Part 6

Analysis of Spectra and Light Curves

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The Disentangling of Stellar Spectra

P. Hadrava

Astronomical Institute, Academy of Sciences,
Boční II 1401, CZ - 141 31 Praha 4, Czech Republic
email: had@sunstel.asu.cas.cz

Abstract. The techniques of disentangling were originally developed to separate spectra of individual components from time series of spectra of binaries and, simultaneously, to determine either the corresponding radial velocities or directly to solve for orbital parameters.

Generalizations of the disentangling method enable us to include also intrinsic line-profile variability of the component spectra into the underlying model, and thus to solve for additional physical parameters of the stars (either single or components of multiple systems). Depending on the problem in question, it may also be helpful to constrain the space of separated spectra by templates or to bound the solution in the parameter space by photometry, interferometry or other observational data.

Keywords. Methods: data analysis, techniques: spectroscopic, stars: atmospheres, binaries: spectroscopic, line: profiles

1. Introduction

The aim of science is to systematize the knowledge gathered from observations and experiments in order to comprehend the natural laws. The understanding of basic principles of nature yields a tool for its exploitation, however, the theoretical conclusions must be tested by new observations to verify, to modify, or to completely disprove and change the starting assumptions and methods of reasoning. In practice, we usually build a theoretical model predicting the expected results of the observations and, from the comparison with real observations, we find values of some free parameters of the model to fit the observations best. An agreement of the observation with the model, however, does not necessarily mean a proof of the theory; there is a danger of a circle in proof because the interpretation of observations is model-dependent. We never expect a perfect agreement with sufficiently decisive observational tests either due to simplifications in the model or due to observational errors. A relatively good agreement thus does not exclude that another, even a completely different model could fit the given observational data better, not to speak about some other data, which could decide on the validity of alternative models. It is thus safer to test a model by a maximum of the available observational data, and to avoid unnecessary limitations of generality of the model. In any case, it is necessary to keep in mind the underlying assumptions of the model and critically judge their consequences.

A good example of the described general problem are the observations of binary stars and their spectroscopy in particular. The photometry yields relative dimensions of eclipsing binaries and the spectroscopy enables us to scale them in absolute physical units (cf. Kallrath & Milone 1999, 2009 for a recent general review). In the classical approach, radial velocities (RVs) of the component stars are measured from time series of spectral observations and the obtained RV-curves are solved for the orbital parameters – cf. Fig. 1. (It is preferable to solve the RV-curves together with the light-curves, which may better constrain some of the orbital parameters, typically the period or the conjunction

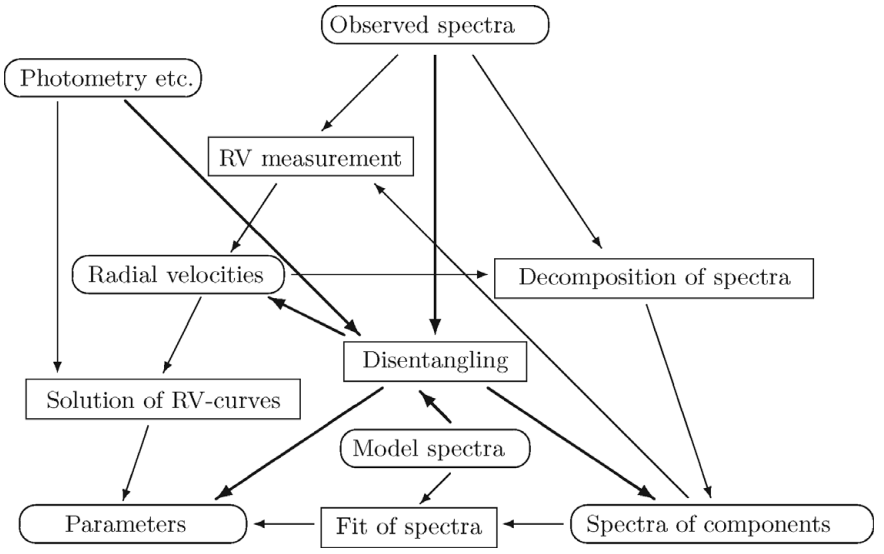

Figure 1. Flow diagram of standard data processing (thin lines) and the disentangling (bold lines) of binaries observations

epoch.) To measure the RVs, we need some model of the component spectra – either the simple assumption that the bottoms of distinguishable line-profiles correspond to the wavelengths of the lines Doppler-shifted for velocity of mass-centers of the component stars, or, in more sophisticated methods like the cross-correlation (cf. Hill 1993) suitable also for blended lines, we need some template spectra of the components. There have been developed methods like the tomographic separation by Bagnuolo & Gies (1991) enabling us to decompose the observed spectra if the RVs of the component stars are known at each exposure. The information about both the RVs and the component spectra is thus entangled in the same set of observed spectra and the original idea of the method of disentangling was to extract it simultaneously without any unsubstantiated *ad hoc* assumptions.

It turns out that, if the RVs can be supposed to obey the laws of Keplerian motion in a multiple stellar system, as we used to assume in the solving the RV-curves, we do not need to include in the fitting of the observed spectra the intermediate step of determining their values, which may be in different phases subjected to different errors (also dependent on S/N), and we can solve directly for the orbital parameters. Regarding the above mentioned restrictions imposed on the orbital parameters from the photometry and/or other data, we can fit these data simultaneously with the observed spectra. Certainly, the RV-curves and their scatter yield an insight into the solution, and for this reason RVs can also be computed, however, it is up to the user which physical model he wants to test with the observational data, eventually, one may try different models and to compare their results. Similarly, if some of the separated component spectra correspond well with computed model spectra, e.g., for plane-parallel stellar atmospheres, it may be preferable to impose such a restriction on the solution already at the stage of the disentangling, to optimize the solution directly with respect to the free physical parameters of the stellar atmosphere and to decrease significantly the number of free parameters to arrive at a safer shape of spectra of the other, possibly peculiar components.

In this generalized view, the disentangling is a versatile tool for testing physical theoretical models with (not-only) spectroscopic observational data, which is open to an additional sophistication in future. Its complete description exceeds the available space

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and consequently intention of the present contribution and readers may find more details in Hadrava (2004b) and Hadrava (2009b). Here, only the basic principles and some improvements of the method of disentangling will be given.

2. Mathematical principle of the disentangling

The spectrum $I(x,t)$ of a multiple system of n stars observable at time t is a superposition of Doppler shifted spectra $I_j|_{j=1}^n$ of all the components. In the simplest case when these spectra do not change with time t, it can be given by expression

$$I(x,t) = \sum_{j=1}^{n} I_j(x) * \Delta_j(x,t,p) \ , \tag{2.1}$$

where $x = c\ln\lambda$ is the logarithmic wavelength scale and

$$\Delta_j(x,t,p) = \delta(x - v_j(t,p)) \tag{2.2}$$

is the Dirac delta-function shifted for v_j, i.e. the instantaneous RV of the component j which depends also on the orbital parameters p. To disentangle a set of N spectra ($N > n$) exposed at times $t_l|_{l=1}^N$ means to fit them by minimizing the residual noise (O–C)^2,

$$0 = \delta \sum_{l=1}^{N} \int \left| I(x,t_l) - \sum_{j=1}^{n} I_j(x) * \Delta_j(x,t_l,p) \right|^2 dx \tag{2.3}$$

with respect to the parameters p (which is an equivalent to the measurement of RVs) as well as with respect to the intrinsic component spectra I_j (which is the separation of spectra of the components). While the fitted expression (2.1) is non-linear with respect to p which, however, represents a modest number of degrees of freedom, it is linear with respect to the highly numerous degrees of freedom of $I_j(x)$. For this reason, it is advantageous to use different numerical methods for the optimization with respect to these variables and to apply them iteratively.

The existing techniques of the disentangling can be divided into two basic groups according to the method of solving for I_j. The methods performing the separation of the spectra in the direct wavelength space (x) can be represented by the method introduced by Simon & Sturm (1994), which is based on the SVD-solution (i.e. the singular value decomposition) of the corresponding set of linear equations given by a huge but sparse matrix. Also, the tomographic separation by Bagnuolo & Gies (1991) or the practically equivalent iterative subtraction developed by Marchenko *et al.* (1998) performs the decomposition of the spectra in their wavelength-domain representation.

Fourier disentangling. The alternative method introduced by Hadrava (1995) profits from the fact that the convolution in Eq. (2.1) is transformed into a simple product

$$\tilde{I}(y,t) = \sum_{j=1}^{n} \tilde{I}_j(y)\ \tilde{\Delta}_j(y,t,p) \tag{2.4}$$

in the Fourier domain (y) and consequently the huge set of coupled linear equations in the optimization reduces into independent equations of dimension n only for each Fourier mode y of the separated spectra $\tilde{I}_j$. The solution is thus much faster in this representation not only for the Doppler shifts (2.2) for which

$$\tilde{\Delta}_j(y,t,p) = \exp(iyv_j(t,p)), \tag{2.5}$$

but for any broadening function $\Delta_j(x,t,p)$.

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Owing to the Parseval theorem, the residual noise can be expressed equivalently to Eq. (2.3) as an integral in the Fourier domain. The optimization with respect to the parameters p can thus be performed by a standard algorithm like the simplex method with the $(O–C)^2$ evaluated in each step by integration in the y-space of the squared residua with respect to the already optimized Fourier modes of the component spectra. Moreover, it is possible to assign different weights to different Fourier modes. This is of great practical use not only for filtering a high-frequency noise but mainly because it is sometimes advantageous to filter out the lower modes which are poorly conditioned (e.g. the constant mode for $y = 0$ is completely singular as follows from Eq. (2.5)), and their instability caused, e.g., by uneven continua may mislead the convergence of orbital parameters, which are determined mainly by the higher Fourier modes corresponding to the widths of the spectral lines. This advantage fairly prevails over the possibility to weight individual pixels of the input spectra in the wavelength-domain disentangling advocated by Ilijić (2004) which, however, may be substituted by careful data-processing before the disentangling is applied.

It should be emphasized that for all methods of disentangling and separation of spectra, a sharp signal with dumped noise is extracted from all spectra at once similarly as explained by Rucinski (2002) for his broadening-function technique (BFT), in contrast to the various cross-correlation techniques where the noise is blurred together with the signal for each spectrum separately, collecting the information at maximum from different lines in the same exposure.

3. Generalized disentangling

The methods of spectra separation as well as the above described simple disentangling are based on the assumption (2.2) of invariability of the component spectra. This, however, is not always satisfied in nature. For instance, the relative strength of component spectra is changed during the eclipses or even the line-profile varies due to the rotational Schlesinger – Rossiter – Mc Laughling effect (Schlesinger 1909 etc.). The proximity effects like the ellipticity or reflection effect influence the line-profiles in the interacting binaries or there may take place intrinsic variations of the component spectra, e.g. due to pulsations or activity of the component stars. One possibility is to exclude the most peculiar exposures (e.g. those taken during the eclipses) from the analysis, but then we lose a very valuable source of information which enables us to test e.g. the structure of the stellar atmospheres. Another possibility is to neglect these effects first, to apply the simple disentangling and then to study the deviations of individual exposures from the simplified model and to discuss the possible influence of the simplification on the results (suppressing the weights of the peculiar exposures, e.g. at eclipses, may be helpful for this approach). However, the best way is to generalize the model which we fit to the data and to disentangle also the free parameters characterizing these additional effects (naturally, keeping in mind that the results are conditioned by an appropriate choice and sophistication of the model).

Fortunately, many of the above mentioned effects may be well-approximated in terms of a convolution of the stellar spectrum with some broadening function (e.g. a rotational broadening) or as a superposition of a few spectral functions (corresponding, e.g., to different layers of the atmosphere) convolved with different broadenings. The basic Eq. (2.1) is thus sufficiently general and the whole procedure of the Fourier disentangling can be applied as before with a generalized form of the broadening functions $\Delta_j(x, t, p)$, which will in addition to the Doppler shift given by Eq. (2.2) also include an intrinsic broadening of the line profile (their Fourier transforms are to be multiplied). From the

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point of view of the underlying physics, the generalized disentangling is related to the BFT designed by Rucinski (2002) for RV measurements. However, the difference is in the question we ask, i.e. the variables we want to solve. In the BFT, a template spectrum is to be chosen (the same for all components) and the broadening function is solved for from the observations, without any *a priori* limitations, to find from it the RVs and possibly also the intrinsic broadening. In the disentangling, we want to solve for the component spectra, hence, we must restrict the space of possible broadening functions by some physical model with a few free parameters only (which can be directly disentangled). It is obvious that if the component spectra are variable, we can solve for some reference spectrum only, which is not uniquely defined because any constant part $b(y)$ of the broadening $\tilde{\Delta}_j(y,t,p) = b(y)\tilde{\Delta}'_j(y,t,p)$ may also be taken as a part of the intrinsic spectrum $\tilde{I}'_j(y) = b(y)\tilde{I}_j(y)$.

Already, the very simple generalization consisting of multiplication of the right-hand side of Eq. (2.2) by a scalar line-strength factor $s_j(t)$ increases significantly possibilities of the method of disentangling (Hadrava 1997). To mention at least some of the examples, it facilitates the fit of spectra for exposures taken during eclipses and thus not only to improve their RVs but also to find limb-darkening in different lines, which show the structure of the stellar atmosphere. It also enables us to disentangle the telluric lines which have variable line-strengths dependent on the atmospheric conditions and air-mass. (The molecular components of the Earth's atmosphere with different variability of abundance can also be mutually separated.) The profit is not only in increased reliability of the disentangled spectra, but also in the possibility to check and to improve the precision of the wavelength scale for the purpose of high-precision RV measurements. (For this purpose also the enhancement of precision to the sub-pixel resolution is important, cf. Hadrava 2009a.) Similarly to the natural telluric lines, the lines from an iodine cell or interstellar and circumstellar lines can also be disentangled. The disentangling of absorption interstellar lines from spectra of binaries or Cepheids yields a unique constraint on the depth structure of the interstellar mass (and simultaneously information about the interstellar extinction needed for a photometric determination of distances of the stars).

A large variety of other models of line-broadening can be included into the generalized disentangling for various purposes. For instance, the already mentioned rotational effect during the eclipses can be disentangled as described by Hadrava (2007). The standard approach to treatment of this effect is to measure RVs by some classical method (e.g., from bisectors or moments of the stellar lines) and to fit the resulting RV-curves with a model which includes a correction of RVs with respect to the Keplerian orbital motion. However, the rotational distortion of the line-profiles depends on the limb-darkening within the profile, and is thus different for different lines. The generalized disentangling offers an alternative possibility to fit the observed spectra directly by appropriately broadened disentangled spectra of the component stars.

Another interesting example is the line-profile variability due to pulsations (Hadrava *et al.* 2010). The RV-variations due to radial pulsations of Cepheids can be measured approximately by disentangling with free RVs (i.e. with switching-off the condition of RVs subjected to an orbital motion). However, the contribution of the parts of atmosphere seen under a non-zero angle results in a distortion of the line-profile, which is moreover dependent again on the limb-darkening in the line. It is thus preferable to model these line-profile variations (LPVs) by an appropriate broadening function and to disentangle the mean intrinsic spectrum of the Cepheid atmosphere in its rest-frame together with the instantaneous pulsational velocities. The variations of line-strengths caused by the changes of the temperature must also be disentangled; eventually the spectrum can be

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disentangled as a linear superposition of two or more spectral functions corresponding to the different temperatures. This application is important because it enables us to perform the Baade – Wesselink calibration of the period – luminosity relation of Cepheids without a need to establish the projection factor (cf. e.g. Nardetto *et al.* 2004). Similar to the application of disentangling to eclipsing binaries (cf. Wilson 2008), this disentangling of single-star spectra also provides primary distance markers on the extragalactic scale. Challenging is its application to Cepheids in binaries (cf. Pietrzyński *et al.* 2010), which could yield a comparison of both these methods and a better insight into the physics of Cepheids. For the non-radially pulsating stars, which are much more common in binaries, the disentangling can be used to solve for the orbital motion, neglecting first the LPVs which are usually of lower amplitude and can be found as residuals from the mean disentangled spectra. However, a true disentangling of non-radial pulsations should include the application of a proper model of the corresponding LPVs.

Disentangling with templates. The general advantage of disentangling including the separation of the component spectra without any *a priori* assumption on their form may turn into a disadvantage in the cases when we have a good reason for accepting such an assumption. For instance, the shape of the telluric spectrum is basically known (up to the possible variations of relative line-strengths of water vapour and other molecules) but, in systems which have a third or a circumstellar component with small RV- amplitude or some peculiar variability, a part of this stellar spectrum may penetrate into the telluric spectrum in the numerical solution. It is thus better to fix some component spectra I_j to appropriate templates J_j if we know them, and to minimize the expression (2.3) with respect to the unknown components only and with respect to the parameters p of the broadening functions Δ_j (including those belonging to the component spectra constrained by the templates).

This option may also be used temporarily to decrease the number of degrees of freedom of the solution before approximate values of orbital parameters are found, and the final tuning of the solution may be performed without constraining by the template. Alternatively, if an unconstrained disentangling gives a component spectrum closely resembling some standard spectral type, it can be constrained by a corresponding model spectrum to disentangle additional parameters of the component (e.g. its rotational broadening and physical parameters of the atmosphere) and to force the solution to distribute the residua between the other, possibly peculiar components. The interpretation of the separated spectra and determination of the physical parameters of the components is the purpose of the whole process and it can be performed on several levels – either by trial-and-error fitting of the decomposed spectra with models which include the possible broadening (cf. e.g. Zverko *et al.* 1997, Pavlovski & Hensberge 2010), or using BFT (Rucinski 2002), or directly in the process of the disentangling. It should be noted that only after a template spectrum is compared with the separated spectra, the systemic (i.e. the γ-) RV can be determined, and that (due to the above mentioned singularity of the zeroth Fourier mode) the continuum cannot be divided between the components from the spectroscopy alone, and hence the separated spectra must be scaled in strength either from the light-curve solution or just from fitting by model spectra.

Disentangling with constrained parameters. As mentioned in the Introduction, different observational data put different limitations on the free parameters of our models. Although it may be encouraging if we arrive at similar results from independent data of different kinds, there is a danger that their independent fits will yield inconsistent values of parameters for which the sensitivity differs. One way is thus to converge in each solution only those parameters on which the data may put stringent limitations and to fix the others to values found from data which are more sensitive to them. However, quite

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often the solutions provide some bounding condition on values of several parameters p, and we should search for the solution of other data in a subspace of the parameter space given by an equation $F(p) = 0$. (For example, photometry of eclipsing binaries usually gives a precise epoch of conjunctions, but the epoch of periastron is correlated with the longitude of periastron for eccentric orbits.) To find numerically the minimum of Eq. (2.3) bounded by such a condition or several conditions, we should add to its right-hand side a term $\sum_k \lambda_k F_k^2(p)$, where $\lambda_k > 0$ are Lagrange multiplicators, and to minimize the overall sum for $\lambda_k \to \infty$. Because in practice the bounding conditions result from some other observations, they are not sharp but admit some scatter given by the $(\mathrm{O–C})^2$ of the data. The disentangling constrained by other observations is thus a simultaneous solution of all available data (e.g. light curves, or published RVs from unavailable spectra, interferometry – either reduced to mutual positions of component stars, or directly the visibility functions, etc.) in addition to the direct fit of the spectra by minimizing a properly weighted sum of squared residua of all the data.

Such a simultaneous solution of all kinds of data enables a direct fit of additional parameters like the distance in the above mentioned solution, together with light-curves or with astrometry (cf. Zwahlen *et al.* 2004). It may also have the advantage of a significant increase in the precision of the obtained parameters values. The errors have to be determined from the Bayesian probability, which should be mapped in the vicinity of the derived best solution in the parameter space. The precision of a common solution for different data may be much higher than that given by an overlap of area restricted by solutions of individual subsets of the data. This is obvious from the example of period searches, where the solutions of data distant in time mutually interfere and their common solution may have an accuracy higher for orders in comparison with the individual solutions.

4. Conclusion

The method of disentangling has been succesfully applied to many studies of individual spectroscopic binaries and multiple stellar systems. Its wavelength-domain versions were independently programmed by several users according to the published descriptions. The Fourier versions of the code: FD-BINARY (Ilijić *et al.* 2004) and KOREL (Hadrava 1995) are available. The latter, which enables us to disentangle up to five components in a two-level hierarchical structure of orbits with their line-strength factors and possible constraining by templates, is now available in the framework of Virtual Observatory (the VO-KOREL at `http://stelweb.asu.cas.cz/vo-korel`, cf. Škoda & Hadrava 2010). There are also versions of KOREL with the pulsational and rotational broadening included, and an implementation of other broadening functions is in progress. The controlling of this version is quite complicated; it is not yet settled to a user-friendly form and hence not yet publically available. The same is true for the new code BAŽANT which is a blend of KOREL with the code FOTEL for solution of light-curves and other data (cf. Hadrava 2004a).

The disentangling simplifies the interpretation of spectroscopic observations of binaries, which is quite laborious in the standard way. This could predestinate this method for an automated application to massive data gathered in large space- or ground-based surveys. At the same time, the simplification of the data-processing enables us to sophisticate the method from the point of view of the involved physics, which, however, increases demands on insight of its users. The future development should thus follow both ways. The latter one actually crosses the borders between the often independent fields of theoretical modelling and methods of interpretation of the data. The development

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of future tools for astrophysics should follow and extrapolate this way to a physical sophistication and versatile applicability.

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Discussion

S. Zucker: How does spectral disentangling deal with gaps in the spectrum? Specifically with multi-order spectra. The question arises because of the Fourier steps that require equidistant sampling in $\log(\lambda)$.

P. Hadrava: A smooth merging of the orders in Echelle spectra is a subtle problem studied, e.g., by Hensberge (2007). It is important for disentangling of wide spectral regions where some unevenesses may complicate disentangling especially of lower Fourier modes. Fortunately, for an accurate determination of radial velocities, we can use narrow regions with higher sampling within separate orders. Equidistant sampling is more critical for the wavelength-domain disentangling than for the Fourier one, but it can be achieved by interpolation or an appropriate primary data reduction.