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NINTH MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

The annual meeting of the Association for Symbolic Logic was held at Columbia University, New York City, on Saturday, February 8, 1947. President Ernest Nagel presided at all sessions. At the morning meeting, Carl G. Hempel presented an hour's address *On inductive logic,* and Frederic B. Fitch read a thirty minute paper entitled *A system of relations and classes.* At the afternoon meeting, W. V. Quine spoke for an hour *On the problem of universals,* and A. R. Turquette presented a thirty minute paper on behalf of J. B. Rosser and himself, *Axiom schemes for m-valued functional calculi of first order.* Abstracts of all papers are appended. Discussion from the floor followed each presentation.

An informal meeting of the Council of the Association was held at luncheon, and a business meeting of the Association preceded the presentation of papers in the afternoon. The election of new officers was reported (as on page 32). It was announced that the members of the Association, by mail vote, (1) had adopted an amendment to the By-Laws increasing annual dues to four dollars (primarily to meet increased costs of printing), and (2) had postponed action on the proposed amendment to the Constitution which would change the name of the Association and the Journal. The latter amendment was the principal subject of discussion at the Council meeting and at the business meeting of the Association. It seemed to be the consensus of both groups that the first step to be taken is to publicize a clear statement of the editorial policy of the Journal, with particular reference to the nature of desirable expository and other non-technical articles.

Charles A. Baylis

Carl G. Hempel. *On inductive logic.*

While in a deductive inference, the premises completely entail the conclusion, or confer certainty upon it, the premises of an inductive inference may be said to stand in a relation of partial entailment to the conclusion, or to confer upon the latter a certain degree of confirmation, or of logical probability. The objective of inductive logic is the construction of a general syntactical and semantical theory of inductive inference.

Most of the research which has been done in this field has proceeded axiomatically and provides no explicit interpretation of the central concept of inductive logic, that of degree of confirmation. Recently, however, explicit definitions of this concept have been proposed, for certain relatively simple types of language, by Carnap (XI 19), and independently, for a narrower class of languages, by Helmer, Hempel, and Oppenheim (XI 17, 18). The paper gives a comparative survey of these two systems of inductive logic and then discusses some of the semantical restrictions which have to be imposed on their application. One of these demands that the atomic sentences, i.e., the full sentences of the primitive predicates, of the language under consideration be logically independent in the sense that no state description may be a logically contradictory sentence; as a consequence, the primitive predicates themselves are logically independent in the sense that none of the logically strongest properties Q_v definable in terms of them is logically void. The need for further semantical restrictions has been pointed out especially by Nelson Goodman (XI 81 (2)), who has shown that the rules of inductive inference entailed by the definitions just referred to lead to counter-intuitive results when applied to certain characteristics which he calls non-projectible. This appears to indicate the need to restrict the application of the rules of inductive logic to languages whose primitive predicates are not only logically independent, but also projectible, and it raises the problem of finding an accurate general definition of projectibility. The clarification of semantical issues of this type seems to constitute one of the most urgent problems in inductive logic.

Frederic B. Fitch. *A system of relations and classes.*

A consistent system K is constructed which deals with relations and classes and is adequate for a large part of mathematics. The formation rules of K are as follows: Every "variable" is a "substantive." The symbols '$=$' and '$\subset$' are substantives. If 'f', 'g', and 'h' are substantives and if 'x' and 'z' are distinct variables, then '$[\hat{x}\hat{z}f]$', '(fgh)', '$\sim f$', and '*f' are substantives. A substantive in which no variables occur free is a "noun." The symbols 'u', 'v', $\cdots$, stand for arbitrary variables, and 'a', 'b', $\cdots$, for arbitrary nouns.

The system K is a class of nouns and is defined by postulates P1–P13 below. It is shown that P1–P12 are consistent. Fairly plausible reasons are also given for supposing that the addition of P13 does not give rise to inconsistency. Quotation marks will be omitted for brevity, and also the usual stipulations about bondage and freedom of variables.

P1. $(a=a)$ is in K.

P2. $(a=b)$ or $\sim(a=b)$ is in K.

P3. $(a\subset b)$ is in K if and only if all c and d are such that $\sim(cad)$ or (cbd) is in K.

P4. $\sim(a\subset b)$ is in K if and only if some c and d are such that (cad) and $\sim(cbd)$ are in K.

P5. $\sim\sim a$ is in K if and only if a is in K.

P6. $(a[\hat{x}\hat{z}(\ldots x\ldots z\ldots)]b)$ is in K if and only if $(\ldots a\ldots b\ldots)$ is in K.

P7. $\sim(a[\hat{x}\hat{z}(\ldots x\ldots z\ldots)]b)$ is in K if and only if $\sim(\ldots a\ldots b\ldots)$ is in K.

P8. (a^*bc) is in K if and only if there is a finite sequence $d_1, d_2, \cdots, d_n$, such that (d_1bd_2), $(d_2bd_3), \cdots, (d_{n-1}bd_n)$ are in K, where d_1 is a and d_n is c and $n>1$.

P9. $\sim(a^*bc)$ is in K if and only if every finite sequence $d_1, d_2, \cdots, d_n$, where d_1 is a and d_n is c and $n>1$, is a sequence such that $\sim(d_1bd_2)$ or $\sim(d_2bd_3)$ or $\cdots$ or $\sim(d_{n-1}bd_n)$ is in K.

P10. a and $\sim a$ are not both in K.

P11. If $(a\subset b)$ and $(b\subset a)$ are in K, so is $(a=b)$.

P12. If a and b are both in K, or if $\sim a$ and $\sim b$ are both in K, then $(a=b)$ is in K.

P13. If (abc) and $(a=d)$ are in K, so is (dbc).

W. V. Quine. *On the problem of universals.*

A theory expressed in terms of quantification presupposes universals, or abstract entities, only if it demands them as values of bound variables. Pure quantification theory presupposes no universals; its unbindable predicate variables can be viewed as *schematic letters* for depicting patterns of true statements, rather than as variables demanding classes or attributes as values.

A theory of concrete entities can often be reconstrued as treating of universals, by identifying entities which are indiscernible within the theory. Such abstraction is reconcilable with *nominalism*, being explicable as a mere figure of speech by redefining '='. But it is inadequate for abstracting other than mutually exclusive classes.

Binding schematic letters is a bolder way of introducing universals. Binding predicate variables turns quantification theory into a theory of attributes or (if we identify indiscernibles) classes. This seems a natural way of reifying all expressible conditions as classes; but it proves to do more. Merely allowing predicate variables all privileges of the 'x', 'y', etc. of quantification theory, and adding no new rules, we can prove that classes outnumber the expressible conditions which they were supposed to reify. Also we can prove Russell's paradox. Restoring consistency then by some *ad hoc* restriction, we have the foundation of classical mathematics. It involves a *platonism* of universals.

But we can limit ourselves to reifying just the expressible conditions, by binding predicate variables but giving them indices and restricting the rule of substitution in a fashion resembling ramified type theory without the axiom of reducibility. This method, inadequate to parts of real-number theory, reflects a *conceptualist* philosophy of universals.

The nominalist can describe the *pursuit of* platonistic or conceptualistic logic, by formulating metalogical rules for those logics but denying meaning to the logics themselves. He can define *formula* and *proof* for these stronger logics in his own terms; difficulties arise, however, in defining *theorem.*

J. B. Rosser and A. R. Turquette. *Axiom schemes for m-valued functional calculi of first order.*

In defining axioms for m-valued propositional calculi ($m \geqq 2$), which are sufficiently general to allow an arbitrary choice of s designated truth-values ($1 \leqq s < m$) and not to require that all truth-functions be definable in terms of primitive truth-functions (i.e., "functional completeness" would not be presupposed), use was made of functions $J_k(\mathrm{P})$

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$(1 \leqq k \leqq m)$ and "partial normal forms" $N_i(P)$ $(1 \leqq i \leqq m)$. It is required that $J_k(P)$ take a designated truth-value when and only when P takes the truth-value k, while if E expresses a formula of the propositional calculus, $N_i(E)$ must take a designated truth-value exactly when $J_i(E)$ takes a designated truth-value, i.e., $N_i(E)$ and $J_i(E)$ must be "weakly equivalent." It is the use of weak equivalence for $N_i(E)$ and $J_i(E)$ rather than strong equivalence which makes it possible to avoid the assumption of functional completeness.

Furthermore, in framing such a general definition of axioms for m-valued propositional calculi, use was made of the following principles: (1) Axioms should be sufficient for the proper handling of partial normal forms. (2) Axioms should make it possible to express the weak equivalence of $N_i(E)$ and $J_i(E)$. (3) Axioms should be sufficient to express a suitable relation between any $J_i(E)$ and E.

The major purpose of the present paper is to define axioms for m-valued functional calculi of first order, which will allow an arbitrary choice of s designated truth-values and will not presuppose functional completeness. Use will again be made of functions $J_k(P)$ and partial normal forms $N_i(P)$. Likewise, use will be made of principles which are essentially the same as (1), (2), and (3). Novelties arise from the added complexity of expressions E* which may now contain functional variables, individual variables, and quantifiers of individual variables.

The Association for Symbolic Logic announces the following elections, each for a term of three years from January 1, 1947:

As President of the Association, Professor Ernest Nagel of Columbia University.

As Vice President of the Association, Professor S. C. Kleene of the University of Wisconsin.

As members of the Executive Committee, Professor Nelson Goodman of the University of Pennsylvania and Professor Max Zorn of Indiana University.

The Executive Committee has appointed Mr. Alfred Glathe as Assistant Secretary of the Association for a term of one year from January 1, 1947.

Institutional contributing subscribers to the Journal for the year 1946 are the following:

HARVARD UNIVERSITY, Cambridge, Massachusetts

PRINCETON UNIVERSITY, Princeton, New Jersey

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THE INSTITUTE FOR ADVANCED STUDY, Princeton, New Jersey

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