A NUMERICAL APPROXIMATION FOR HIERARCHICAL TRIPLES

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Abstract. This paper describes a numerical method for following the evolution of the orbit of a perturbed binary (e.g. the inner binary of a hierarchical triple) by means of averaging.

1. Introduction

Interactions between primordial binaries in star clusters frequently give rise to long-lived hierarchical triple systems. These are a troublesome feature of N-body simulations. In many cases the relative motion of the inner components cannot be treated as unperturbed: perturbations by the outer body can radically alter the probability of a physical collision (Marchal 1990; this paper, Fig.1). In this paper we analytically average over the fast motion of the binary. Then it is necessary only to integrate numerically the equations for the secular evolution.

2. Outline and Illustration of the Method

If the method of averaging is applied to the motion of the inner binary, its semi-major axis, a, is constant (Marchal 1990). Therefore the orientation and shape of its orbit are determined by its angular momentum vector $\mathbf{h}$ and the Laplace vector $\mathbf{e}$, whose magnitude is the eccentricity, e.

Let m_3 be the mass of the third body, and $\mathbf{R}$ its position vector relative to the barycentre of the binary. Then in the quadrupole (tidal) approximation, the average rate of change of $\mathbf{h}$ is given by $\langle\dot{\mathbf{h}}\rangle = \langle r_2^2\rangle f_{23}\mathbf{u}_1 - \langle r_1^2\rangle f_{13}\mathbf{u}_2 + (\langle r_1^2\rangle - \langle r_2^2\rangle)f_{12}\mathbf{u}_3$, where $\langle r_1^2\rangle = a^2(1/2 + 2e^2)$, $\langle r_2^2\rangle = a^2(1 -$

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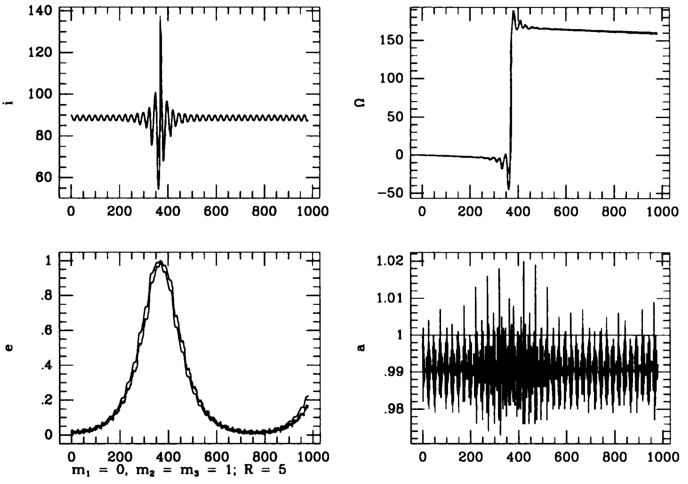

Figure 1. Illustration of the method. The masses are as indicated. Initially $e = 0$, $a = 1$ and the orbital planes are orthogonal. The third body is *forced* to move on a circular orbit of radius 5. Each graph shows the variation of one element of the inner binary with time, calculated in two ways: (i) the averaged equations described here, and (ii) an "exact" integration of the equation of motion of the binary, with the exact perturbation by m_3. Where the two graphs can be distinguished, the latter is the one with high-frequency oscillations. *Upper left*: the inclination between the orbital planes. Large oscillations occur when $e \simeq 1$, but the two integrations are generally in satisfactory agreement. *Upper right*: the longitude of the line of intersection of the orbital planes. At the time when $e \simeq 1$ the motion of the inner binary changes sense. *Lower right*: the semi-major axis. The systematic offset could be corrected by taking into account the periodic oscillations in a in setting up the initial conditions. *Lower left*: the eccentricity. When $e \simeq 1$ a collision between the components of the inner binary is possible.

$e^2)/2$, the unit vectors $\mathbf{u}_i$ are parallel to $\mathbf{e}$, $\mathbf{h}$ and $\mathbf{h} \times \mathbf{e}$ (respectively), and $f_{ij} = 3Gm_3R_iR_j/R^5$ if $i \neq j$.

Derivation of the simplest form of the method is completed by carrying out a similar treatment of the Laplace vector $\mathbf{e}$. In fact, however, two further developments are necessary before a satisfactory method is obtained: inclusion of the octupole perturbation, and allowance for periodic perturbations when setting up the initial conditions for $\mathbf{e}$ and $\mathbf{h}$.

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References

Marchal C., 1990, The Three-Body Problem. Elsevier, Amsterdam.