The 20-knot diagram shows that the first bearing should be taken at about 62° relative bearing, the second at about 112°. The 10-knot diagram shows that the first should be taken at about 64°, the second at about 104°. The reduced change of bearing reflects the need, in the interests of accuracy, to shorten the run if the speed is such that uncertainties in the run have greater effect. If circumstances dictate a less than ideal choice for the first, or the second, bearing, the diagrams can still be used to indicate the optimum choice for the other bearing.

Next one may question the authors' estimate of the run accuracy. It is hard to believe that the error in the run can be given by one formula regardless of the circumstances. Off a prominent headland where the tides run hard (e.g. Portland Bill) the run must be much less accurate than in an area where the tidal streams are weaker and vary little from point to point (e.g. the late-lamented buoy in the middle of Lyme Bay). The existence or reliability of tidal data also varies from one region to another.

The difference between the two diagrams arises from the ratio of run error to bearing error. The run error can be expressed in terms of an 'angle', obtained by dividing the standard (or probable) error at the end of one hour by the distance run in an hour. Dividing this 'angle' by the standard (or probable) error of bearing gives a measure, r, of the ratio of run error to bearing error. The figures given by the authors yield $r = 4{\cdot}4$ for 10 knots and $2{\cdot}2$ for 20 knots. The navigator who prefers to estimate his own errors can calculate r and choose the appropriate diagram.

The Edge of a Needless Collision

Frank Coffman Bell

COMMANDANT L. OUDET in his contribution 'Lessons from a Needless Collision' (*Journal*, **20**, 30) may not have gone so far as to suggest that at no time ought either *Tenacious* or *Placid* to have altered course or speed then obtaining. Yet from his account this is plainly the case, a fact for which there must be some technical explanation, although he does not fully disclose it, saying rather, 'We have not sought to teach but only to stimulate thought among Masters and their shipowners.' No doubt others are not excluded. His pars. 2 & 3 give the facts in summary and the statement concerning the verdicts of *Tenacious* that 'at 1330 she finds the bearing is steady' plus some muttering about even worse than steady at 1340. This summary he later on amplifies slightly as to the facts:

(1) in par. 16, 'between 1344 and 1349. . . *Placid* made no (careful) observation.'
(2) in par. 18, '*Placid* did not detect this alteration (by *Tenacious* at 1340, from 130° to 160°).'
(3) in par. 20, 'In the first four minutes after the first radar observation, the range of the echo detected by *Placid* went from 4 miles to 2·6 miles, that is to say a reduction of 1·4 miles and a rate of closing of 21 knots.'

(4) in Table I, 'Bearings of the two Ships from each other', it does not seem allowable to assume that all entries were known at the times to the respective ships.

(5) in Fig. 1, 'The Collision between *Placid* and *Tenacious*', are presented three plots to different stated scales and in an absolutely fixed coordinate system (an enormous merit) of the tracks of the two ships. These plots may with Table I represent a construction from relatively more sketchy information. Noteworthy is the fact that Fig. 1 shows not one mutual line of bearing (hereafter called for short The Line); and that there is no statement that these plots were kept on board by either or both of the ships. but rather several implications suggesting the contrary. The Line is not interchangeable with 'bearing'—a matter of great if somewhat subtle importance. The Line in its motion or its locus envelopes an edge of regression, hereafter The Edge for short, and the rotation of The Line is actually the curvature of The Edge. 'Bearing' is merely a direction. Fig. 1 is perfectly suited to the constructions which I shall give below. One has only to pencil in some easy projections and some instantaneous positions of The Line.

At $13^h\ 40^m\ 40^s$ a tangency of *Tenacious*' track with The Edge would have taken place (she reported altering at 1340) at which epoch the time-rate of rotation of The Line was slightly greater than $10°/5^m$ and increasing. In his par. 20 Commandant Oudet suggests that at this epoch *Placid* either could have calculated or perhaps actually did calculate the closing rate along The Line, and could have deduced or did deduce that the speed of *Tenacious* was 7 knots, that of *Placid* being known on board her to be 13 knots. His argument in par. 20 applies with even more force to the situation earlier on at about 1315, when also *Tenacious*' track had come tangent to The Edge, at which epoch the time-rate of rotation of The Line was $2°/5^m$ and increasing. Surely the same argument can be extended to the situation as seen from *Tenacious* at these two epochs so far as to admit that she could have deduced or did deduce a speed of 13 knots for *Placid*, the speed of *Tenacious* being known on board her to be 7 knots. It may be noted also that his pars. 16 and 17 give the weighty opinion of Commandant Oudet that 5^m is more than enough for a careful observation and the necessary deductions therefrom. On this sure basis, one can assume that at the end of any era of at least 5^m duration, either ship could have plotted both tracks for the era in an absolutely fixed coordinate system.

From 1310 until collision occurred at 1350, I judge from the Commandant's account (a) that *Tenacious* was concerned only with the obligation to alter if the absolute rotation of The Line should tend to vanish, (b) that she was basing her verdict about this vanishing on rotation measured in a coordinate system moving with the ship, (c) that she was not at all concerned with such obligation as there might be to maintain course and speed if for these the miss had already occurred, and (d) that she was, in fact, not aware of any danger in a 'conservative' view on assessing a tendency of The Line's rotation to vanish—i.e. she held the conviction that if such a tendency could be read by any 'reasonable' allowance in the matter of accuracy, the safest thing to assume would be the vanishing. I suggest that no frame of mind could be more dangerous, and especially when coupled with measurements referred to a moving rather than to an absolutely fixed coordinate system. It seems from the Commandant's legal discussion that *Tenacious*' alterations should be held to be in violation of

Rule 27 if it could be shown to the court's satisfaction that a miss will certainly occur if two ships maintain course and speed while The Line's rotation does not vanish—a showing which I intend to make here shortly.

Placid made her first observation, by radar, at 1336, range of 4 miles, and made only one other reported accurate measurement at 1340, of an echo a point to starboard, range nearly 3 miles. She had no basis for stopping, and could not have been hit if she had maintained course and speed. There was no difference between the situations at 1310 and at 1340. The orders 'Slow' at 1342 and 'All Stop' at 1344 were not justifiable (nor, of course, justified), being in the circumstances at least as dangerous as 'Maintain Course and Speed'. On this point one may well compare the first sentences of the Commandant's par. 10. A miss had already occurred and *Placid* did make or could have made the measurements necessary to this determination. But it appears that if she had had these measurements, her view of their usage would have been identical with that of *Tenacious*, resulting in the same legal position with respect to the inevitable collision.

I now propose to show that at 1330 *Tenacious* ought not to have altered. I shall refer to facts about the parabola as assembled in concise form for practical persons by Edward Vermilye Huntington, Ph.D., the late Professor of Mathematics and Mechanics in Harvard University, in Marks, L. S.(Ed.) (1941) *Mechanical Engineers' Handbook*, 4th ed. McGraw-Hill Book Co. Inc., New York and London, pp. 138–140, esp. Fig. 20. Professor Huntington there deals briefly with this problem: To construct a parabola, given two tangents and their points of contact, *P* and *Q*. I shall modify this treatment by making use of the assumption that speeds and courses are constant over each part of a composite collision situation. By 1325 at the latest, *Tenacious* easily estimates and plots in an absolutely fixed coordinate system her point of tangency *Q* as having occurred at 1315, with *Placid* on The Line instantaneously bearing 117° and at range 17 intervals of 5^m into 7 knots. Plotting the track of *Placid* in the same coordinate system, *Tenacious* marks its point of intersection, zero, with The Line 117°. It is then evident that the track of *Placid* projected 17 intervals of 5^m into 13 knots from point zero will come tangent to a non-singular parabola at point 17, *P*, and that this same parabola is also tangent to the track of *Tenacious* at point *Q*. This non-singular parabola is The Edge of all The Lines, and therefore collision is impossible if only each ship maintains course and speed. Taking the view of *Placid*, by 1325 she is able to demonstrate on her plot the tangency at *Q*, 1315, of the track of *Tenacious* with The Edge and then proceed to the same conclusion. If at 1330 any order is required in either ship, it reads, 'Maintain Course and Speed'. About 1350, the two ships will pass each other Green to Green and about one mile apart, without those delays against which Captain Wepster so rightly inveighs. But it is important to realize that the miss was already a certainty after 1315, provided only that neither ship altered course or speed.

There is no difficulty in applying the same analysis and construction to the situation which in historical fact obtained at 1340, at which epoch a miss also occurred provided only that neither ship altered. But *Placid*, neglecting the last clear chance to avoid collision, stopped and by so doing caused an unprofitable delay in trade and risked being run down while dead in the water by other traffic, matters which no doubt came to the thoughtful attention of the owners.