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A Hypergraph with Commuting Partial Laplacians

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Abstract. Let F be a totally real number field and let GL_n be the general linear group of rank n over F. Let $\mathfrak{p}$ be a prime ideal of F and $F_{\mathfrak{p}}$ the completion of F with respect to the valuation induced by $\mathfrak{p}$. We will consider a finite quotient of the affine building of the group GL_n over the field $F_{\mathfrak{p}}$. We will view this object as a hypergraph and find a set of commuting operators whose sum will be the usual adjacency operator of the graph underlying the hypergraph.

1 Introduction

For a finite graph, consider all eigenvalues of the adjacency matrix. The eigenvalue with the second largest absolute value plays a key part in the estimation of different invariants of the graph. Thus, one is interested in finding an upper bound for its absolute value. With this motivation in mind, we consider the affine building of the group GL_n over the completion $F_{\mathfrak{p}}$ of a totally real field F at a nonarchimedean place $\mathfrak{p}$. Then we will consider a discrete co-compact arithmetic subgroup Γ of $\mathrm{GL}_n(F_{\mathfrak{p}})$ that acts without fixed points on the vertices of the building and we will view the finite building quotient of $\Gamma \setminus \mathrm{GL}_n(F_{\mathfrak{p}})$ as a hypergraph. Its underlying graph will be a finite regular graph. The main theorem of this article shows that the adjacency operator of this graph can be expressed as the sum of the generators of the Hecke algebra of $\mathrm{GL}_n(F_{\mathfrak{p}})$ with respect to a maximal compact subgroup. These operators are well understood. Since the Hecke algebra is commutative, estimating the eigenvalues of the adjacency matrix of the graph becomes equivalent to estimating the eigenvalues of the generators of the Hecke algebra. We will undertake this estimation in a paper to follow.

2 Combinatorics

The basic notations and definitions about graphs are taken from [2]. A *graph* G is a pair of sets $\bigl(V(G), E(G)\bigr)$ such that $E(G) \subset \{Y : Y \subset V(G), |Y| = 2\}$ and $V(G) \neq \varnothing$. The set $V(G)$ is the set of *vertices* of G and $E(G)$ is the set of *edges* of G. The vertices x and y are said to be *adjacent* if $\{x, y\}$ is an edge. Denote by $A(x)$ the set of vertices adjacent to x. Then the cardinality $|A(x)|$ of $A(x)$ is denoted by $d(x)$ and is said to be the *degree* of x. If every vertex of G has degree s, then G is said to be s-*regular*. If G is a graph with a finite number of vertices $\{x_1, \ldots, x_n\}$, the *adjacency*

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matrix $\delta = (\delta_{ij})$ of G is the $n \times n$ matrix with entries δ_{ij} equal to 1 if x_i is adjacent to x_j and 0 otherwise.

One can define a combinatorial Laplacian for graphs and, in fact, also for CW-complexes, as follows [7]. For the finite graph $G = \big(V(G), E(G)\big)$ fix some arbitrary orientation on the edges. For $e \in E(G)$ denote by e^- its origin and by e^+ its target. The operator $d\colon L^2\big(V(G)\big) \to L^2\big(E(G)\big)$ is defined as $df(e) = f(e^+) - f(e^-)$. If $|V(G)| = n$ and $|E(G)| = m$, then the matrix of d with respect to the standard bases of $L^2\big(V(G)\big)$ and $L^2\big(V(E)\big)$, respectively, is an $m \times n$ matrix D indexed by the pairs (e, v), where $v \in V(G)$ and $e \in E(G)$, such that

$$D_{e,v} = \begin{cases} 1 & \text{if } v = e^+ \\ -1 & \text{if } v = e^- \\ 0 & \text{otherwise} \end{cases}$$

Let D^* be the transpose of D. Then D^*D is an $n \times n$ matrix. The operator Δ on $L^2\big(V(G)\big)$ whose matrix is $\Delta = D^*D$ is called the *Laplacian operator* of the graph G. Observe that while D depends on the orientation on $E(G)$, Δ does not.

A *hypergraph* X is a set V together with a family Σ of subsets of V. The elements of V and Σ are called respectively the *vertices* and the *faces* of the hypergraph. If $S \in \Sigma$, the *rank* of S is the cardinality $|S|$ of S and the *dimension* of S is given by $|S| - 1$. Vertices are faces of dimension 0. A *simplicial complex* with vertex set V is a collection Σ of finite subsets of V, called *simplices*, such that every singleton $\{v\}$ is a simplex and every subset of a simplex S is a simplex, called a *face* of S. We include the empty set as a simplex. It has rank 0 and dimension -1.

Remark A simplicial complex is a special case of hypergraph. For the rest of the article all hypergraphs considered will be simplicial complexes.

A simplex that is not contained in any other simplex is called a *maximal simplex*. Two maximal simplices C and C' of the same dimension are *adjacent* if they have in common a co-dimension 1 face. A *gallery* is a sequence of maximal simplices of the same dimension $\Xi = (C_0, \dots, C_d)$ such that consecutive maximal simplices C_{i-1} and C_i are adjacent for $i = 1, \dots, d$. The *length* of Ξ is d. We say that Ξ connects C_0 to C_d.

Two simplices A and B will be called *joinable* if there exists another simplex C such that A and B are both faces of C. We say that two simplices are *disjoint* if their intersection is the empty simplex. The *link* of a simplex A, denoted lk_A, is a subcomplex of Σ consisting of the simplices B which are disjoint from A and which are joinable to A.

A finite-dimensional simplicial complex Σ is called a *chamber complex* if all maximal simplices have the same dimension and any two can be connected by a gallery. The maximal simplices are called *chambers*. The hypergraph in Figure 1 is a chamber complex, the hypergraph in Figure 2 is not a chamber complex. A *labelling* of the chamber complex Σ by a set I is a function which assigns to each vertex of Σ an element of I in such a way that the vertices of every chamber are mapped bijectively onto I. Labelling one chamber completely determines the labelling on any adjacent

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chamber and therefore on the entire chamber complex. Thus any two labellings of the same chamber complex are isomorphic. A chamber complex Σ is said to be *labellable* if there is a labelling of Σ. We will say that a hypergraph is *labellable* if it is a chamber complex which is labellable. The hypergraph in Figure 3 is a labellable hypergraph. The hypergraph in Figure 4 is not labellable.

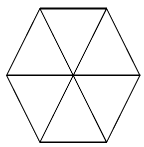

Figure 1

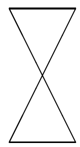

Figure 2

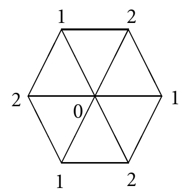

Figure 3

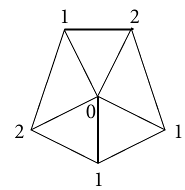

Figure 4

By omitting all faces of dimension higher or equal to 2 from a hypergraph X we obtain the *underlying graph* of X, denoted by $\underline{X}$.

3 The Partial Laplacians

Assume now that X is a finite labellable hypergraph with rank $n+1$ such that $\underline{X}$ is s-regular. The dimension of the chambers is then equal to n. Fix a labelling $\{0, 1, \dots, n\}$ of X. Let $x^{(i)}$, $i = 0, 1, \dots, n$, denote a vertex labelled i. If x is any vertex in $\underline{X}$, we denote by $A_i(x)$ the subset of $A(x)$ consisting of vertices labelled i. As an operator on $L^2\big(V(\underline{X})\big)$, the combinatorial Laplacian Δ on $\underline{X}$ is an averaging operator. It is defined by

$$\Delta f(x) = d(x) f(x) - \sum_{y \in A(x)} f(y), \quad f \in L^2\big(V(\underline{X})\big).$$

Since $\underline{X}$ is s-regular, $\Delta = sI - \delta$, where δ is the adjacency matrix of $\underline{X}$ and I is the identity matrix. We will also denote by δ the adjacency operator on $L^2\big(V(\underline{X})\big)$ whose matrix is the adjacency matrix. For each $i = 1, \dots, n$ we define an operator φ_i on $L^2\big(V(\underline{X})\big)$ in such a way that for $f \in L^2\big(V(\underline{X})\big)$ the value of $\varphi_i(f)$ at a vertex x labelled 0 is the sum of the values of f at vertices in $A_i(x)$. The value of $\varphi_i(f)$ at the other vertices is determined by permuting the labelling.

$$\varphi_i(f)(x^{(k)}) = \sum_{y \in A_\alpha(x^{(k)})} f(y), \quad \text{where } \alpha = (i+k) \bmod (n+1).$$

Definition For each $i = 1, 2, \dots, n$, the operator φ_i defined above will be called the *i-th partial Laplacian* of the hypergraph X.

4 The Affine Building

Following [10] we introduce the notion of the affine building. If L is a field endowed with a nontrivial discrete valuation val, let $\mathfrak{O}_L$ be its ring of integers and $\mathfrak{p}_L$ its prime

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ideal. Assume that L is complete and the residue field $\mathfrak{O}_L/\mathfrak{p}_L$ is finite. In this section alone let G be a connected reductive algebraic group. We denote by T a maximal L-split torus of G, by $\mathcal{N}$ (resp. $\mathcal{Z}$) the normalizer (resp. the centralizer) of T in G. The group of rational points of $\mathcal{N}$ (resp. $\mathcal{Z}$) over L will be denoted by $\mathcal{N}(L)$ (resp. $\mathcal{Z}(L)$). We will denote the group of characters (resp. co-characters) of T by $X^* = X^*(T) = \mathrm{Hom}_L(T, \mathrm{Mult})$ (resp. $X_* = X_*(T) = \mathrm{Hom}_L(\mathrm{Mult}, T)$). Then $\mathcal{V}$ will denote the real vector space $X_* \otimes \mathbb{R}$, $\phi = \phi(G, T) \subset X^*$ will be the set of roots of G relative to T and U_a, for $a \in \phi$, will be the unipotent subgroup of G normalized by T and corresponding to the root a. Since G is connected, the group $W = \mathcal{N}(L)/\mathcal{Z}(L)$ is the *Weyl group* of the root system.

Associated to G, T and L there is a canonical affine space $\mathcal{A} = \mathcal{A}(G, T, L)$ under $\mathcal{V}$ on which $\mathcal{N}(L)$ operates, a system $\phi_{af} = \phi_{af}(G, T, L)$ of affine functions on $\mathcal{A}$, and a mapping $\alpha \to X_\alpha$ of ϕ_{af} onto a set of subgroups of $G(L)$, such that $s^{-1}X_\alpha s = X_{\alpha \circ s}$, for $s \in \mathcal{N}(L)$, that the vector parts $\mathfrak{v}(\alpha)$ of the functions $\alpha \in \phi_{af}$ are elements of ϕ, and that, for $a \in \phi$, the groups X_α with $\mathfrak{v}(\alpha) = a$ form a filtration of $U_a(L)$. The system ϕ_{af} is called the *affine root system* and its elements are called *affine roots*.

Let $\nu \colon \mathcal{Z}(L) \to \mathcal{V}$ be the homomorphism defined by

$$\chi\big(\nu(z)\big) = -\operatorname{val}\big(\chi(z)\big) \quad \text{for } z \in \mathcal{Z}(L) \text{ and } \chi \in X^*(\mathcal{Z}).$$

There is an extension of ν to a homomorphism, which we will also denote by ν, of $\mathcal{N}$ in the group of affine transformations of $\mathcal{A}$. The group $\mathcal{N}(L)$ operates on $\mathcal{A}$ through ν. The affine space $\mathcal{A}$ is called the *apartment* of T relative to G and L.

For every affine function α such that $a = \mathfrak{v}(\alpha) \in \phi$, we denote by $\mathcal{A}_\alpha$ the set $\alpha^{-1}([0, \infty])$ and by $\partial\mathcal{A}_\alpha$ the set $\alpha^{-1}(0)$. The sets $\mathcal{A}_\alpha$ (resp. $\partial\mathcal{A}_\alpha$) for $\alpha \in \phi_{af}$ are called the *half-apartments* (resp. the *walls*). The *chambers* are defined as the connected components of the complement in $\mathcal{A}$ of the union of walls. The facets of the chambers are also called the *facets* of $\mathcal{A}$. Chambers are facets of maximum dimension. If G is quasi-simple the facets are simplices, if G is semisimple they are polysimplices and in general they are direct products of a polysimplex and a real affine space.

There is the notion of an affine reflection r_α. Its vector part is the reflection r_a associated with $a = \mathfrak{v}(\alpha) \in \phi$ and its fixed hyperplane is $\partial\mathcal{A}_\alpha$. The group W_{af} generated by all r_α, $\alpha \in \phi_{af}$, is called the *Weyl group* of the affine root system. The affine root system, ϕ_{af}, is stable by the group $\tilde{W}_{af} = \nu\big(\mathcal{N}(L)\big)$. It follows that the half-apartments, the walls and the chambers are permuted by $\tilde{W}_{af}$, and that W_{af} is a normal subgroup of $\tilde{W}_{af}$. The Weyl group W_{af} is simply transitive on the set of all chambers. Attached to the apartment there is a Dynkin diagram as in [10, 1.8].

For $x \in \mathcal{A}$, we denote by ϕ_x the subset of ϕ consisting of the vector parts of all affine roots vanishing in x, and by $W_{af,x}$ the group generated by all reflections r_α for $\alpha \in \phi_{af}$ with $\alpha(x) = 0$. The point x is called *special* for ϕ_{af} if every element of the root system ϕ is proportional to some element of ϕ_x, that is, if ϕ and ϕ_x have the same Weyl group. If x is special, then W_{af} is the semidirect product of $W_{af,x}$ by the group of all translations contained in W_{af}.

The *building* $\mathcal{B} = \mathcal{B}(G, L)$ of G over L can be constructed by "gluing together" the apartments of the various maximal L-split tori. Below we give a more precise definition. By a "$G(L)$-set" we mean a set with a left action of $G(L)$ on it.

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Definition Let $\mathcal{A}$ be the affine space introduced before. Then there exists one and, up to unique isomorphism, only one $G(L)$-set $\mathcal{B}$ containing $\mathcal{A}$ and having the following properties: $\mathcal{B} = \bigcup_{g\in G(F)} g\mathcal{A}$, the group $\mathcal{N}(L)$ stabilizes $\mathcal{A}$ and operates on it through ν, and, for every affine root α, the group X_α fixes the half-apartment $\mathcal{A}_\alpha$ pointwise. The set $\mathcal{B}$ is called the *building* attached to the group G.

The sets $g\mathcal{A}$ with $g \in G(L)$ are called the *apartments* of the building. Let ${}^gT$ be the torus obtained from T by conjugating the elements in T by g and let ${}^g\mathcal{N}(L)$ be the group obtained from $\mathcal{N}(L)$ by conjugating all elements by g. The apartment $g\mathcal{A}$ can be identified with "the" apartment of the maximal split torus ${}^gT$. This gives a one-to-one correspondence between the apartments of $\mathcal{B}$ and the maximal L-split tori of G. The apartment $g\mathcal{A}$ is the only one stable by ${}^gT$, and ${}^g\mathcal{N}(L)$, which determines ${}^gT$, is the stabilizer of $g\mathcal{A}$ in $G(L)$.

Since the stabilizer $\mathcal{N}(L)$ of $\mathcal{A}$ in $G(L)$ preserves the affine structure and its partition into facets, each apartment $g\mathcal{A}$ of $\mathcal{B}$ is endowed with a natural structure of real affine space and a partition into facets. These structures agree on intersections. According to [10, 2.2.1], if $\mathcal{A}'$ and $\mathcal{A}''$ are two apartments, there is an element of $G(L)$ which maps $\mathcal{A}'$ onto $\mathcal{A}''$ and fixes the intersection $\mathcal{A}' \cap \mathcal{A}''$ pointwise. Moreover, $\mathcal{A}' \cap \mathcal{A}''$ is a closed convex union of facets in $\mathcal{A}'$, hence also in $\mathcal{A}''$. Therefore, there is a partition of $\mathcal{B}$ into *facets*. The facets which are open in apartments are called *chambers*. If G is quasi-simple, $\mathcal{B}$ is a simplicial complex. If G is semisimple, $\mathcal{B}$ is a polysimplicial complex. Also, given two facets of $\mathcal{B}$, there is an apartment containing them both.

One can choose in $\mathcal{V}$ a scalar product invariant under W. If G is quasi-simple, such a scalar product is unique up to a scalar factor. If G is semisimple, the scalar product can be chosen canonically. Then we have an Euclidean distance on $\mathcal{A}$ and, through the action of $G(L)$, on any apartment. Hence the building is endowed with a distance and in fact $\mathcal{B}$ is a complete metric space.

Attached to the building $\mathcal{B}$ there is a Dynkin diagram as in [10, 2.4]. We can also talk about *special* points of $\mathcal{B}$. For every subset Ω of the building $\mathcal{B}$, we denote by $G(L)^\Omega$ the group of all elements of $G(L)$ fixing Ω pointwise. If Ω is reduced to one point x, we write $G(L)^x$ for $G(L)^\Omega$. The stabilizers $G(L)^x$ of special points $x \in \mathcal{B}$ are called *special subgroups* of $G(L)$. The stabilizers of facets are called *parahoric subgroups*, and the stabilizers of chambers are called *Iwahori subgroups*. The Iwahori (resp. parahoric) subgroups can also be defined as the inverse images in the stabilizers $G(L)^x$, for $x \in \mathcal{B}$, of the $\mathfrak{O}_L/\mathfrak{p}_L$-Borel (resp. $\mathfrak{O}_L/\mathfrak{p}_L$-parabolic) subgroups under the reduction (mod $\mathfrak{p}_L$)-homomorphism.

We would like to introduce two more facts from [10]. First, the special subgroups of $G(L)$ are maximal compact subgroups, and secondly, if G is semisimple and simply connected, the maximal compact subgroups of $G(L)$ are precisely the stabilizers of the vertices of the building $\mathcal{B}$.

5 The Affine Building of $\mathrm{GL}_n(F_\mathfrak{p})$

Let F be a totally real number field and $\mathfrak{O}$ its ring of integers. Let $\mathfrak{p}$ be a prime ideal of F and $F_\mathfrak{p}$ the completion of F with respect to the valuation induced by $\mathfrak{p}$. We will denote by $\mathfrak{O}_\mathfrak{p}$ the ring of integers of $F_\mathfrak{p}$. The residue field $\mathrm{k}_\mathfrak{p}$ of $F_\mathfrak{p}$ is finite. Let π be

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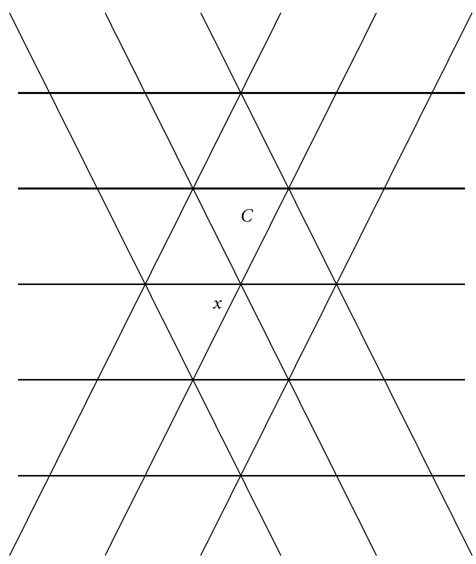
the uniformizer of $F_\mathfrak{p}$, *i.e.* $\pi \in \mathfrak{O}_\mathfrak{p}$ with $\mathfrak{p} = \pi\mathfrak{O}_\mathfrak{p}$. For the remainder of the article let G be the general linear group of rank n over the field $F_\mathfrak{p}$. Then $G(F_\mathfrak{p}) = \mathrm{GL}_n(F_\mathfrak{p})$ will be the group of $F_\mathfrak{p}$-rational points in G. We denote by T a maximal $F_\mathfrak{p}$-split torus of G. The Weyl group W is isomorphic to the symmetric group on n letters.

Consider the affine building $\bar{\mathcal{B}}$ of $\mathrm{GL}_n(F_\mathfrak{p})$. For the rest of the article building will mean affine building, unless otherwise specified.

Proposition *The building $\bar{\mathcal{B}}$ of $\mathrm{GL}_n(F_\mathfrak{p})$ is the direct product of the building of $\mathrm{SL}_n(F_\mathfrak{p})$ and an affine line.*

Proof See [1].

Below we give a picture of the apartment of $\mathrm{SL}_3(F_\mathfrak{p})$. The building is obtained by "ramifying" along every edge, each edge belonging to $q + 1$ triangles, where q is the cardinality of the residue field $k_\mathfrak{p}$ of $F_\mathfrak{p}$.



All vertices of the building of $\mathrm{SL}_n(F_\mathfrak{p})$ are special [10]. Since $\mathrm{SL}_n(F_\mathfrak{p})$ is semisimple and simply connected, the vertices of its building are in one-to-one correspondence with the maximal compact subgroups of $\mathrm{SL}_n(F_\mathfrak{p})$ [10]. The maximal compact subgroups of $\mathrm{SL}_n(F_\mathfrak{p})$ are the group $\mathrm{SL}_n(\mathfrak{O}_\mathfrak{p})$ and its conjugates under $\mathrm{GL}_n(F_\mathfrak{p})$. The affine line has only one vertex which corresponds to $\mathfrak{O}_\mathfrak{p}^*$, the only maximal compact subgroup of $F_\mathfrak{p}^*$. Thus, the group $\mathrm{GL}_n(F_\mathfrak{p})$ acts transitively on the vertices of the building of $\mathrm{SL}_n(F_\mathfrak{p})$ and it fixes the vertex of the affine line.

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It is well known that the maximal compact subgroups of $\mathrm{GL}_n(F_\mathfrak{p})$ are the group $\mathrm{GL}_n(\mathfrak{O}_\mathfrak{p})$ and its conjugates. Since $\mathrm{GL}_n(\mathfrak{O}_\mathfrak{p}) = \mathrm{SL}_n(\mathfrak{O}_\mathfrak{p}) \rtimes \mathfrak{O}_\mathfrak{p}^*$, it follows that the vertices of $\bar{\mathcal{B}}$ are in one-to-one correspondence with the maximal compact subgroups of $\mathrm{GL}_n(F_\mathfrak{p})$. Also, all vertices of $\bar{\mathcal{B}}$ are special and hence all maximal compact subgroups of $\mathrm{GL}_n(F_\mathfrak{p})$ are special subgroups.

In what follows we will make use of the action of the Weyl group $W = \mathcal{N}(F_\mathfrak{p})/\mathcal{Z}(F_\mathfrak{p})$ on $\bar{\mathcal{B}}$, and in particular on the set of vertices. The group W acts trivially on the affine line and for the remainder of the discussion we will ignore the affine line when we refer to the building $\bar{\mathcal{B}}$. The building $\bar{\mathcal{B}}$ is a labellable hypergraph [5, IV.1].

6 Finite Building Quotients of $\bar{\mathcal{B}}$

Consider the unitary group in n variables $U(n)$. For the precise definition see [8]. Let $\mathbb{A}$ be the ring of adeles of F, $\mathbb{A}_f$ the ring of finite adeles defined as the restricted direct product that defines $\mathbb{A}$ but without the infinite factor, and $\mathbb{A}_f^\mathfrak{p}$ the ring of finite adeles at all places except $\mathfrak{p}$ defined as the restricted direct product that defines $\mathbb{A}_f$ but without the term at the place $\mathfrak{p}$. Let G' be a F-form of $U(n)$ such that $G'(F_\mathfrak{p}) \cong \mathrm{GL}_n(F_\mathfrak{p})$ and $G'(\mathbb{R})$ is compact. For the definition of a form of $U(n)$ see again [8]. Denote by $K_\mathfrak{p}$ the maximal compact subgroup $G'(\mathfrak{O}_\mathfrak{p}) \cong \mathrm{GL}_3(\mathfrak{O}_\mathfrak{p})$ of $G'(F_\mathfrak{p})$. For each finite place $v \neq \mathfrak{p}$, let K_v be a compact open subgroup of $G'(F_v)$ chosen to be small and such that the group

$$K_f = \prod_{q \text{ finite}} K_q$$

is a compact open subgroup of $G'(\mathbb{A}_f)$. Then the group

$$K_f^\mathfrak{p} = \prod_{\substack{q \neq \mathfrak{p} \\ q \text{ finite}}} K_q$$

is a compact open subgroup of $G'(\mathbb{A}_f^\mathfrak{p})$. By [4, Theorem 5.1], the number of double cosets in

$$G'(F) \setminus G'(\mathbb{A})/G'(\mathbb{R})K_f$$

is finite and thus the number of double cosets in

$$G'(F) \setminus G'(\mathbb{A})/G'(\mathbb{R})G'(F_\mathfrak{p})K_f^\mathfrak{p}$$

is finite. Let $\{x_1, \dots, x_k\}$ be a set of representatives of these cosets. Then we have

$$G'(\mathbb{A}) = \bigcup_{i=1}^{k} G'(F)x_i\big(G'(\mathbb{R})G'(F_\mathfrak{p})K_f^\mathfrak{p}\big).$$

For each $i = 1, \dots, k$, consider the group $\Gamma_i' = G'(\mathbb{R})G'(F_\mathfrak{p})K_f^\mathfrak{p} \cap x_i G'(F) x_i^{-1}$. Each group Γ_i' is a discrete co-compact subgroup of $G'(\mathbb{R})G'(F_\mathfrak{p})$. Since $G'(\mathbb{R})$ is

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compact, the projection of Γ_i' on $G'(F_\mathfrak{p})$, which we also denote by Γ_i', remains a discrete subgroup. It is not difficult to see that Γ_i' is finitely generated. Then, according to [9, Lemma 8], Γ_i' has a normal subgroup Γ_i of finite index which has no nontrivial element of finite order. This implies that any element of Γ_i different from the identity acts on $G'(F_\mathfrak{p})/K_\mathfrak{p}$ without fixed points.

Denote by $\mathcal{B}_j$ the building quotient of the building of $G(F_\mathfrak{p})$ by Γ_j. It is a finite labellable hypergraph with rank n and labelling $\{0, 1, \dots, n-1\}$. Its underlying graph is regular.

7 The Partial Laplacians of $\mathcal{B}_j$

Let K denote the maximal compact subgroup $G(\mathfrak{O}_\mathfrak{p}) = \mathrm{GL}_n(\mathfrak{O}_\mathfrak{p})$ of $G(F_\mathfrak{p})$ and consider a vertex x in $\bar{\mathcal{B}}$ whose stabilizer is K. Since $G(F_\mathfrak{p})$ acts transitively on the set of vertices of $\bar{\mathcal{B}}$ we have

$$[G(F_\mathfrak{p}) : \mathrm{Stab}_{G(F_\mathfrak{p})}\, x] = |\,\mathrm{Orb}_{G(F_\mathfrak{p})}\, x| = |\text{Set of vertices}|.$$

Thus there is a one-to-one correspondence between the set of vertices of $\bar{\mathcal{B}}$ and the quotient group $G(F_\mathfrak{p})/K$ and therefore a one-to-one correspondence between the set of vertices of $\mathcal{B}_j$ and $\Gamma_j \backslash G(F_\mathfrak{p})/K$, the set of orbits of the action of Γ_j on the quotient group $G(F_\mathfrak{p})/K$.

Let V denote the vector space of compactly supported functions on the vertices of $\mathcal{B}_j$, $V = C_c\big(\Gamma_j \backslash G(F_\mathfrak{p})/K\big)$. It consists of the compactly supported functions on $\Gamma_j \backslash G(F_\mathfrak{p})$ that are right invariant under K. Consider now the Hecke algebra $\mathcal{H}_\mathfrak{p}$ of $G(F_\mathfrak{p})$ with respect to K,

$$\mathcal{H}_\mathfrak{p} = \mathcal{H}_\mathfrak{p}(G, K) = C_c\big(K \backslash G(F_\mathfrak{p})/K\big).$$

It is the set of complex valued, compactly supported functions on $G(F_\mathfrak{p})$ which are bi-invariant under K, endowed with the convolution given by

$$(f_1 * f_2)(g) = \int_{G(F_\mathfrak{p})} f_1(x) f_2(x^{-1}g)\, dx, \quad f_1, f_2 \in \mathcal{H}_\mathfrak{p} \text{ and } g \in G(F_\mathfrak{p}).$$

The Hecke algebra $\mathcal{H}_\mathfrak{p}$ acts on V by the induced algebra representation attached to the right regular representation. We denote this action by $\star$. For $\varphi \in \mathcal{H}_\mathfrak{p}$, $f \in V$ and x a representative in $G(F_\mathfrak{p})$ of a coset in $\Gamma_j \backslash G(F_\mathfrak{p})/K$, the action $\star$ is given by

$$(f \star \varphi)(x) = \int_{G(F_\mathfrak{p})} \varphi(y) f(xy)\, dy.$$

Let $d(a_1, \dots, a_n)$ denote the $n \times n$ diagonal matrix with entries $a_1, \dots, a_n$. We denote by φ_i, $i = 0, 1, \dots, n$, the characteristic function of the double coset $K t_i K$, where

$$t_i = d(\overbrace{1, \dots, 1}^{n-i}, \overbrace{\pi, \dots, \pi}^{i}) \in G(F_\mathfrak{p}).$$

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The functions φ_i, $i = 0, 1, \dots, n$, are sometimes called the *fundamental Hecke functions* and they generate the Hecke algebra $\mathcal{H}_\mathfrak{p}$.

Theorem *The fundamental Hecke functions φ_i, $i = 1, \dots, n-1$, are the partial Laplacians of $\mathcal{B}_j$.*

Corollary *The partial Laplacians of $\mathcal{B}_j$ commute with each other and their sum is the adjacency operator δ.*

Proof of the Corollary The corollary follows immediately from the fact that the Hecke algebra $\mathcal{H}_\mathfrak{p}$ is commutative [6].

Before we continue with the proof of the theorem, let us introduce some notation. We denote by $\mathcal{M}_{n,m}$ the set of $n \times m$ matrices and by I_n the $n \times n$ identity matrix. For every $i = 1, \dots, n$ the group

$$P_{n-i,i} = \left\{ \begin{pmatrix} A & B \\ 0 & C \end{pmatrix} \in G : A \in \mathcal{M}_{n-i,n-i}, B \in \mathcal{M}_{n-i,i}, C \in \mathcal{M}_{i,i} \right\}$$

is a maximal proper parabolic subgroup of G. The subgroup $M_{n-i,i}$ of $P_{n-i,i}$ consisting of matrices for which the block B has all entries equal to 0 is the Levi component of $P_{n-i,i}$. We have $M_{n-i,i} \cong \mathrm{GL}_{n-i} \times \mathrm{GL}_i$. The subgroup $N_{n-i,i}$ of $P_{n-i,i}$ consisting of matrices with the blocks A and C each equal to the identity matrix of the corresponding size is the unipotent radical of $P_{n-i,i}$. We have the Levi decomposition $P_{n-i,i} = M_{n-i,i} \cdot N_{n-i,i}$. Up to conjugation, the groups $P_{n-i,i}(F_\mathfrak{p})$ are all the proper maximal parabolic subgroups of $G(F_\mathfrak{p})$.

Proof of the Theorem Let φ_i be a fundamental Hecke function, f a function in the vector space V and x a representative in $G(F_\mathfrak{p})$ of a coset in $\Gamma_j \setminus G(F_\mathfrak{p})/K$. Then x corresponds to a unique vertex of $\mathcal{B}_j$ which we also denote by x. We will also denote by x the vertex of $\bar{\mathcal{B}}$ corresponding to the coset xK. Let $\mathcal{A}$ be an apartment of $\bar{\mathcal{B}}$ that contains x and let T be the maximal $F_\mathfrak{p}$-split torus corresponding to $\mathcal{A}$. We fix the apartment $\mathcal{A}$, and therefore the torus T, for now. Without loss of generality, we can assume that T is the subgroup of diagonal matrices of $G(F_\mathfrak{p})$.

We want to determine the value of $(f \star \varphi_i)(x)$. If $i = 0$, φ_0 is the characteristic function of K and $(f \star \varphi_0)(x) = f(x)\mathrm{vol}\,(K)$. The measure on $G(F_\mathfrak{p})$ is normalized such that $\mathrm{vol}\,(K) = 1$. Then $(f \star \varphi_0)(x) = f(x)$ and we see that φ_0 is not a candidate for one of the partial Laplacians. For the rest of the proof we will assume that $i \neq 0$.

Since φ_i is the characteristic function of Kt_iK and f is right invariant under K, we have

$$(f \star \varphi_i)(x) = \int_{Kt_iK/K} f(xy)\, dy.$$

The group K is compact open and so is Kt_iK. Then Kt_iK/K is finite and $(f \star \varphi)(x)$ is, in fact, a finite sum,

$$(f \star \varphi_i)(x) = \sum_{y \in Kt_iK/K} f(xy).$$

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We make the change of variables $y = ut_i$. Since $y = k_1 t_i k_2 K$ with $k_1, k_2 \in K$, we have $u = yt_i^{-1} = k_1 t_i K t_i^{-1}$ and therefore

$$(f \star \varphi_i)(x) = \sum_{u \in K/t_i K t_i^{-1} \cap K} f(xut_i).$$

Consider now the reduction (mod $\mathfrak{p}\mathfrak{O}_\mathfrak{p}$)-homomorphism $\zeta : K = G(\mathfrak{O}_\mathfrak{p}) \to G(\mathrm{k}_\mathfrak{p})$. It is a surjective homomorphism [10, 3.4.4] and its kernel is $\ker \zeta = \{H \in K \mid H \equiv I_n \pmod{\mathfrak{p}\mathfrak{O}_\mathfrak{p}}\}$.

Claim $\ker \zeta \subset t_i K t_i^{-1} \cap K$.

Proof Write any matrix $A \in K$ in block form as $A = (A_{ij})_{1 \le i,j \le 2}$ with $A_{11} \in \mathcal{M}_{n-i,n-i}(\mathfrak{O}_\mathfrak{p})$, $A_{12} \in \mathcal{M}_{n-i,i}(\mathfrak{O}_\mathfrak{p})$, $A_{21} \in \mathcal{M}_{i,n-i}(\mathfrak{O}_\mathfrak{p})$, $A_{22} \in \mathcal{M}_{i,i}(\mathfrak{O}_\mathfrak{p})$. Then the matrices in $t_i K t_i^{-1}$ are of the form $t_i A t_i^{-1} = (A'_{ij})_{1 \le i,j \le 2}$ with $A'_{11} = A_{11}$, $A'_{12} = \pi^{-1} A_{12}$, $A'_{21} = \pi A_{21}$, and $A'_{22} = A_{22}$.

Let $B \in K$ be a matrix in $\ker \zeta$. Then B is of the form $(B_{ij})_{1 \le i,j \le 2}$ with $B_{11} \in \mathcal{M}_{n-i,n-i}(\mathfrak{O}_\mathfrak{p})$, $B_{12} = \pi B'_{12}$ with $B'_{12} \in \mathcal{M}_{n-i,i}(\mathfrak{O}_\mathfrak{p})$, $B_{21} = \pi B'_{21}$ with $B'_{21} \in \mathcal{M}_{i,n-i}(\mathfrak{O}_\mathfrak{p})$, $B_{22} \in \mathcal{M}_{i,i}(\mathfrak{O}_\mathfrak{p})$ and such that $B_{11} \equiv I_{n-i} \pmod{\mathfrak{p}\mathfrak{O}_\mathfrak{p}}$ and $B_{22} \equiv I_i \pmod{\mathfrak{p}\mathfrak{O}_\mathfrak{p}}$. We can rewrite B_{12} as $\pi^{-1}(\pi^2 B'_{12})$ and then $B = t_i C t_i^{-1}$ where $C = (C_{ij})_{1 \le i,j \le 2}$ with $C_{11} = B_{11}$, $C_{12} = \pi^2 B_{12}$, $C_{21} = B'_{21}$, and $C_{22} = B_{22}$. Hence $\ker \zeta \subset t_i K t_i^{-1} \cap K$. ■

The matrices in $t_i K t_i^{-1} \cap K$ are of the block form $\begin{pmatrix} A_{11} & A_{12} \\ \pi A_{21} & A_{22} \end{pmatrix}$, where A_{ij}, $1 \le i, j \le 2$, are as in the proof of the claim. Then the image of $t_i K t_i^{-1} \cap K$ under ζ is the subgroup of $\mathrm{GL}_n(\mathrm{k}_\mathfrak{p})$ consisting of matrices in block form $\begin{pmatrix} A_1 & A_2 \\ A_3 & A_4 \end{pmatrix}$, such that all entries of $A_3 \in \mathcal{M}_{i,n-i}(\mathrm{k}_\mathfrak{p})$ are equal to 0,

$$\zeta(t_i K t_i^{-1} \cap K) = \left\{ \begin{pmatrix} * & * \\ 0 & * \end{pmatrix} \in \mathrm{GL}_n(\mathrm{k}_\mathfrak{p}) \right\}.$$

Hence, $\zeta(t_i K t_i^{-1} \cap K)$ is the proper maximal parabolic subgroup $P_{n-i,i}(\mathrm{k}_\mathfrak{p})$ of $G(\mathrm{k}_\mathfrak{p})$.

Since the reduction (mod $\mathfrak{p}\mathfrak{O}_\mathfrak{p}$) is surjective, by the second isomorphism theorem $G(\mathrm{k}_\mathfrak{p}) \cong K/\ker \zeta$. Since the image of $t_i K t_i^{-1} \cap K$ under ζ is $P_{n-i,i}(\mathrm{k}_\mathfrak{p})$, we have $P_{n-i,i}(\mathrm{k}_\mathfrak{p}) \cong t_i K t_i^{-1} \cap K/\ker \zeta$. Hence,

$$G(\mathrm{k}_\mathfrak{p})/P_{n-i,i}(\mathrm{k}_\mathfrak{p}) \cong (K/\ker \zeta)/(t_i K t_i^{-1} \cap K/\ker \zeta) \cong K/t_i K t_i^{-1} \cap K.$$

Denote by $\mathcal{N}_{(M_{n-i,i},T)}(F_\mathfrak{p})$ the normalizer of T in $M_{n-i,i}(F_\mathfrak{p})$ and let $W_{n-i,i} = W(T, M_{n-i,i}) = \mathcal{N}_{(M_{n-i,i},T)}(F_\mathfrak{p})/T(F_\mathfrak{p})$ be the Weyl group of $M_{n-i,i}$ relative to T. The group $W_{n-i,i}$ is generated by reflections corresponding to the set of simple roots from which one root has been removed. Then $P_{n-i,i}(F_\mathfrak{p}) = B(F_\mathfrak{p}) \cdot W_{n-i,i} \cdot B(F_\mathfrak{p})$. Denote by $N(\mathrm{k}_\mathfrak{p})$ the subgroup of upper triangular unipotent matrices in $G(\mathrm{k}_\mathfrak{p})$.

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The Bruhat decomposition [3, 21.16] holds and we have

$$G(\mathrm{k}_{\mathfrak{p}})/P_{n-i,i}(\mathrm{k}_{\mathfrak{p}}) = \bigcup_{w\in W/W_{n-i,i}} N(\mathrm{k}_{\mathfrak{p}})\cdot w\cdot P_{n-i,i}(\mathrm{k}_{\mathfrak{p}})/P_{n-i,i}(\mathrm{k}_{\mathfrak{p}}).$$

Therefore

$$K/t_iKt_i^{-1}\cap K \cong G(\mathrm{k}_{\mathfrak{p}})/P_{n-i,i}(\mathrm{k}_{\mathfrak{p}}) = \bigcup_{w\in W/W_{n-i,i}} N(\mathrm{k}_{\mathfrak{p}})wP_{n-i,i}(\mathrm{k}_{\mathfrak{p}})/P_{n-i,i}(\mathrm{k}_{\mathfrak{p}}),$$

and $(f\star\varphi_i)(x)$ becomes

$$(f\star\varphi_i)(x) = \sum_{w\in W/W_{n-i,i}} \quad \sum_{u\in N(\mathrm{k}_{\mathfrak{p}})wP_{n-i,i}(\mathrm{k}_{\mathfrak{p}})/P_{n-i,i}(\mathrm{k}_{\mathfrak{p}})} f(xut_i).$$

We make the change of variables $u = dw$. Since $u \in N(\mathrm{k}_{\mathfrak{p}})wP_{n-i,i}(\mathrm{k}_{\mathfrak{p}})/P_{n-i,i}(\mathrm{k}_{\mathfrak{p}})$, $dw = nwpP_{n-i,i}(\mathrm{k}_{\mathfrak{p}})$ with $n \in N(\mathrm{k}_{\mathfrak{p}})$ and $p \in P_{n-i,i}(\mathrm{k}_{\mathfrak{p}})$ and thus $d = nwP_{n-i,i}(\mathrm{k}_{\mathfrak{p}})w^{-1}$. With this change of variables the expression for $(f\star\varphi_i)(x)$ becomes

$$(f\star\varphi_i)(x) = \sum_{w\in W/W_{n-i,i}} \quad \sum_{d\in N(\mathrm{k}_{\mathfrak{p}})/wP_{n-i,i}(\mathrm{k}_{\mathfrak{p}})w^{-1}\cap N(\mathrm{k}_{\mathfrak{p}})} f(xdwt_i).$$

Now, if $i = n$, $M_{0,n}(F_{\mathfrak{p}}) = P_{0,n}(F_{\mathfrak{p}}) = G(F_{\mathfrak{p}})$ and $W_{0,n} = W$ is the entire Weyl group. Then $(f\star\varphi_n)(x) = f(xt_n)$ and again we see that φ_n is not a candidate for one of the partial Laplacians. Thus, for the rest of the proof $i = 1, 2, \dots, n-1$.

We can choose the vertex x such that its stabilizer is K. We want to investigate the vertices $xdwt_i$ as w ranges through a set of representatives of the cosets in $W/W_{n-i,i}$ and d ranges through $N(\mathrm{k}_{\mathfrak{p}})/wP_{n-i,i}(\mathrm{k}_{\mathfrak{p}})w^{-1}\cap N(\mathrm{k}_{\mathfrak{p}})$.

Fix a chamber C_0 in the apartment $\mathcal{A}$ such that x is one of its vertices and the other vertices in C_0 are stabilized by the maximal compact subgroups $t_iKt_i^{-1}$, $i = 1, 2, \dots, n-1$. We refer to C_0 as the standard chamber. We can assume that the stabilizer of C_0 is the Iwahori subgroup which is the inverse image under the reduction (mod $\mathfrak{p}\mathfrak{O}_{\mathfrak{p}}$)-homomorphism of the $\mathrm{k}_{\mathfrak{p}}$-Borel subgroup $B(\mathrm{k}_{\mathfrak{p}})$ of upper triangular matrices in $G(\mathrm{k}_{\mathfrak{p}})$. The dimension 1 face determined by the vertices x and xt_i, corresponding to K and $t_iKt_i^{-1}$ respectively, is stabilized by the parahoric subgroup which is the inverse image under the reduction (mod $\mathfrak{p}\mathfrak{O}_{\mathfrak{p}}$)-homomorphism of the $\mathrm{k}_{\mathfrak{p}}$-parabolic subgroup $P_{n-i,i}(\mathrm{k}_{\mathfrak{p}})$.

For $w\in W$ let $n_w \in \mathcal{N}(\mathrm{k}_{\mathfrak{p}})$ be a representative of w in the normalizer of T and denote by $w\big(B(\mathrm{k}_{\mathfrak{p}})\big)$ the conjugate $n_w\cdot B(\mathrm{k}_{\mathfrak{p}})\cdot n_w^{-1}$. Then, by [3, 21.23], $w \to w\big(B(\mathrm{k}_{\mathfrak{p}})\big)$ is a bijection of $W_{n-i,i}$ onto the set of minimal parabolic subgroups of $P_{n-i,i}(\mathrm{k}_{\mathfrak{p}})$ containing $T(\mathrm{k}_{\mathfrak{p}})$. Hence we have a bijection of $W_{n-i,i}$ onto the set of chambers C of $\mathcal{B}_j$ which are in the apartment $\mathcal{A}$ corresponding to T and which contain the dimension 1 face determined by x and xt_i.

Suppose that x is a vertex labelled 0. The link of x in $\bar{\mathcal{B}}$, and therefore in $\mathcal{B}_j$, is canonically isomorphic with the spherical building of $G(\mathrm{k}_{\mathfrak{p}})$ [10, 3.5.4]. If $w = I_n$,

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then $wP_{n-i,i}(\mathrm{k}_\mathfrak{p})w^{-1} \cap N(\mathrm{k}_\mathfrak{p}) = N(\mathrm{k}_\mathfrak{p})$ and $d \in \{I_n\}$. In this case $xdwt_i = xt_i$ is the vertex stabilized by $t_i K t_i^{-1}$. After an eventual relabelling of the hypergraph, xt_i is the vertex labelled i in the standard chamber C_0.

Consider now the case when w is a representative of a coset in $W/W_{n-i,i}$ such that $w \neq I_n$. Then, if $d = I_n$, the vertex xwt_i is a vertex in the apartment $\mathcal{A}$ in a chamber that does not contain the face of dimension 1 determined by x and xt_i. Thus, for $w \neq I_n$ and $d = I_n$, the summands in $(f \star \varphi_i)(x)$ are the values of f at the vertices in $A_i(x)$ that are in the chambers $C \neq C_0$ belonging to the apartment $\mathcal{A}$.

Let now T_1 and T_2 be two maximal $\mathrm{k}_\mathfrak{p}$-split tori which are both Levi components of the Borel subgroup $B(\mathrm{k}_\mathfrak{p})$. Then they are conjugate by a unique element in the unipotent radical of $B(\mathrm{k}_\mathfrak{p})$ which is $N(\mathrm{k}_\mathfrak{p})$ [3, 20.5]. Denote by $\mathcal{A}_l$ the apartment in the spherical building corresponding to the torus T_l, $l = 1, 2$. Since T_1 and T_2 are both Levi components of $B(\mathrm{k}_\mathfrak{p})$, the chamber stabilized by $B(\mathrm{k}_\mathfrak{p})$ belongs to both apartments: $\mathcal{A}_1$ and $\mathcal{A}_2$. Then, for a representative w of a coset in $W/W_{n-i,i}$ such that $w \neq I_n$ and $d \in N(\mathrm{k}_\mathfrak{p})/wP_{n-i,i}(\mathrm{k}_\mathfrak{p})w^{-1} \cap N(\mathrm{k}_\mathfrak{p})$, $d \neq I_n$, the vertex $xdwt_i$ belongs to an apartment different from $\mathcal{A}$ and in this case the summands in $(f \star \varphi_i)(x)$ are the values of f at the vertices in $A_i(x)$ belonging to apartments different from $\mathcal{A}$. By taking left cosets of $wP_{n-i,i}(\mathrm{k}_\mathfrak{p})w^{-1} \cap N(\mathrm{k}_\mathfrak{p})$ multiple countings of the value at the same vertex are avoided.

Thus $(f \star \varphi_i)(x)$ is the sum of the values of f at all vertices in $A_i(x)$. The value of $(f \star \varphi_i)(x)$ at the other vertices is obtained by permuting the labelling. Therefore, for $i = 1, 2, \dots, n-1$, each fundamental Hecke function φ_i is the i-th partial Laplacian for $\mathcal{B}_j$. This concludes the proof of the theorem. ■

Thus, as stated in the corollary, the adjacency operator is a sum of fundamental Hecke operators. These operators are well understood. Estimating the eigenvalues of the adjacency matrix of the underlying graph of $\mathcal{B}_j$ reduces to estimating the eigenvalues of the fundamental Hecke operators. This can be achieved using the representation theory of the Hecke algebra and its connection with the theory of unramified representations of the group $\mathrm{GL}_n(F_\mathfrak{p})$. We will undertake this estimation for the case $n = 3$ in a paper to follow.

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