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# Inferring the dense nuclear matter equation of state with neutron star tides

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**Abstract.** During the late stages of a neutron star binary inspiral finite-size effects come into play, with the tidal deformability of the supranuclear density matter leaving an imprint on the gravitational-wave signal. As demonstrated in the case of GW170817—the first direct detection of gravitational waves from a neutron star binary—this can lead to strong constraints on the neutron star equation of state. As detectors become more sensitive, effects which may have a smaller influence on the neutron star tidal deformability need to be taken into consideration. Dynamical effects, such as oscillation mode resonances triggered by the orbital motion, have been shown to contribute to the tidal deformability, especially close to the neutron star coalesence. We calculate the contribution of the various stellar oscillation modes to the tidal deformability and demonstrate the (anticipated) dominance of the fundamental mode, showing what the impact of the matter composition is on the tidal deformability.

**Keywords.** dense matter, equation of state, gravitational waves, relativity, methods: analytical, (stars:) binaries: general, stars: neutron, stars: oscillations (including pulsations)

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## 1. Introduction

The detections of gravitational waves from binary neutron stars (Abbott *et al.* 2017, 2020) have led to renewed focus on the elusive neutron star equation of state. Much of the recent focus has been on the neutron star tidal deformability, essentially the extent to which the tidal interaction with a binary companion deforms the neutron star fluid. This is a useful measure as it can be extracted from (or, at least, constrained by) the gravitational-wave signal (Flanagan & Hinderer 2008). The deformability (often expressed in terms of the Love number, $k_l$) represents the static contribution to the neutron star's tidal response (equilibrium tide). In addition, there is a dynamical tide. This is traditionally represented by the excitation of the oscillation modes of the star. The associated effect on the inspiral signal is weak, but its inclusion has been demonstrated to improve waveform models (Schmidt & Hinderer 2019; Pratten *et al.* 2021).

An interesting question to pose is to what extent the composition of the neutron star matter enters the problem. As the star is deformed by the tidal interaction, matter is driven out of equilibrium and it is easy to argue that the relevant nuclear reactions are too slow to re-establish equilibrium on the time scale of the inspiral. The upshot of this argument is that the equation of state is no longer barotropic, as has been assumed in virtually every previous analysis of the tidal problem. Hence, we want to establish to what extent a "frozen" matter composition leads to a noticeable effect on the Love number and whether this affects the extraction of neutron star parameters from an observed signal.

In the following, we outline the work presented in Andersson & Pnigouras (2020). Work in the same direction can be found in Andersson & Pnigouras (2021) and Passamonti *et al.* (2021).

## 2. The effective Love number

The tide raised by a binary companion (here treated as a point particle) induces a linear response in the primary. In order to quantify this response, we solve the linearised fluid equations in Newtonian gravity. Assuming that the star is non-rotating, the tidal perturbation is described by the Euler equation

$$-\omega^2\xi_i + \frac{1}{\rho}\nabla_i\delta p - \frac{1}{\rho^2}\delta\rho\nabla_i p + \nabla_i\delta\Phi = -\nabla_i\chi, \tag{2.1}$$

where $\xi_i$ denotes the tidal displacement vector, $\rho$ is the density, $p$ is the pressure, $\Phi$ is the gravitational potential, $\delta$ denotes Eulerian perturbations, and $\omega$ is the tidal frequency. In a coordinate system centred on the primary, which we will take to have mass $M_\star$, the tidal potential $\chi$ is given by

$$\chi = -GM' \sum_{l\geq 2}\sum_{m=-l}^{l} \frac{W_{lm}r^l}{D^{l+1}(t)} Y_{lm} e^{-im\psi(t)}, \tag{2.2}$$

where $M'$ is the mass of the secondary. The orbit of the companion is taken to be in the plane $[D(t), \pi/2, \psi(t)]$ where $D$ is the binary separation and $\psi$ is the orbital phase. For $l = 2$ (which leads to the main contribution to the gravitational-wave signal), we have $W_{20} = -\sqrt{\pi/5}$, $W_{2\pm 2} = \sqrt{3\pi/10}$, and $W_{2\pm 1} = 0$.

We now aim to express the driven response of the fluid in terms of a set of normal modes, corresponding to solutions $\xi_n$ (where $n$ is a label that identifies the modes, say in terms of the number of nodes in the radial eigenfunction and the corresponding spherical harmonics). Letting the mode frequency be $\omega_n$, we have

$$\xi^i = \sum_n a_n \xi_n^i. \tag{2.3}$$

The Love number is generally defined as $k_l = \delta\Phi(R)/2\chi(R)$. Then, after expressing the perturbed gravitational potential in terms of the displacement vector, we get

$$k_l = -\frac{1}{2} + \frac{2\pi}{2l+1}\sum_n \frac{\tilde{Q}_n^2}{\tilde{\omega}_n^2 - \tilde{\omega}^2}\left[1 - \tilde{\omega}^2\left(\frac{V_n}{W_n}\right)_R\right]\left[1 - \tilde{\omega}_n^2\left(\frac{V_n}{W_n}\right)_R\right]^{-1}, \tag{2.4}$$

where frequencies have been normalised as $\tilde{\omega} = \omega/(GM_\star/R^3)$, each mode is decomposed into its radial and horizontal components as

$$\xi_n^i = \left(W_n \frac{\nabla^i r}{r}\right) Y_{lm} + V_n \nabla^i Y_{lm}, \tag{2.5}$$

and we have introduced the "overlap integral"

$$\tilde{Q}_n = -\frac{1}{M_\star R^l}\int \delta\rho_n^* r^{l+2}\mathrm{d}r. \tag{2.6}$$

In the low-frequency limit (i.e., in the equilibrium tide approximation), we get

$$k_l \approx -\frac{1}{2} + \frac{2\pi}{2l+1}\sum_n \frac{\tilde{Q}_n^2}{\tilde{\omega}_n^2}\left[1 - \tilde{\omega}_n^2\left(\frac{V_n}{W_n}\right)_R\right]^{-1}. \tag{2.7}$$

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**Table 1.** The accumulated contribution to the Love number from the different modes, in order of relevance of the contribution, for the three different models we consider.

| $\Gamma_1 = 2$ | | $\Gamma_1 = 2.05$ | | $\Gamma_1 = 7/3$ | |
|---|---|---|---|---|---|
| mode | $k_l$ | mode | $k_l$ | mode | $k_l$ |
| $f$ | 0.27528 | $f$ | 0.27055 | $f$ | 0.24685 |
| $+p_1$ | 0.25887 | $+p_1$ | 0.25526 | $+g_1$ | 0.26115 |
| $+p_2$ | 0.26021 | $+p_2$ | 0.25653 | $+p_1$ | 0.25052 |
| $+p_3$ | 0.26015 | $+g_1$ | 0.25878 | $+g_2$ | 0.25556 |
| | | $+g_2$ | 0.25960 | $+p_2$ | 0.25653 |
| | | $+g_3$ | 0.25993 | $+g_3$ | 0.25856 |
| | | $+g_4$ | 0.26008 | $+g_4$ | 0.25944 |
| | | | | $+g_5$ | 0.25983 |
| | $9 \times 10^{-4}$ | | $7 \times 10^{-4}$ | | $3 \times 10^{-4}$ |

As a quantitative test of Eq. (2.7), we compare results for three models corresponding to a background configuration with a polytropic equation of state of the form $p \propto \rho^{\Gamma}$, with $\Gamma = 2$. Since, as noted before, the nuclear reactions required to establish chemical equilibrium are too slow to act on the inspiral timescale, it would be reasonable to assume that the composition of a perturbed fluid element is held frozen, as the system sweeps through the sensitivity band of a ground based detector. This changes the response of the stellar fluid to the tidal driving which, in turn, allows us to estimate the impact the matter composition has on the problem. Composition gradients can be accounted for by adjusting the adiabatic index $\Gamma_1 = (\partial \ln p / \partial \ln \rho)_{\text{ad}}$, which is evaluated at fixed composition. Our reference model is barotropic ($\Gamma_1 = \Gamma$) and we compare it to two stratified models, with $\Gamma_1 = 2.05$ and $7/3$, respectively.

The numerical results, listed in Table 1, demonstrate the relative importance of $g$-modes (buoyancy modes) for strongly stratified models. In each case, the mode sum converges to the expected value for the Love number which, for a barotropic model with $\Gamma = 2$, should be $k_l \approx 0.259909$ and the results from Table 1 do, indeed, converge towards this number. This tells us that the sum over the star's different oscillation modes provides an alternative representation for the Love number. The $f$-mode provides the dominant contribution in all cases, but in order to have a precise representation we need to account for both $p$-modes (pressure modes) and $g$-modes.

Having demonstrated that the sum over the star's oscillation modes provides a precise description of the tidal response in the static limit, let us turn to the dynamical response associated with finite frequencies, given by Eq. (2.4). It provides a closed expression for the frequency dependent tidal response (encoded in $k_l$). This allows us to quantify the level at which each individual mode contributes to the overall result. Results for the three different values of $\Gamma_1$ we have considered are presented in Fig. 1. The different panels show the relative contributions to the tidal deformability (compared to that of the $f$-mode alone). The resonances associated with each mode, which occur when $\tilde{\omega} = \tilde{\omega}_n$, are easily distinguishable in each case and the resonance associated with the $f$-mode leads to a common feature in all panels. The results tell us that modes other than the $f$-mode contribute to the overall result at the few percent level. These results are important as they provide the first demonstration of the level at which frozen matter composition impacts on the tidal response across the range of frequencies relevant for a binary inspiral.

## 3. Implications

We have discussed the tidal response of a neutron star during the late stages of neutron star binary inspiral. In particular, we have focussed on the role of the matter composition. This issue has previously been ignored as studies have almost exclusively focussed of barotropic fluid models. However, it is natural to argue (given the time scale involved)

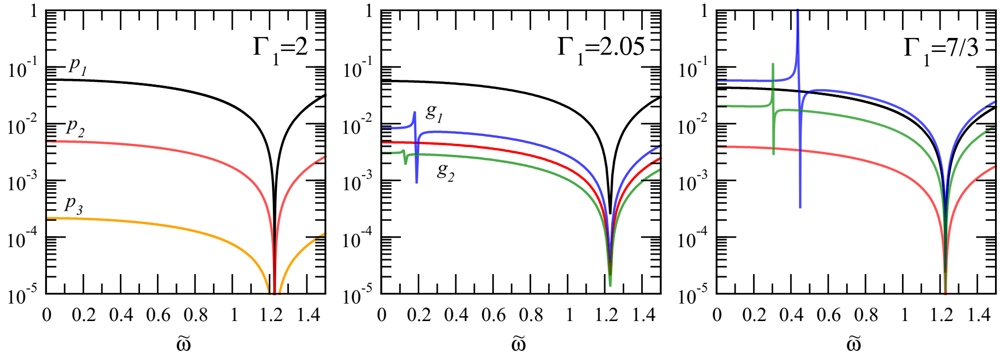


**Figure 1.** Relative contributions to the tidal deformability (compared to that of the $f$-mode alone), for the three different models we consider. Individual modes are colour coded (as indicated in the panels), with the same colour representing the same mode in all panels.

that the matter composition should remain "frozen" during the late stages of binary inspiral, leading to a stratified perturbation problem (where the adiabatic index of the perturbation is different from that of the equilibrium background). This connects with previous work on tidal resonances, which has quantified the role of the $g$-modes (which rely on stratification for their existence).

Our numerical results indicate that the difference is at (or below) the level of a few percent. However, it is nevertheless important to quantify this contribution. We need to do this in order to understand systematic "errors" associated with the assumed physics, which ultimately determines the accuracy with which we can hope to extract stellar parameters like the radius from observations. Today's gravitational-wave detectors are not at a level where a change of a percent in the tidal response makes much difference, but one might want to keep an eye on these issues for future reference.

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