We complete the argument in Chapter 8, where we show that if in any σ-closed extension of V there is no $X \subseteq \omega_1$ codable along a tree T, then $\aleph_1^V$ must be remarkable in L.

In Chapter 9 we review Schindler's proof of generic absoluteness from a remarkable cardinal to show that the argument gives a level-by-level upper bound: a strongly λ^+-remarkable cardinal is enough to get $L(\mathbf{R})$-absoluteness for λ-linked proper posets.

Chapter 10 is devoted to partially reversing the level-by-level upper bound of Chapter 9. Adapting the methods of Neeman, *Hierarchies of forcing axioms II*, we are able to show that $L(\mathbf{R})$-absoluteness for $|\mathbf{R}| \cdot |\lambda|$-linked posets implies that the interval $[\aleph_1^V, \lambda]$ is Σ_1^2-remarkable in L.

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SANDRO MÁRCIO DA SILVA PRETO, ***Semantics modulo satisfiability with applications: function representation, probabilities and game theory***, Institute of Mathematics and Statistics, University of São Paulo, São Paulo, Brazil, 2021. Supervised by Marcelo Finger. MSC: 03B70, 68T27, 60A86, 91A40. Keywords: Coherence of constraints, formal methods, function representation, Łukasiewicz infinitely-valued logic, Nash equilibrium, neural networks, non-classical probabilities, piecewise linear functions, probabilistic constraints, probabilistic satisfiability, propositional logics, rational McNaughton functions, uncertain games, valuation semantics.

Abstract

In the context of propositional logics, we apply semantics modulo satisfiability—a restricted semantics which comprehends only valuations that satisfy some specific set of formulas—with the aim to efficiently solve some computational tasks. Three possible such applications are developed.

We begin by studying the possibility of implicitly representing rational McNaughton functions in Łukasiewicz Infinitely-valued Logic through semantics modulo satisfiability. We theoretically investigate some approaches to such representation concept, called representation modulo satisfiability, and describe a polynomial algorithm that builds representations in the newly introduced system. An implementation of the algorithm, test results and ways to randomly generate rational McNaughton functions for testing are presented. Moreover, we propose an application of such representations to the formal verification of properties of neural networks by means of the reasoning framework of Łukasiewicz Infinitely-valued Logic.

Then, we move to the investigation of the satisfiability of joint probabilistic assignments to formulas of Łukasiewicz Infinitely-valued Logic, which is known to be an NP-complete problem. We provide an exact decision algorithm derived from the combination of linear algebraic methods with semantics modulo satisfiability. Also, we provide an implementation for such algorithm for which the phenomenon of phase transition is empirically detected.

Lastly, we study the game theory situation of observable games, which are games that are known to reach a Nash equilibrium, however, an external observer does not know what is the exact profile of actions that occur in a specific instance; thus, such observer assigns subjective probabilities to players actions. We study the decision problem of determining if a set of these probabilistic constraints is coherent by reducing it to the problems of satisfiability of probabilistic assignments to logical formulas both in Classical Propositional Logic and Łukasiewicz Infinitely-valued Logic depending on whether only pure equilibria or also mixed equilibria are allowed. Such reductions rely upon the properties of semantics modulo satisfiability. We provide complexity and algorithmic discussion for the coherence problem and, also, for the problem of computing maximal and minimal probabilistic constraints on actions that preserves coherence.

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Luka Mikec, ***On Logics and Semantics for Interpretability***, University of Barcelona, Spain, and University of Zagreb, Croatia (cotutelle), 2021. Supervised by Joost J. Joosten and Mladen Vuković. MSC: 03B45, 03B60, 03F45. Keywords: modal logic, metamathematics, formalised interpretability, interpretability logics, generalised Veltman semantics.

Abstract

We study various properties of formalised relativised interpretability. In the central part of this thesis we study for different interpretability logics the following aspects: completeness for modal semantics, decidability and algorithmic complexity.

In particular, we study two basic types of relational semantics for interpretability logics. One is the Veltman semantics, which we shall refer to as the regular or ordinary semantics; the other is called generalised Veltman semantics. In the recent years and especially during the writing of this thesis, generalised Veltman semantics was shown to be particularly well-suited as a relational semantics for interpretability logics. In particular, modal completeness results are easier to obtain in some cases; and decidability can be proven via filtration in all known cases. We prove various new and reprove some old completeness results with respect to the generalised semantics. We use the method of filtration to obtain the finite model property for various logics.

Apart from results concerning semantics in its own right, we also apply methods from semantics to determine decidability (implied by the finite model property) and complexity of provability (and consistency) problems for certain interpretability logics.

From the arithmetical standpoint, we explore three different series of interpretability principles. For two of them, for which arithmetical and modal soundness was already known, we give a new proof of arithmetical soundness. The third series results from our modal considerations. We prove it arithmetically sound and also characterise frame conditions w.r.t. ordinary Veltman semantics. We also prove results concerning the new series and generalised Veltman semantics.

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Pedro Pinto, ***Proof Mining with the Bounded Functional Interpretation***, Faculdade de Ciências, Universidade de Lisboa, Lisboa, Portugal, 2019. Supervised by Fernando Ferreira. MSC: 03F10, 47H09, 47H10, 47J25. Keywords: bounded functional interpretation, majorants, metastability, weak compactness, fixed points.

Abstract

In this doctoral thesis, we show how the bounded functional interpretation of F. Ferreira and P. Oliva can be used and contribute to the Proof Mining program, a program which aims to extract computational information from mathematical theorems using proof-theoretic techniques. We present a method for the elimination of sequential weak compactness arguments from the quantitative analysis of certain mathematical results. This method works as a "macro" and allowed us to obtain quantitative versions of important results of F. E. Browder, R. Wittmann, and H. H. Bauschke in fixed point theory in Hilbert spaces. Although the theorems of Browder and Wittmann were previously analyzed by U. Kohlenbach using the monotone functional interpretation, it was not clear why such analyses did not require the use of functionals defined by bar recursion. This phenomenon is now fully understood by a theoretical justification for the elimination of sequential weak compactness in the context