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CRITICAL TIMES OF HEAT AND MASS TRANSPORT THROUGH MULTIPLE LAYERS

ROSLYN HICKSON

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Heat and mass transport through multiple layers has applications to a wide range of areas, including industrial, geological, and medical problems [8, 9]. An important aspect of multilayer transport is the phenomenon of 'critical time', which is a measure of how long the diffusive process takes. For example, how long until the coldest point of a steel coil reaches the annealing temperature during heating? The goals of this thesis are to:

(1) explore the effect of layered structures on the critical time, thus demonstrating the limitations of the traditional averaging method for multilayer diffusion;
(2) find simple approximate expressions of critical time that accurately reflect the multilayer behaviour, for both diffusion and reaction diffusion processes;
(3) conduct a thorough comparison of prominent definitions currently in the literature, in particular finding when they are equivalent;
(4) determine an averaging method which simplifies the multilayer reaction diffusion model to an analogous single layer system.

In this thesis, an exact solution for multilayer diffusion is used to demonstrate the limitations of traditional averaging methods, which are only accurate for a large number of layers [1–3, 5]. The averaging method does not capture the asymmetric behaviour of the multilayer system. To verify the 'exact' solution, finite difference schemes are developed for multilayered materials with a range of matching conditions between the layers, including a jump matching condition [4]. The exact solution is used to find elegant approximations for seven definitions of critical time, which

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accurately capture the multilayer behaviour. Each critical time definition is appropriate for different physical applications, with the most important difference being whether the definition is spatially dependent or independent. A thorough comparison between the critical time definitions is conducted.

Reaction diffusion processes are considered, with the focus on a linear reaction function. An exact multilayer solution is found for a constant reactivity in all layers, which is then used to explore three critical time definitions. The outcome of the analysis is a simple, novel relationship between the reaction and diffusion timescales [6, 7]. An appropriate averaging of the reactivities for multilayer reaction diffusion is found, which is accurate for large numbers of layers or in the steady state [7]. Therefore, the multilayer reaction diffusion problem with different reactivities in each layer reduces to the single constant reactivity case, for which an exact solution and simple approximations are found.

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ROSLYN HICKSON, National Centre for Epidemiology and Population Health, Australian National University, Canberra, ACT 0200, Australia
e-mail: Roslyn.Hickson@anu.edu.au

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