 0;$$

or, in polar co-ordinates,

$$r_1^2 - P_1 r_1 + P_2 = 0,$$
$$r_2^2 - Q_1 r_2 + Q_2 = 0,$$

where $$P_1 = -\frac{u_1 r}{u_2},\ P_2 = \frac{u_0 r_2}{u_2},$$

with similar values for Q_1 and Q_2.

Then
$$a_1 = P_1,$$
$$a_2 = P_1^2 - 2P_2,$$
$$a_3 = P_1^3 - 3P_1P_2 \qquad \text{\&c.},$$

and similarly for β_r.

(1.) If a line through the origin cut the conic U in A, and V in B, to find the equation of the locus of a point P which divides AB in the ratio $\lambda : \mu$, $(\lambda + \mu = 1)$.

Here the required equation is

$$\begin{vmatrix} r^4 & r^3 & r^2 & r & 1 \\ 2(\lambda a_1 + \mu\beta_1), & 1 & 0 & 0 & 0 \\ 2(\lambda^2 a_2 + \lambda\mu a_1\beta_1 + \mu^2\beta_2), & s_1 & 2 & 0 & 0 \\ 2\lambda^3 a_3 + 3\lambda^2\mu a_2\beta_1 + 3\lambda\mu^2 a_1\beta_2 + 2\mu^3\beta_3, & s_2 & s_1 & 3 & 0 \\ 2\lambda^4 a_4 + 4\lambda^3\mu a_3\beta_1 + 6\lambda^2\mu^2 a_2\beta_2 + 4\lambda\mu^3 a_1\beta_3 + 2\lambda^4\beta_4, & s_3 & s_2 & s_1 & 4 \end{vmatrix}$$

This, when finally reduced, gives

$$\begin{aligned} & u_2^2(v_2 + \mu v_1 + \mu^2 v_0)^2 + \lambda^2 u_1^2 v_2(v_2 + \mu v_1 + \mu^2 v_0) \\ & + \lambda^2 u_0 u_2(2v_2^2 + 2\mu v_1 v_2 - 2\mu^2 v_0 v_2 + \mu^2 v_1^2) \\ & + \lambda u_1 u_2(\mu v_1 + 2v_2)(v_2 + \mu v_1 + \mu^2 v_0) \\ & + \lambda^3 u_0 u_1 v_2(\mu v_1 + 2v_2) \\ & + \lambda^4 u_0^2 v_2^2 = 0. \end{aligned}$$

(2.) If $OP^2 = OA.\ OB$, we get by the same method,

$$\begin{aligned} & u_2^2 v_2^2(u_2 v_2 - u_0 v_0)^2 - u_1 v_1 u_2^2 v_2^2(u_2 v_2 + u_0 v_0) \\ & + (u_2^2 v_2^2 - 4u_0 v_0 u_2 v_2)(u_1^2 v_0 v_2 + u_0 u_2 v_1^2) \\ & + u_1^4 v_0^2 v_2^2 + u_0^2 u_2^2 v_1^4 = 0. \end{aligned}$$

(3.) Finally, if P be the harmonic conjugate of O with respect to A and B, we get for the equation to the locus of P,

$$\begin{aligned} & (u_2 v_0 - u_0 v_2)^2 + (u_2 v_1 + u_1 v_2)(u_1 v_0 + u_0 v_1) \\ & + (4u_2 v_0 + u_1 v_1 + 4u_0 v_2)(u_1 v_0 + u_0 v_1) \\ & + 4(u_1 v_0 + u_0 v_1)^2 + 4u_0 v_0 u_1 v_1 \\ & + 8u_0 v_0(u_2 v_0 + u_0 v_2) \\ & + 16u_0 v_0(u_1 v_0 + u_0 v_1) \\ & + 16u_0^2 v_0^2 = 0. \end{aligned}$$

A Trigonometrical Note.

By Dr J. S. Mackay.
