

## SIMPLICITY OF CATEGORIES DEFINED BY SYMMETRY AXIOMS

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**ABSTRACT.** We consider two generalizations  $R_{0w}$  and  $R_0$  of the usual symmetry axiom for topological spaces to arbitrary closure spaces and convergence spaces. It is known that the two properties coincide on Top and define a non-simple subcategory. We show that  $R_{0w}$  defines a simple subcategory of closure spaces and  $R_0$  a non-simple one. The last negative result follows from the stronger statement that every epireflective subcategory of  $R_0$  Conv containing all  $T_1$  regular topological spaces is not simple. Similar theorems are shown for the topological categories Fil and Mer.

A full and isomorphism closed epireflective subcategory  $\mathbf{L}$  of a topological category  $\mathbf{H}$  is called simple if there exists a single object  $E \in |\mathbf{L}|$  such that  $\mathbf{L}$  is the epireflective hull of  $\{E\}$  in  $\mathbf{H}$ . This also means that every object of  $\mathbf{L}$  is a subobject of a power of  $E$ .

In this context “ $Y$  is a subobject of  $X$ ” means that there exists an embedding from  $Y$  to  $X$ , so that  $Y$  is an extremal subobject in the categorical sense. For further details on these notions we refer to [9].

Simplicity problems for TOP and its subcategories defined by means of separation axioms  $T_0, T_1, T_2$  and by the symmetry axiom  $R_0$  have been settled for quite some time [6], [7], [15]. Pretop is known to be simple. Simplicity of its subcategories defined by  $T_0, T_1, T_2$  properties has recently been studied by the authors in [13].

In the first section of this paper we show that while  $R_{0w}$  and  $R_0$  both are extensions of the same symmetry axiom in TOP, they define subcategories of Pretop, where  $R_{0w}$  Pretop is simple and  $R_0$  Pretop is not. The negative result for  $R_0$  Pretop is a consequence of a generalization of a well known result of Herrlich [7].

In the second section we generalize this result yet one step further to  $R_0$  convergence spaces. As a consequence we can conclude that every epireflective subcategory of  $R_0$  Conv containing all  $T_1$  regular topological spaces is not simple.

In the last section we consider epireflective subcategories of the categories Fil of filter spaces and Mer of merotopic spaces. Our main theorem in this section states that every epireflective subcategory of Fil (of Mer) containing all spaces that are both  $T_1$  regular filterspaces and  $c^\wedge$  embedded Cauchy spaces, is not simple in Fil (in Mer).

### 1. Closure spaces.

For topological spaces it is well known that the following properties are equivalent ( $\mathcal{x}$  denotes the filter generated by  $\{x\}$  and  $\mathcal{W}(x)$  is the neighborhoodfilter in  $x$ ).

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- (i)  $\dot{x} > \mathcal{W}(y) \Rightarrow \mathcal{W}(x) = \mathcal{W}(y)$
- (ii)  $\dot{x} > \mathcal{W}(y) \Rightarrow \dot{y} > \mathcal{W}(x)$

Each of them defines the usual  $R_0$  axiom for topological spaces. Spaces satisfying this axiom are also called symmetric topological spaces. It was shown in [8] that the  $R_0$  topological spaces are exactly those embeddable into merotopic spaces.

For closure spaces with a non-idempotent closure operator, conditions (i) and (ii) are not longer equivalent. Condition (i) was used in [16] as a definition of  $R_0$  closure spaces and it was shown that  $R_0$  closure spaces are exactly those embeddable into merotopic spaces. Condition (ii) defines a weaker property which we will denote by  $R_{0_w}$ . Both  $R_0$  and  $R_{0_w}$  are extensions of the topological  $R_0$  axiom.

Let  $R_0$  Pretop and  $R_{0_w}$  Pretop be the full subcategories of Pretop defined by the properties  $R_0$  and  $R_{0_w}$  respectively. Both  $R_0$  Pretop and  $R_{0_w}$  Pretop are bireflective subcategories of Pretop.

It is known (see e.g. [15]) that  $R_0$  Top is not simple in TOP. We will show that the two different extensions of the  $R_0$  property to Pretop give different answers with regard to simplicity in Pretop.

**THEOREM 1.1.**  *$R_{0_w}$  Pretop is simple.*

**PROOF.** On  $X = \{0, 1, 2\}$  we define a closure structure by means of the neighborhoodfilters of its points in the following way:  $\mathcal{W}(0) = \{X\}$ ,  $\mathcal{W}(1) = \langle \{0, 1\} \rangle$ ,  $\mathcal{W}(2) = \langle \{0, 2\} \rangle$ .

Clearly  $X$  has the  $R_{0_w}$  property. Moreover, if  $Y$  is any closure space having the  $R_{0_w}$ -property we define the following maps.

For  $y \in Y$  and for any  $U \in \mathcal{W}(y)$  let

$$f_{y,U}(z) = \begin{cases} 1 & \text{if } y = z \\ 0 & \text{if } z \in U \setminus \{y\} \\ 2 & \text{if } z \in Y \setminus U \end{cases}$$

It is easily checked that these functions are continuous. Moreover the source  $(f_{y,U} : Y \rightarrow X)_{y \in Y, U \in \mathcal{W}(y)}$  is point separating and  $Y$  has the initial Pretop-structure.

It follows that  $Y$  belongs to the epireflective hull of  $X$ . ■

Next we consider the stronger symmetry axion  $R_0$ . In order to investigate the simplicity of  $R_0$  Pretop we need the following notions.

**DEFINITION 1.2.** If  $Y$  is any closure space the relation

$$y_1 \sim y_2 \Leftrightarrow \mathcal{W}(y_1) = \mathcal{W}(y_2)$$

is an equivalence relation.

A map  $f: X \rightarrow Y$  between two closure spaces is quasiconstant if  $f(x_1) \sim f(x_2)$  whenever  $x_1, x_2 \in X$ .

If  $\phi: Y \rightarrow Y|_{\sim}$  is the canonical surjection associated with the equivalence relation then clearly  $f: X \rightarrow Y$  is quasiconstant if and only if  $\phi \circ f$  is constant. When  $Y$  is an  $R_0$  closure space we have

$$\mathcal{W}(y_1) = \mathcal{W}(y_2) \Leftrightarrow \text{cl}\{y_1\} = \text{cl}\{y_2\}$$

and the equivalence classes of the relation above are exactly the closures of singletons of  $Y$ . Moreover in the case of an  $R_0$  space non-equivalent points can be separated in a  $T_1$  way i.e. each of them has a neighborhood not containing the other point. However, contrary to the topological case the Pretop-quotient  $Y|_{\sim}$  of an  $R_0$  closure space  $Y$  need not be  $T_1$ , as follows from the next example.

EXAMPLE 1.3.  $Y = \{0\} \cup \{\frac{1}{n} | n \geq 1\}$  and a closure structure on  $Y$  is defined by the neighborhoods

$$\begin{aligned} \mathcal{W}(0) &= \left\langle \left\{ \{0\} \cup \left\{ \frac{1}{n} \mid n \geq m \right\} \mid m \geq 1 \right\} \right\rangle \\ \mathcal{W}\left(\frac{1}{n}\right) &= \langle Y \setminus \{0\} \rangle \quad n \geq 1 \end{aligned}$$

Clearly  $Y$  is an  $R_0$  space. However the Pretop quotient is the Sierpinski space and hence it is not  $T_1$ .

The fact that the  $R_0$  property alone is not sufficient to guarantee that the quotient is  $T_1$  also follows from the next result.

PROPOSITION 1.4. *If  $Y$  is a closure space and  $Y|_{\sim}$  is the Pretop quotient of the identification of Definition 1.2, then  $Y|_{\sim}$  is a  $T_1$  space if and only if both of the following conditions are fulfilled:*

- (a)  $Y$  is an  $R_0$  space
- (b)  $\text{cl}\{y\}$  is closed whenever  $y \in Y$ .

PROOF. Suppose  $Y|_{\sim}$  has the  $T_1$  property and  $y \in \mathcal{W}(z)$ ,  $y$  and  $z \in Y$ . If  $y$  and  $z$  are not equivalent then there exists  $W \in \mathcal{W}(z)$  such that  $\phi(y) \notin \phi(W)$  and of course this is impossible. So (a) clearly is fulfilled.

In order to prove (b) suppose  $z \in \text{cl}\{y\}$ . Since  $Y$  is  $R_0$  the set  $\text{cl}\{y\}$  is exactly the  $y$ -equivalence class. Therefore  $\phi(y) \in \phi(W)$  whenever  $W \in \mathcal{W}(z)$ . In view of the  $T_1$  property of  $Y|_{\sim}$  we can conclude that  $\phi(z) = \phi(y)$  and then  $z \in \text{cl}\{y\}$ .

Next suppose  $Y$  satisfies conditions (a) and (b) and  $\phi(y) \neq \phi(z)$  then  $z \notin \text{cl}\{y\}$ . Moreover since  $\text{cl}\{y\}$  is closed there exists a neighborhood  $W$  of  $z$  such that  $\text{cl}\{y\} \cap W = \emptyset$ . Since  $Y$  is  $R_0$  the set  $\text{cl}\{y\}$  is exactly the equivalence class of  $y$ . Therefore  $\phi(W)$  is a neighborhood of  $\phi(z)$  not containing  $y$ . The same way a neighborhood of  $\phi(y)$  not containing  $\phi(z)$  can be constructed. ■

In [7] Herrlich has shown that a topological space  $Y$  is  $T_1$  if and only if there exists a  $T_1$  regular topological space  $X$  with at least two points such that every continuous function from  $X$  to  $Y$  is constant.

Our purpose is to find a generalization of this theorem for  $R_0$  closure spaces. In spite of the fact that by Proposition 1.4 a generalization cannot be obtained by straightforward application of Herrlich's result to the identification  $Y|_{\sim}$  of the given  $R_0$  closure space  $Y$ , the proof in [7] can be modified in order to get the generalized result for the  $R_0$ -case.

**THEOREM 1.5.** *If  $Y$  is a closure space then the following are equivalent*

- (a)  $Y$  is an  $R_0$  space
- (b) there is a regular  $T_1$  topological space  $X$  (containing at least two points) such that every continuous map from  $X$  to  $Y$  is quasiconstant.

**PROOF.**

(b)  $\Rightarrow$  (a). If  $Y$  is not an  $R_0$  space then there exist  $y$  and  $y'$  in  $Y$  such that  $y > \mathcal{W}(y')$  and  $\mathcal{W}(y) \neq \mathcal{W}(y')$ . Let  $X$  be any regular  $T_1$  topological space with at least two points and  $B$  an open non-empty and proper subset of  $X$  then the function  $f: X \rightarrow Y$  mapping  $B$  to  $y$  and  $X \setminus B$  to  $y'$  is continuous and not quasiconstant.

(a)  $\Rightarrow$  (b). The same construction as the one applied by Herrlich in [7] works here. We use the same notations and we will only indicate the modifications that have to be made.

1. Let  $Y$  be an  $R_0$  closure space and suppose  $\text{card } Y \leq \aleph_\alpha$ . For  $i = 1, 2$  we put  $R_i$  a set of cardinality  $\aleph_{\alpha+i}$ ,  $r_i$  a fixed point of  $R_i$ . We endow  $R_i$  with the same topology as in [7]: a subset  $B$  of  $R_i$  is open if  $r_i \in B$  implies  $\text{card}(R_i \setminus B) < \aleph_{\alpha+i}$ .

Now if  $f: R_i \rightarrow Y$  is continuous and  $f(r_i) = y_0$  then  $f^{-1}(Y \setminus \{y\})$  is a neighborhood of  $r_i$  whenever  $y_0 \notin \text{cl}\{y\}$ . But then  $\cap\{f^{-1}(Y \setminus \{y\}) \mid y_0 \notin \text{cl}\{y\}\}$  is a neighborhood too. Moreover using the  $R_0$  property of  $Y$  we have

$$\begin{aligned} \cap\{f^{-1}(Y \setminus \{y\}) \mid y_0 \notin \text{cl}\{y\}\} &= \cap\{f^{-1}(Y \setminus \{y\}) \mid y \notin \text{cl}\{y_0\}\} \\ &= f^{-1}(\text{cl}\{y_0\}) \end{aligned}$$

and  $\text{cl}\{y_0\}$  is exactly the equivalence class of  $y_0$ . Hence  $f$  is quasiconstant on this neighborhood of  $r_i$ .

2. All constructions towards the final construction of  $X$  have to be repeated exactly as in [7]. Whenever it is shown in [7] that two points have equal images through a continuous function, the corresponding result will now be that the images are equivalent. As in [7] a sequence  $X_0 \subset X_1 \subset X_2 \subset \dots$  of regular  $T_1$  topological spaces is constructed and whenever  $f$  is a continuous function from  $X_{n+1}$  to  $Y$  we now have that it is quasiconstant on  $X_n$ . On  $X = \cup\{X_n \mid n = 0, 1, \dots\}$  one takes the final topological structure for the sink  $(j_n : X_n \rightarrow X)_{n=0,1,\dots}$  where  $j_n$  is the canonical injection of  $X_n$  to  $X$ . Then  $X$  is regular  $T_1$  and when  $f$  is any continuous function from  $X$  to the given closure space  $Y$  the compositions  $f \circ j_n$  are all continuous. Therefore  $f$  is quasiconstant on  $X$ . ■

**COROLLARY 1.6.** *Every epireflective subcategory  $\mathbf{L}$  of  $\text{Pretop}$  such that  $T_1 \text{ reg Top} \subset \mathbf{L} \subset R_0 \text{ Pretop}$  is not simple.*

PROOF. Let  $Y$  be any space in  $|\mathbf{L}|$  then if  $X$  is the space as constructed in the previous theorem, all continuous maps from  $X$  to  $Y$  are quasiconstant. But then, since  $X$  is a  $T_1$  space, it cannot be initial for any source

$$(x \xrightarrow{f_i} Y)_{i \in I}. \quad \blacksquare$$

### 2. Convergence spaces

In this section we consider convergence spaces in the sense of Fisher [4]. A strong symmetry axiom for convergence spaces was introduced in [16] in the following way.

DEFINITION 2.1. A convergence space is  $R_0$  if

$$\dot{x} \rightarrow y \Rightarrow x \text{ and } y \text{ have the same convergent filters.}$$

For closure spaces this definition coincides with the  $R_0$  property defined in the previous section.

$R_0$  convergence spaces are exactly those embeddable in the category of merotopic spaces. The category  $R_0 \text{ Conv}$  of  $R_0$  convergence spaces is bireflective in  $\text{Conv}$ . For convergence spaces we use the equivalence relation as introduced in *Definition 1.2*:  $y_1$  and  $y_2$  in  $Y$  are equivalent ( $y_1 \sim y_2$ ) if  $y_1$  and  $y_2$  have the same convergent filters.

Moreover, as before, a map  $f : X \rightarrow Y$  between two convergence spaces is called quasiconstant if  $f(x_1) \sim f(x_2)$  whenever  $x_1, x_2 \in X$ .

Whenever  $Y$  is a convergence space let  $\psi Y$  be its Pretop reflection. The neighborhoodfilters of  $\psi Y$  are given by

$$\mathcal{W}(y) = \cap \{ \mathcal{F} \mid \mathcal{F} \xrightarrow{Y} y \} = \cap \{ \mathcal{U} \mid \mathcal{U} \text{ ultra, } \mathcal{U} \xrightarrow{Y} y \}.$$

PROPOSITION 2.2. *Let  $Y$  be an  $R_0$  convergence space. Then the following properties hold*

- (a)  $\psi Y$  is  $R_0$
- (b)  $\mathcal{W}(y_1) = \mathcal{W}(y_2) \Leftrightarrow y_1$  and  $y_2$  have the same  $Y$ -convergent filters.

PROOF.

(a) If  $\dot{x} > \mathcal{W}(y)$  then  $\dot{x} > \{ \mathcal{U} \mid \mathcal{U} \text{ ultra, } \mathcal{U} \xrightarrow{Y} y \}$ . It follows that in the collection of ultrafilters on the right there is one member containing  $\{x\}$ . Hence  $\dot{x} \xrightarrow{Y} y$ . Since  $Y$  is  $R_0$  the points  $x$  and  $y$  have the same  $Y$ -convergent filters and then  $\mathcal{W}(x) = \mathcal{W}(y)$ .

(b) Suppose  $\mathcal{W}(y_1) = \mathcal{W}(y_2)$ . Since  $\dot{y}_1 > \mathcal{W}(y_2)$  we can conclude as in (a) that  $\dot{y}_1 \xrightarrow{Y} y_2$  and since  $Y$  is an  $R_0$  space this again implies that  $y_1$  and  $y_2$  have the same convergent filters. Since the other implication is trivial, we are done. ■

Using this result we now can derive the following generalization of Theorem 1.5.

THEOREM 2.3. *If  $Y$  is a convergence space then the following are equivalent*

- (a)  $Y$  is an  $R_0$  space
- (b) there is a regular  $T_1$  topological space  $X$  (containing at least two points) such that every continuous map from  $X$  to  $Y$  is quasiconstant.

PROOF.

(a)  $\Rightarrow$  (b) goes exactly as in the proof of Theorem 1.5.

(b)  $\Rightarrow$  (a). Suppose  $Y$  is an  $R_0$  convergence space. From Proposition 2.2 we know that  $\psi Y$  is an  $R_0$  closure space. Then we can apply Theorem 1.5 to construct the  $T_1$  regular topological space  $X$ . If  $f : X \rightarrow Y$  is continuous then  $f : X \rightarrow \psi Y$  is continuous and hence quasiconstant to  $\psi Y$ . Moreover since by Proposition 2.2 (b) the equivalence classes for  $Y$  and  $\psi Y$  are the same,  $f$  is quasiconstant to  $Y$ . ■

COROLLARY 2.4. *Every epireflective subcategory  $\mathbf{L}$  of  $\text{Conv}$  such that*

$$T_1 \text{ Reg Top} \subset \mathbf{L} \subset R_0 \text{ Conv}$$

*is not simple.*

The result of Theorem 2.3 should be compared with the next one which was obtained by R. Lowen together with the first author in [14].

THEOREM 2.5 [14]. *For every convergence space  $Y$  there exists a  $T_1$   $c$ -embedded convergence space  $X$  such that for any source  $(f_i : X \rightarrow Y)_{i \in I}$  the space  $X$  is not initial in  $\text{Conv}$ .*

For the definition of  $c$ -embedded spaces we refer to [3]. Note that Theorem 2.5 implies that every epireflective subcategory of  $\text{Conv}$  containing all  $T_1$   $c$ -embedded convergence spaces is not simple. In particular if  $R_{0_w}$  is the weak symmetry axiom for convergence spaces defined by

$$R_{0_w} : \dot{x} \rightarrow y \Leftrightarrow \dot{y} \rightarrow x$$

then it coincides with  $R_{0_w}$  for closure spaces and defines an epireflective subcategory  $R_{0_w} \text{ Conv}$ . By the previous result  $R_{0_w} \text{ Conv}$  is not simple.

REMARK 2.6. The space  $X$  constructed in Theorem 2.5 is  $T_1$  and  $c$ -embedded. It is not a closure space and it does not satisfy the strong regularity condition in the sense of [8].

The space  $X$  used in Theorem 2.3 which was constructed by Herrlich in [7] is  $T_1$  regular and topological. In general it is not an  $\omega$ -regular space and so it is not necessarily  $c$ -embedded [11].

### 3. Filterspaces and merotopic spaces

Both Theorem 2.3 and Theorem 2.5 have immediate consequences with regard to simplicity of certain subcategories of the category  $\text{Fil}$  of all filter merotopic spaces.

For definitions and notations on  $\text{Fil}$  and its subcategories we refer to [1], [2], [9], [10] or [12]. We recall the following notations.

**C** is the full subcategory of **Fil** whose objects are those filter spaces  $X$  which satisfy: if  $\mathcal{A}$  and  $\mathcal{B}$  are micromeric in  $X$  and if for some point  $x \in X$  we have  $\mathcal{A} \rightarrow x$  and  $\mathcal{B} \rightarrow x$  then  $\{A \cup B \mid A \in \mathcal{A}, B \in \mathcal{B}\}$  is micromeric in  $X$ .

$T_1 \hat{c} \text{ emb Chy}$  is the full subcategory of the category **Chy** of all Cauchy spaces whose objects are those  $T_1$  Cauchy spaces  $X$  for which the source

$$X \xrightarrow{j} \text{Hom}(\text{Hom}(X, \mathbf{R}), \mathbf{R})$$

is initial.

$T_1 \text{ Reg Fil}$  is the full subcategory of **Fil** whose objects are the  $T_1$  regular spaces in the sense of [8].

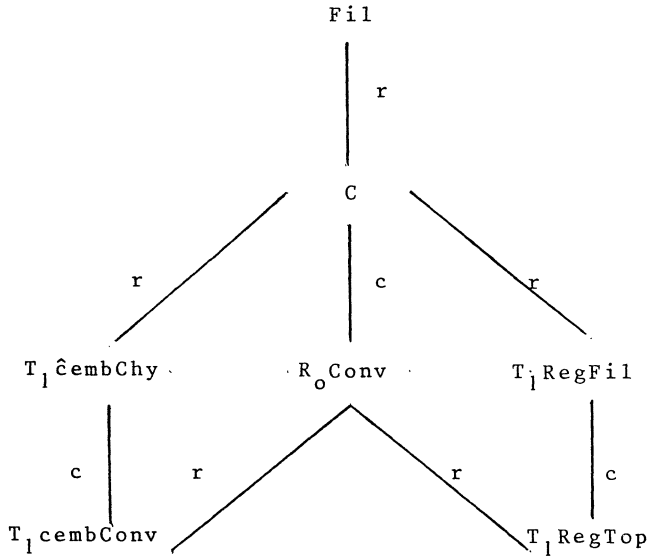
If we denote *sub* for the subspaces taken in the category **Mer** of merotopic spaces (or equivalently in **Fil**) we have [2], [12]

$$\mathbf{C} = \text{sub } R_0 \text{ Conv}$$

$$T_1 \hat{c} \text{ emb Chy} = \text{sub } T_1 \hat{c} \text{ emb Conv}$$

$$T_1 \text{ Reg Fil} = \text{sub } T_1 \text{ Reg Top.}$$

Lines in the following diagram indicate subcategories, *r* stands for epireflective, *c* stands for coreflective. All coreflections are restrictions of the  $R_0 \text{ Conv}$  coreflection of a **C**-space.



Theorems 2.3 and 2.5 now immediately imply that every epireflective subcategory **L** of **Fil** such that  $T_1 \hat{c} \text{ emb Chy} \subset \mathbf{L} \subset \text{Fil}$  or  $T_1 \text{ Reg Fil} \subset \mathbf{L} \subset \mathbf{C}$  is not simple.

This result can be considerably improved by means of the following result.

**THEOREM 3.1.** *For every filter space  $Y$  there exists a space  $X$  in*

$$|T_1 \text{ Reg Fil} \cap \hat{c} \text{ emb Chy}|$$

such that for any source  $(f_i: X \rightarrow Y)_{i \in I}$  the space  $X$  is not initial in  $\text{Fil}$ .

PROOF. The proof is analogous to the proof of the main theorem in [14]. Let  $Y$  be an arbitrary filter space. Take an infinite set  $X$  with cardinality strictly larger than the cardinality of the underlying set of  $Y$ . Further we fix a uniform ultrafilter  $\mathcal{U}$  on  $X$ . We make  $X$  a filterspace in the following way.

A filter  $\mathcal{F}$  on  $X$  is micromeric if  $\mathcal{F} = \dot{x}$  for some point  $x \in X$  or  $\mathcal{F}$  is a non principal ultrafilter different from  $\mathcal{U}$ . Clearly  $X$  is a  $T_1$  Cauchy space. Let  $\mathcal{F}$  be any micromeric filter on  $X$ . It is easy to calculate that with the notations of [8] we have  $\mathcal{F}(\prec) = \mathcal{F}$ . Hence  $X$  is regular in the sense of [8].

Every bounded real valued function is Cauchy continuous and  $\mu X$  is the discrete topology. Using the characterization theorem in [5] we can conclude that  $X$  is  $c^\wedge$  embedded.

Now let  $(f_i: X \rightarrow Y)_{i \in I}$  be a source in  $\text{Fil}$ , then by the proposition in [14] for every  $i \in I$  there exists an ultrafilter  $\mathcal{W}_i \neq \mathcal{U}$  such that  $\text{stack}_Y f_i(\mathcal{W}_i) = \text{stack}_Y f_i(\mathcal{U})$ . Hence  $\text{stack}_Y f_i(\mathcal{U})$  is micromeric for every  $i \in I$ . It follows that  $\mathcal{U}$  is micromeric in the initial filter space of the source. So finally we can conclude that  $X$  cannot be initial in  $\text{Fil}$ . ■

At this point it is natural to ask whether an analogous improvement of Theorems 2.3 and 2.5 can also be obtained in the category of convergence spaces. This question is formulated in the following:

PROBLEM 3.2. Given any  $R_0$  convergence space  $Y$  can one always construct a space  $X \in |T_1 \text{ reg Top} \cap c \text{ emb Conv}|$  such that for any source  $(f_i: X \rightarrow Y)_{i \in I}$  the space  $X$  is not initial in  $\text{Conv}$ .

COROLLARY 3.3. Every epireflective subcategory  $\mathbf{L}$  of  $\text{Fil}$  such that  $T_1 \text{ Reg Fil} \cap c^\wedge \text{ emb Chy} \subset \mathbf{L}$  is not simple in  $\text{Fil}$ .

The same construction can be used to formulate the corresponding result about non-simplicity in the category  $\text{Mer}$  of all merotopic spaces. The proof goes completely analogous to the proof of Theorem 3.1.

THEOREM 3.4. For every merotopic space  $Y$  there exists a space  $X$  in  $|T_1 \text{ Reg Fil} \cap c^\wedge \text{ emb Chy}|$  such that for any source  $(f_i: X \rightarrow Y)_{i \in I}$  the space  $X$  is not initial in  $\text{Mer}$ .

COROLLARY 3.5. Every epireflective subcategory  $\mathbf{L}$  of  $\text{Mer}$  such that  $T_1 \text{ Reg Fil} \cap c^\wedge \text{ emb Chy} \subset \mathbf{L}$  is not simple in  $\text{Mer}$ .

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