for the C 30 autogiro The bracket has extreme values $x \pm \mu$ The damping is always positive when x is greater than μ, *i e*, at fractional distances x from the root greater than the ratio of the forward speed of the machine (parallel to the plane of the rotor disc) to the blade tip speed Considering motion in the first harmonic only, *i e*, $n = 1$, from (41) it is evident that the damping term may rise to 1 9 (0 4 + 0 3) = 1 3 times the inertia loading associated with bending at a radius of 0 4 and a tip speed ratio of 0 3 At the blade tip the damping loading amplitude rises to 2 5 times the inertia loading Considering third harmonic components however their relative magnitude falls to 0 44 and 0 82 at the 0 4 point and the blade tip respectively

Over considerable portions of the rotor disc damping loads will then be of the same order as the inertia loads , but, particularly at high speed in the root region on a retreating blade the damping values may become negative

Aerodynamic damping of drag bending deflection is negligible and when estimations of the resultant rigid blade loads w_y in drag are accurate there may be some justification for introducing inertia load corrections due to elastic deflections If these become appreciable, however, it is possible that the drag damper will come into play and then elastic deflections will need to be considered in the equations of blade motion about the drag hinge

When higher harmonic distortion is being investigated there is, however, no justification for introducing elastic inertia terms without also introducing aerodynamic damping terms in considering blade bending in the flapping plane The region in which damping of motion in the flapping plane is least is that region in which the blade is moving downwind and here the air flow over the blade may be from trailing to leading edge This region increases with the forward speed of the helicopter In this region the aerodynamic forces may not be simply related to blade incidence and possibly step by step calculations are desirable to establish whether appreciable higher harmonic distortion is likely

Conclusion

From the preceding illustrations and arguments it will be appreciated that even in steady forward flight rotor blades are subjected to fluctuating stresses These tend to cause fatigue Now the fatigue life of almost identical structures is somewhat variable and affected considerably by incidental imperfections Considerable accuracy in the estimation of the fluctuating stresses is then hardly necessary Transverse load calculations need be no more elaborate than those arising in estimating the loads on an inflexible blade, using simple assumptions Stresses may then be estimated by fitting a simple type solution approximately to represent the essential features of the estimated load system This procedure breaks down if there is appreciable dynamic response to higher harmonic components in the transverse loading system More work on this problem may be necessary Step by step calculations of the motion resulting after a blade has been given an arbitrary deflection in its primary mode may indicate whether the problem is of practical significance If this motion is rapidly damped out, then the exact calculation of the magnitude of the higher harmonics of the loading on a flexible blade, which will be a tedious process, would not seem physically essential

Fatigue conditions may not be those which are of the greatest significance in designing rotor blades Peak load conditions associated perhaps with a jerky start of the rotor or with taxying over rough ground with the rotor stopped may be more critical In these cases, while the design might be criticised, a rough estimate only of the blade stresses in forward flight is essential

It might be that peak loading conditions in forward flight, combined with a sudden up-gust, may be critical for a rotor blade Then more accurate calculation of blade loads would seem desirable, as well as a more accurate assessment of the stresses they cause by combining together several type solutions or otherwise

If uncertainties in the maximum load to which a rotor blade may be subjected in forward flight force one to assume arbitrarily, for example, that the blade stalls all along its length, then great accuracy in stressing is hardly justifiable One type solution may indicate the order of the stresses with sufficient accuracy

DISCUSSION

Opening the discussion on the two papers given at the morning session, **Mr R Hafner** (*Member—Bristol Aeroplane Co*) said that he had listened to both papers with great interest

Prof OWEN's paper reminded Mr HAFNER very much of another paper written some time ago by a young man in Farnborough, a paper which was full of terms such as "type solution," "transverse and longitudinal loading systems," etc It reminded him of the days of the old giroplane Since then much had changed, Mr OWEN had grown older and very much wiser, he had left the rotor business and become a Professor in Liverpool! But those of us who had remained with helicopters have become wiser too We had learned that the problem of stressing rotor blades was extremely complex and had begun to doubt if a theoretical approach, with the unavoidable abstractions in order to simplify the mathematics, would be of practical value

The major abstractions in Prof OWEN's paper were in the assumptions that the induced velocity was uniform over the rotor disc and, further, that the rotor was in a quasi-static condition Then the problem was only in finding the solution to the differential equations for the elastic line of the blade in such a condition, *i e*, a combination of extensional and transverse static loading systems Critical stresses would then be obtained at certain points defined by such co-ordinates as the distance from the rotor centre and the azimuth angle

We had learned since that the induced velocity was far from uniform and the rotor blade was not in a quasi-static equilibrium In fact we knew that the blade was capable of swinging in various modes and natural frequencies, and, if fluctuating forces acted upon the blade at frequencies close to those above, then serious oscillations might ensue Indeed, we knew that in practice the third rotor order could be quite close to the natural frequency of the second flexual mode of the blade in the vertical plane, and thus the dynamic amplification factor could be considerable

It was obvious, therefore, that the above phenomenon could not be neglected in a theoretical treatment for the stressing of blades, and if, on the other hand, it was included, then the consequent mathematical complexity presented a major problem for "six men and a boy"

As a practical engineer, he was therefore inclined to seek the solution by the empirical approach, *i e*, by making less calculations and more guesses in the preliminary design stages For this purpose some simple theoretical treatment giving a rough picture of the elastic line of the blade would suffice and Dr OWEN's use of "type solutions" was then very useful When the prototype blade was built it would be "strain gauged," whilst the flight envelope was explored during the preliminary test flying stage The third stage of development was on the rotor tower where the rotor was run in order to establish fatigue properties Thus the airworthiness of the rotor was established by type test rather than by calculations

Referring to Mr SQUIRE's paper Mr HAFNER thought it touched upon many features which were of interest to the helicopter engineer He agreed with Mr SQUIRE on the value of blade twist It simply meant shaping the blade so as to give the optimum angle of attack from tip to root for the inflow conditions pertaining at the rotor An autogiro rotor with little axial flow did not require much blade twist, but a helicopter rotor did The helicopter designer was particularly concerned with the retreating blade, as this appeared to lead to a critical design condition The retreating blade was capable of producing lift from the blade tip to about the mid point section The inner half of the retreating blade was substantially ineffective, owing to insufficient air flow Thus, the blade designer could twist the outer half of the blade only, such as to conform to the inflow conditions for the retreating blade, whereas the inner half of the blade could preferably be shaped such as to meet the inflow conditions of the advancing blade

Mr SQUIRE's suggested using tip tabs in order to produce cyclic twisting of the blade, thereby controlling the flapping of the blade in forward flight This was exactly what had already been done at Bristol, and they would hear later in the day, from Mr TURNER, more about this work They would see that multiple tabbing of rotor blades represented a powerful means of controlling rotor characteristics

Mr SQUIRE finally talked about compressibility and showed curves giving Mach numbers for drag divergence for various aerofoils Mr HAFNER considered these curves to be slightly optimistic They were probably valid for the assumption of no lift However, the lift coefficient for the advancing blade tip was of the order of 2 which would give lower Mach numbers than those shown on the slide That was the reason why he preferred a C/T ratio for the blade tip of about 08 rather than 10

The Bristol Helicopter had frequently been criticised for its high tip speed and it was suggested that Glauert's figure of merit was very poor for this helicopter

He thought that Mr Squire's paper had clearly shown that this figure of merit was not a suitable criterion for a practical helicopter What mattered in practice was avoiding stalling at the retreating side and compressibility problems at the advancing side of the rotor

Mr P E Q Shunker (*The Fairey Aviation Co, Ltd*) The impression I have received from Prof Owen's talk, with particular reference to the estimation of fluctuating stresses of the blades, is that, provided dynamic response to the higher harmonics may be neglected, the effort required by a more elaborate analysis than, say, " Type " solutions, would not be justified in view of the inaccuracy of the applied aerodynamic loading as predicted at present

In the design stage of a new blade with radially varying characteristics, I know of no easy means by which to establish whether there will be appreciable dynamic response or not, except by calculation Designers in general rely on a knowledge of the natural frequencies of the blade in the flapping and azimuth planes to indicate the possibility of appreciable dynamic response To determine such frequencies considerable work must be done when using, for instance, methods such as Myklestad's A method, however, involving the representation of the blade by a number of mass points and the use of the elegant processes of the matric algebra to reduce the solution to formal arithmetical computation (which incidentally is easy to check) promises to be less laborious in its direct application to a blade under a given aerodynamic forcing In the particular case of zero forcing, solution of the above problem leads of course to the natural frequencies, but the resulting frequency equation is not easily evaluated

I feel that blade loading (below the stall) is reasonably predictable distributionally (*i e*, radially and in azimuth) so that the above analysis can and should be done to give an indication of any undue dynamic response Steps may then be taken to avoid such responses

It is both logical and desirable that, having arrived thus far, one should include the predicted aerodynamic loadings in actual magnitude (Albeit, perhaps, these may be somewhat inaccurate, though I wonder, as a result of recent efforts to relate rotor theory to practice by various investigators, if they are too inaccurate for useful design work)

The order of the fluctuating stresses can be thus determined, assessed against the background of fatigue tests and later related to strain gauge results obtained during flight

Professor J B B Owen (*in reply*) I have known Mr Hafner for a number of years and have already taken him to task for some of the remarks he made this morning I think he would now agree with my interpretation of what he should have said

You will remember that I started this morning by reminding you of the simple equations relating transverse loading, shear, bending moment and deflexion I proceeded to show stage by stage how these simple equations were modified as we proceeded to consider the helicopter blade bent in the lift and drag planes

What Mr Hafner should have said is, that when he extends this fundamental analysis to the blades of his particular helicopter, he is forced to introduce into these equations the further " elastic-inertia " and damping terms I mentioned, and then the six men and a boy, to which his colleague referred, are at present unable to solve these equations I suspect that his main difficulty lies in estimating the aerodynamic excitation but I think, in parallel with other work which his famous company has done, it will be found, in a few years' time, that one boy will be able to do it all right ! (*Laughter*) There is evidence in the remarks of the second speaker this morning that, using matrices, he has dealt with dynamic effects As to the best method of dealing with the problem, it is undoubtedly the way one knows best

There may be one thing I did not make too clear, and that was in relation to the damping and inertia terms due to elastic deflexion One term does not strictly directly cancel out the other The effect I was trying to illustrate is the resonance effect, which I think you all know If you have no damping the motion diverges rapidly as resonance is approached If appreciable damping is present the increase in the amplitude of the motion may be negligible

I think Mr Hafner will find in my paper quite a lot that is not present in the earlier papers

(At this point the conference adjourned for lunch)