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BOOK REVIEWS

WHITELAW, T. A., *Introduction to abstract algebra* (Blackie, 2nd edition, 1988) pp. viii + 200, 0 216 92259 3, Paper £10.95.

The most obvious difference between this new edition and the first edition (1978: reviewed in Vol. 22 (1979), p. 68) is the insertion of a chapter on the symmetric group. This is certainly a good idea, and the new chapter has all the virtues (and all the delicious schoolmasterly quirks, as in "Then, as the student should verify in detail,...") of the rest of the book. Throughout, the exercises are a particularly strong feature, being varied, interesting and far from trivial. Those who learn and those who teach abstract algebra at this level have reason to be grateful to Dr. Whitelaw for disseminating his skill and experience in this area beyond the walls of Glasgow University.

J. M. HOWIE

WHITE, NEIL (ed.), *Combinatorial geometries* (Encyclopaedia of Mathematics and its applications, Vol. 29, Cambridge University Press, 1987), xii + 212 pp., 0 521 33339 3, £25.

This is the second of a three-volume series intended to cover matroids and combinatorial geometries. Like the first volume, it consists of a collection of expositions by various experts, edited to ensure uniformity of presentation throughout.

The first volume was described as a "primer in the basic axioms and constructions of matroids". Historically, matroids arose as a generalization of linear dependence, and later they were seen to be closely related to geometric lattices and combinatorial geometries. The result is a subject which can be approached from varying starting points using different sets of axioms. The approaches and axiom systems are described in the first volume. The present second volume begins with three chapters on coordinatization, or the representation of matroids as vector space matroids, including the particular cases of binary matroids (those representable over GF(2), discussed by J. C. Fournier), and unimodular matroids (those representable over every field, discussed by Neil White, the editor). Then Richard Brualdi deals with connections with matching theory. Matching matroids arise from matchings in graphs, while transversal matroids arise from matchings in bipartite graphs; remarkably these two classes of matroids are identical. There then follows a chapter by R. Cordovil and B. Lindström on simplicial matroids.

Attempts to generalise graph theory to matroid theory have produced analogues of chromatic polynomials, called characteristic polynomials. These are studied by T. Zaslavsky, along with some invariants of matroids such as the Tutte–Grothendieck invariant. Martin Aigner then studies geometric lattices, on which a rank function can be defined, and their Whitney numbers. For some lattices these numbers are unimodal, in fact log-concave.

The final chapter, written by Ulrich Faigle, is the longest. It deals with matroids in combinatorial optimization; this is the aspect of matroids which has been of widespread interest in recent years. The greedy algorithm works precisely in the matroid setting. Other optimization problems with constraints presented by integer-valued submodular functions (the discrete analogue of convex functions; the rank function of a matroid is submodular) can be considered from the point of view of integral matroids, which are collections of vectors in integral polyhedral matroids. The author's aim is to show how matroids are not only abstractable from optimization